H.W-5 Solutions.

Problem 2.66.

1) The spectrum of the output signal $y(t)$ is the product of $X(f)$ and $H(f)$, thus,

$$Y(f) = X(f)H(f) = X(f)A(f_0)e^{j(\theta(f_0)+(f-f_0)\theta'(f)|_{f=f_0})}$$

$y(t)$ is narrowband centered at $f = \pm f_0$. Hence to get the Lowpass equivalent, we have to shift $y(t)$ to the right by f_0.

$$Y_l(f) = u(f+f_0)X(f+f_0)A(f_0)e^{j(\theta(f_0)+f\theta'(f)|_{f=f_0})}$$

$$= X_l(f)A(f_0)e^{j(\theta(f_0)+f\theta'(f)|_{f=f_0})}$$

2) $$y_l(f) = F^{-1}\left[X_l(f)A(f_0)e^{j\theta(f_0)}e^{jf\theta'(f)|_{f=f_0}}\right]$$

$$= A(f_0)x_l\left(t+\frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)$$

With $y(t) = \text{Real}\left(y_l(t)e^{j2\pi f_0 t}\right) - V_x(t)e^{j\theta_x(t)}$

$$y(t) = \text{Real}\left[A(f_0)x_l\left(t+\frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)e^{j\theta(f_0)}e^{j2\pi f_0 t}\right]$$

$$= \text{Real}\left[A(f_0)V_x\left(t+\frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)e^{j2\pi f_0 t}e^{j\theta_x\left(t+\frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)}\right]$$

$$= A(f_0)V_x(t-t_g)\cos\left(2\pi f_0 t+\theta(f_0)+\theta_x\left(t+\frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)\right)$$

$$= A(f_0)\, V_x(t - t_g)\cos\left(2\pi f_0 (t - t_p) + \theta_x\left(t + \frac{1}{2\pi}\theta'(f)\Big|_{f=f_0}\right)\right)$$

$$t_g = \frac{1}{2\pi}\theta'(f)\Big|_{f=f_0}$$

$$t_p = -\frac{1}{2\pi}\frac{\theta(f_0)}{f_0} = -\frac{1}{2\pi}\frac{\theta(f)}{f}\Big|_{f=f_0}$$

3) t_g can be looked as the time lag of the envelope of the signal, whereas t_p is the corresponding phase delay.

9.2.

We have

$$y = \begin{cases} a + n - \frac{1}{2} & \text{with prob. } \frac{1}{4} \\ a + n + \frac{1}{2} & \text{with prob. } \frac{1}{4}. \\ a + n & \text{with prob. } \frac{1}{2}. \end{cases}$$

By symmetry $P_e = P(e|a=1) = P(e|a=-1)$.

$$P_e = p(e|a=-1) = \frac{1}{2}P(n-1>0) + \frac{1}{4}P(n-3/2>0)$$
$$+ \frac{1}{4}P(n-\tfrac{1}{2}>0)$$
$$= \frac{1}{2}Q\left(\frac{1}{\sigma_n}\right) + \frac{1}{4}Q\left(\frac{3}{2\sigma_n}\right) + \frac{1}{4}Q\left(\frac{1}{2\sigma_n}\right)$$

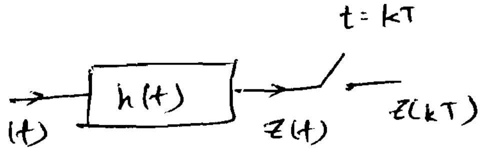
9.4.

1) Taking the inv. Fourier transform of $H(f)$ we obtain.

$$h(t) = \mathcal{F}^{-1}(H(f)) = \delta(t) + \alpha/2\,\delta(t-t_0) + \alpha/2\,\delta(t+t_0)$$

$$y(t) = s(t) * h(t) = s(t) + \alpha/2\, s(t-t_0) + \alpha/2\, s(t+t_0)$$

2) ~~At the sig.~~



$$h(t) = s(T-t)$$

$$H(f) = S^*(f)\, e^{-j2\pi f T}$$

$$Z(f) = Y(f)\,H(f)$$

$$= S(f)S^*(f)e^{-j2\pi f T} + \alpha/2\,\left[|S(f)|^2\left(e^{-j2\pi f(T+t_0)} + e^{-j2\pi f(T-t_0)}\right]\right.$$

$$z(t) = \int_{-\infty}^{\infty} Z(f)\, e^{j2\pi f t}\, df, \quad \text{Assume } R_{ss}(t) = \mathcal{F}^{-1}(|S(f)|^2)$$

$$= R_{ss}(t-T) + \alpha/2\,\left(R_{ss}(t-T-t_0) + R_{ss}(t-T+t_0)\right)$$

$$Z(kT) = R_{ss}((k-1)T) + \alpha/2\,[R_{ss}((k-1)T + t_0) + R_{ss}((k-1)T - t_0))$$

3) At $t_0 = T$

$$Z(kT) = \underline{R_{ss}((k-1)T)} + \alpha/2\,[R_{ss}((k-2)T) + R_{ss}(kT)]$$

$\therefore$ ISI is $\alpha/2\, R_{ss}((k-2)T) + \alpha/2\, R_{ss}(kT)$

As Matched filter brought a delay of $\underline{T}$ units.

9.5.

The pulse $x(t)$ having raise cosine spectrum is

$$x(t) = \frac{\text{sinc}(t/T)\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$

$\text{sinc}(t/T) = 1$ if $t = 0$

and 0 when $t = nT$.

and,

$$g(t) = \frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2} = \begin{cases} 1, & t = 0 \\ \text{bounded} & t \neq 0 \end{cases}$$

The function $g(t)$ needs to be checked for only those values of t such that $4\alpha^2 t^2/T^2 = 1$

or $\alpha t = T/2$.

$$\lim_{t \to T/2\alpha} \frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2} = \lim_{x \to 1} \frac{\cos\left(\frac{\pi x}{2}\right)}{1 - x}.$$

By L' Hospital rule

$$\lim_{x \to 1} \frac{\cos \pi x}{1 - x} = \lim_{x \to 1} \pi/2 \cdot \sin\left(\frac{\pi x}{2}\right) = \pi/2 < \infty$$

Hence,

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$\Rightarrow$ $x(t)$ is Nyquist

9.8.

1) Channel bandwidth = 1400 Hz

$\therefore R_{max} = 2800$

If an M-ary PAM modulation is used for transmission, then in order to acheive bit rate of 9600 bps, with maximum symbol rate of R_{max}, the minimum size of Constellation is $M = 2^k = 16$, In this case the symbol rate is

$$R = \frac{9600}{k} = \underline{2400 \text{ sym/sec}}$$

and symbol interval $= 1/R = \frac{1}{2400}$ sec

2) To find excess bandwidth,

$$1200(1+\alpha) = 1400$$

$$\boxed{\alpha = 0.166.}$$