Homework-3 Solutions

8.10.

$R = 10^5$ bits/sec $\Rightarrow T_b = 10^{-5}$ sec.

$$P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$E_b = A^2 T_b$

$P(e) = P_2 = 10^{-6}$

$$\sqrt{\frac{2E_b}{N_0}} = 4.75 \Rightarrow E_b = \frac{4.75^2 N_0}{2}$$

$$= 0.112813$$

Then, $A^2 T_b = 0.112813 \Rightarrow A = \sqrt{0.112813 \times 10^5} = \underline{106.21}$

8.12

Assume $E(n^2(t)) = \sigma_n^2 = \frac{N_0}{2}$.

$$E[n_1 n_2] = E\left[\int_0^T s_1(t) n(t)\, dt \int_0^T s_2(\tau) n(\tau)\, d\tau\right]$$

$$= \left[\int_0^T \int_0^T s_1(t) s_2(\tau) E(n(t) n(\tau))\, dt\, d\tau\right]$$

$$= \frac{N_0}{2} \int_0^T s_1(t) s_2(t)\, dt = 0$$

as $s_1(t)$ and $s_2(t)$ are orthogonal.

8.13

1) The optimum thresholding is given by

$$\alpha^* = \frac{N_0}{4\sqrt{E_b}} \ln\left(\frac{1-p}{p}\right) = \frac{N_0}{4\sqrt{E_b}} \ln 2$$

2) The average probability of error is

$$P(e) = P(a_m=-1)\int_{\alpha^*}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-(r+\sqrt{E_b})/N_0}\, dr + P(a_m=1)\int_{-\infty}^{\alpha^*} \frac{1}{\sqrt{\pi N_0}} e^{-(r-\sqrt{E_b})^2/N_0}\, dr$$

$$= \frac{2}{3} Q\left(\frac{\alpha^* + \sqrt{E_b}}{\sqrt{\frac{N_0}{2}}}\right) + \frac{1}{3} Q\left(\frac{\sqrt{E_b} - \alpha^*}{\sqrt{\frac{N_0}{2}}}\right)$$

$$= \frac{2}{3} Q\left[\frac{\sqrt{\frac{2N_0}{E_b}}\ln 2}{4} + \sqrt{\frac{2E_b}{N_0}}\right] + \frac{1}{3} Q\left[\sqrt{\frac{2E_b}{N_0}} - \frac{\sqrt{\frac{2N_0}{E_b}}\ln 2}{4}\right]$$

(3) Substitute $E_b = 1$ and $N_0 = 0.1$

we get

$$P_e = \frac{2}{3} Q\left(\frac{\sqrt{0.2} \times \ln 2}{4} + \sqrt{20}\right) + \frac{1}{3} Q\left(\sqrt{20} + \frac{\sqrt{0.02}\ln 2}{4}\right)$$

$$= \frac{2}{3} Q(4.5496) - \frac{1}{3} Q(4.3946)$$

$$= \boxed{3.64 \times 10^{-6}.}$$