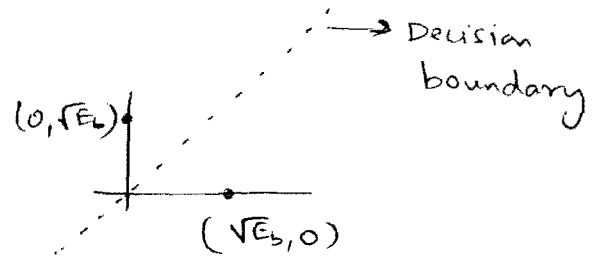


1.

$$y_1 = n_1$$

$$y_2 = \sqrt{E_b} + n_2$$



$P(e | \text{signal 2 transmitted})$

$$= P(n_1 > \sqrt{E_b} + n_2)$$

$$= P(n_1 - n_2 > \sqrt{E_b})$$

Note: $n_1 = n_2 = N(0, \frac{N_0}{2})$

$$\Rightarrow n_1 - n_2 = N(0, N_0)$$

$$\therefore \Rightarrow P(n_1 - n_2 > \sqrt{E_b})$$

$$= P\left(\frac{n_1 - n_2 - 0}{\sqrt{N_0}} > \sqrt{\frac{E_b}{N_0}}\right)$$

$$= \underline{Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

8.22

Three symbols transmitted = $A, 0, -A$ with equal probability

\Rightarrow Decision thresholds = $A/2, -A/2$ and decision is

such that

$$A : r > A/2$$

$$0 : -A/2 < r < A/2$$

$$-A : r < -A/2$$

If the variance of AWGN noise is σ_n^2 , then the avg. probability of error

$$\begin{aligned}
 P(e) &= \frac{1}{3} \int_{-\infty}^A \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(r-A)^2/2\sigma_n^2} dr \\
 &\quad + \frac{1}{3} \left(1 - \int_{-A/2}^{A/2} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-r^2/2\sigma_n^2} dr \right) \\
 &\quad + \frac{1}{3} \int_{-A/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(r+A)^2/2\sigma_n^2} dr \\
 &= \frac{1}{3} Q\left(\frac{A}{2\sigma_n}\right) + \frac{1}{3} \cdot 2 Q\left(\frac{A}{2\sigma_n}\right) + \frac{1}{3} Q\left(\frac{A}{2\sigma_n}\right) \\
 &= \underline{\underline{\frac{4}{3} Q\left(\frac{A}{2\sigma_n}\right)}}
 \end{aligned}$$

8.23 The pdf of noise n is

$$f(n) = \frac{\gamma}{2} e^{-\gamma|n|}$$

The optimal receiver uses the criterion

$$\frac{f(r|A)}{f(r|-A)} = e^{-\gamma[|r-A| - |r+A|]} \begin{matrix} A \\ \geq 1 \\ -A \end{matrix} \Rightarrow r \begin{matrix} A \\ > \\ < \\ -A \end{matrix} 0$$

The average probability of error is

$$\frac{1}{2} P(e|A) + \frac{1}{2} P(e|-A)$$

$$= \frac{1}{2} \int_{-\infty}^0 f(r|A) dr + \frac{1}{2} \int_0^{\infty} f(r|-A) dr$$

$$= \frac{1}{2} \int_{-\infty}^0 \frac{\lambda}{2} e^{-\lambda|r-A|} dr + \frac{1}{2} \int_0^{\infty} \frac{\lambda}{2} e^{-\lambda|r+A|} dr.$$

$$= \frac{\lambda}{4} \int_{-\infty}^{-A} e^{-\lambda|x|} dx + \frac{\lambda}{4} \int_A^{\infty} e^{-\lambda|x|} dx.$$

$$= \frac{1}{2} e^{-\lambda A}$$

2.(i) The variance of the noise is

$$\sigma_n^2 = \frac{\lambda}{2} \int_{-\infty}^{\infty} e^{-\lambda|x|} x^2 dx.$$

$$= \lambda \int_0^{\infty} e^{-\lambda x} x^2 dx = \frac{\lambda \cdot 2!}{\lambda^3} = \underline{\underline{\frac{2}{\lambda^2}}}$$

Hence, the SNR is

p.4

$$\text{SNR} = \frac{A^2}{2/T^2} = \frac{A^2 T^2}{2}$$

and the probability of error is given by.

$$P(e) = \frac{1}{2} e^{-\sqrt{\gamma^2 A^2}} = \frac{1}{2} e^{-\sqrt{2 \text{SNR}}}$$

For $P(e) = 10^{-5}$, we obtain.

$$\ln(2 \times 10^{-5}) = -\sqrt{2 \text{SNR}} \Rightarrow \text{SNR} = \underline{\underline{17.6741 \text{ dB}}}$$

If noise was Gaussian, then

$$P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{\text{SNR}})$$

$$\Rightarrow \text{for } P(e) = 10^{-5},$$

$$\text{SNR} = \underline{\underline{12.594 \text{ dB}}}$$

The required SNR is 5dB lesser when

Additive noise is Gaussian.

The points in constellation = $\pm d, \pm 3d, \dots, \pm(M-1)d$

$$E_{av} = \frac{1}{M} \sum_i E_i = \frac{2d^2}{M} (1^2 + 9 + \dots + (M-1)^2)$$

we know

$$1^2 + 2^2 + 3^2 + \dots + M^2 = \frac{M(M+1)(2M+1)}{6} \rightarrow \textcircled{1}$$

Now

$$\begin{aligned} 2^2 + 4^2 + \dots + M^2 &= 4 \left(1^2 + 2^2 + \dots + \left(\frac{M}{2}\right)^2 \right) \\ &= \frac{M(M+1)(M+2)}{6} \rightarrow \textcircled{2} \end{aligned}$$

Subtracting $\textcircled{1} - \textcircled{2}$

$$\begin{aligned} \Rightarrow 1^2 + 3^2 + \dots + (M-1)^2 &= \frac{M(M+1)(2M+1)}{6} - \frac{M(M+1)(M+2)}{6} \\ &= \frac{M(M^2-1)}{6} \end{aligned}$$

$$\therefore E_{av} = \frac{2d^2}{M} \times \frac{M(M^2-1)}{6} = \frac{d^2(M^2-1)}{3}$$