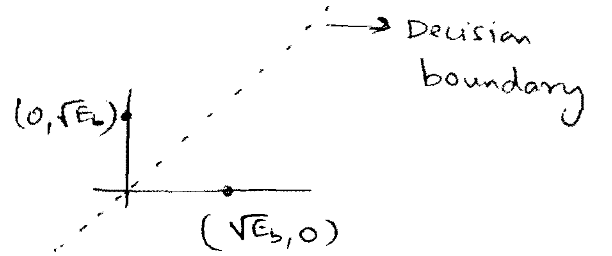
# HW4 Solutions

1. $y_1 = n_1$

$y_2 = \sqrt{E_b} + n_2$.




P(e | signal 2 transmitted)

$$= P(n_1 > \sqrt{E_b} + n_2)$$

$$= P(n_1 - n_2 > \sqrt{E_b})$$

Note : $n_1 = n_2 = N(0, \frac{N_0}{2})$

$n_1 - n_2 = N(0, N_0)$

$$\therefore \Rightarrow P(n_1 - n_2 > \sqrt{E_b})$$

$$= P\left(\frac{n_1 - n_2 - 0}{\sqrt{N_0}} > \sqrt{\frac{E_b}{N_0}}\right)$$

$$= \underline{Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

2.

8.22 Three symbols transmitted = A, 0, −A with equal probability

$\Rightarrow$ Decision thresholds = $A/2$, $-A/2$ and decision is such that

$A : r > A/2$

$0 : -A/2 < r < A/2$

$-A : r < -A/2$

If the variance of AWGN noise is $\sigma_n^2$, then the avg. probability of error

$$P(e) = \frac{1}{3}\int_{-\infty}^{A} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(r-A)^2/2\sigma_n^2}\,dr$$

$$+ \frac{1}{3}\left(1 - \int_{-A/2}^{A/2} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-r^2/2\sigma_n^2}\,dr\right)$$

$$+ \frac{1}{3}\int_{-A/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(r+A)^2/2\sigma_n^2}\,dr.$$

$$= \frac{1}{3} Q\left(\frac{A}{2\sigma_n}\right) + \frac{1}{3}\cdot 2 Q\left(\frac{A}{2\sigma_n}\right) + \frac{1}{3} Q\left(\frac{A}{2\sigma_n}\right).$$

$$= \frac{4}{3} Q\left(\frac{A}{2\sigma_n}\right)$$

8.23 The pdf of noise n is

$$f(n) = \frac{\lambda}{2} e^{-\lambda|n|}$$

The optimal receiver uses the criterion

$$\frac{f(r/A)}{f(r|-A)} = e^{-\lambda[|r-a| - |r+A|]} \underset{-A}{\overset{A}{\gtrless}} 1 \Rightarrow r \underset{-A}{\overset{A}{\gtrless}} 0$$

The average probability of error is

$$\tfrac{1}{2} P(e|A) + \tfrac{1}{2} P(e|-A)$$

$$= \tfrac{1}{2} \int_{-\infty}^{0} f(r|A)\,dr + \tfrac{1}{2} \int_{0}^{\infty} f(r|-A)\,dr$$

$$= \tfrac{1}{2} \int_{-\infty}^{0} \frac{\lambda}{2} e^{-\lambda|r-A|}\,dr + \tfrac{1}{2} \int_{0}^{\infty} \frac{\lambda}{2} e^{-\lambda|r+A|}\,dr.$$

$$= \lambda/4 \int_{-\infty}^{-A} e^{-\lambda|x|}\,dx + \lambda/4 \int_{A}^{\infty} e^{-\lambda|x|}\,dx.$$

$$= \underline{\tfrac{1}{2} e^{-\lambda A}}$$

2.(ii) The variance of the noise is

$$\sigma_n^2 = \lambda/2 \int_{-\infty}^{\infty} e^{-\lambda|x|} x^2\,dx.$$

$$= \lambda \int_{0}^{\infty} e^{-\lambda x} x^2\,dx = \frac{\lambda \cdot 2!}{\lambda^3} = \underline{\underline{2/\lambda^2}}$$

Hence, the SNR is

$$SNR = \frac{A^2}{2/\lambda^2} = \frac{A^2\lambda^2}{2}$$

and the probability of error is given by

$$P(e) = \frac{1}{2} e^{-\sqrt{\lambda^2 A^2}} = \frac{1}{2} e^{-\sqrt{2SNR}}$$

For $P(e) = 10^{-5}$, we obtain.

$$\ln(2 \times 10^{-5}) = -\sqrt{2SNR} \Rightarrow SNR = \underline{17.6741 \text{ dB}}$$

If noise was Gaussian, then

$$P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{SNR}\right)$$

$\Rightarrow$ for $P(e) = 10^{-5}$,

$$SNR = \underline{12.594 \text{ dB}}$$

The required SNR is 5dB lesser when Additive noise is Gaussian.

8.24

The points in constellation = $\pm d, \pm 3d, \cdots \pm (M-1)d$

$$E_{av} = \frac{1}{M} \sum_i E_i = \frac{2d^2}{M}\left(1+9+\cdots+(M-1)^2\right)$$

we know

$$1^2+2^2+3^2+\cdots+M^2 = \frac{M(M+1)(2M+1)}{6} \longrightarrow ①$$

Now

$$2^2+4^2+\cdots+M^2 = 4\left(1^2+2^2+\cdots+\left(\frac{M}{2}\right)^2\right)$$

$$= \frac{M(M+1)(M+2)}{6}. \longrightarrow ②$$

Subtracting ① − ②

$$\Rightarrow \quad 1^2+3^2+\cdots+(M-1)^2 = \frac{M(M+1)(2M+1)}{6} - \frac{M(M+1)(M+2)}{6}.$$

$$= \frac{M(M^2-1)}{6}.$$

$$\therefore \quad E_{av} = \frac{2d^2}{M} \times \frac{M(M^2-1)}{6} = \frac{d^2(M^2-1)}{3}$$