

Problem 2.66.

1) The spectrum of the output signal $y(t)$ is the product of $X(f)$ and $H(f)$, thus,

$$Y(f) = X(f)H(f) = X(f)A(f_0)e^{j(\theta(f_0) + (f-f_0)\theta'(f)|_{f=f_0})}$$

$y(t)$ is narrowband centered at $f = \pm f_0$. Hence to get the Lowpass equivalent, we have to shift $y(t)$ to the right by f_0 .

$$\begin{aligned} Y_L(f) &= u(f+f_0)X(f+f_0)A(f_0)e^{j(\theta(f_0) + f\theta'(f)|_{f=f_0})} \\ &= X_L(f)A(f_0)e^{j(\theta(f_0) + f\theta'(f)|_{f=f_0})} \end{aligned}$$

$$2) \quad y_L(f) = F^{-1} \left[X_L(f)A(f_0)e^{j\theta(f_0)} e^{j f \theta'(f)|_{f=f_0}} \right]$$

$$= A(f_0)x_L\left(t + \frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)$$

$$\text{With } y(t) = \text{Real}(y_L(t)e^{j2\pi f_0 t}) = V_x(t)e^{j\theta_x(t)}$$

$$y(t) = \text{Real} \left[A(f_0)x_L\left(t + \frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)e^{j\theta(f_0)} e^{j2\pi f_0 t} \right]$$

$$\Rightarrow \text{Real} \left[A(f_0)V_x\left(t + \frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)e^{j2\pi f_0 t} e^{j\theta_x\left(t + \frac{1}{2\pi}\theta'(f)|_{f=f_0}\right)} \right]$$

$$= A(f_0)V_x(t-t_g)\cos(2\pi f_0 t + \theta(f_0) + \theta_x\left(t + \frac{1}{2\pi}\theta'(f)|_{f=f_0}\right))$$

$$= A(f_0) V_x(t - t_g) \cos\left(2\pi f_0(t - t_p) + \theta_x\left(t + \frac{1}{2\pi} \theta'(f) \Big|_{f=f_0}\right)\right)$$

$$t_g = \frac{1}{2\pi} \theta'(f) \Big|_{f=f_0}$$

$$t_p = -\frac{1}{2\pi} \frac{\theta(f_0)}{f_0} = -\frac{1}{2\pi} \frac{\theta(f)}{f} \Big|_{f=f_0}$$

3) t_g can be looked as the time lag of the envelope of the signal, whereas t_p is the corresponding phase delay.

9.2.

We have

$$y = \begin{cases} a + n - \frac{1}{2} & \text{with prob. } \frac{1}{4} \\ a + n + \frac{1}{2} & \text{with prob. } \frac{1}{4} \\ a + n & \text{with prob. } \frac{1}{2} \end{cases}$$

By symmetry $P_e = P(e|a=+1) = P(e|a=-1)$.

$$\begin{aligned} P_e &= P(e|a=-1) = \frac{1}{2} P(n-1 > 0) + \frac{1}{4} P(n-3/2 > 0) \\ &\quad + \frac{1}{4} P(n-1/2 > 0) \\ &= \frac{1}{2} Q\left(\frac{1}{\sigma_n}\right) + \frac{1}{4} Q\left(\frac{3}{2\sigma_n}\right) + \frac{1}{4} Q\left(\frac{1}{2\sigma_n}\right) \end{aligned}$$

9.4. ② Taking the inv. Fourier transform of $H(f)$ we obtain.

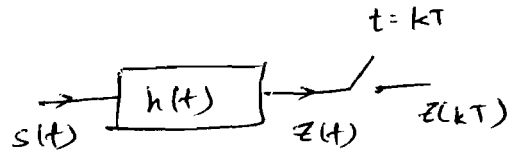
$$h(t) = F^{-1}(H(f)) = \delta(t) + \alpha_{\frac{1}{2}} \delta(t-t_0) + \alpha_{\frac{1}{2}} \delta(t+t_0)$$

$$y(t) = s(t) * h(t) = s(t) + \alpha_{\frac{1}{2}} s(t-t_0) + \alpha_{\frac{1}{2}} s(t+t_0)$$

2) ~~sig~~ ~~sig~~ sig

$$h(t) = s(T-t)$$

$$H(f) = S^*(f) e^{-j2\pi fT}$$



$$Z(f) = Y(f) H(f)$$

$$= S(f) S^*(f) e^{-j2\pi fT} + \alpha_{\frac{1}{2}} [|S(f)|^2 (e^{-j2\pi f(T+t_0)} + e^{-j2\pi f(T-t_0)})]$$

$$z(t) = \int_{-\infty}^{\infty} Z(f) e^{j2\pi ft} df, \quad \text{Assume } R_{ss}(t) = \mathcal{F}^{-1}(|S(f)|^2)$$

$$= R_{ss}(t-T) + \alpha_{\frac{1}{2}} (R_{ss}(t-T-t_0) + R_{ss}(t-T+t_0))$$

$$z(kT) = R_{ss}((k-1)T) + \alpha_{\frac{1}{2}} [R_{ss}((k-1)T+t_0) + R_{ss}((k-1)T-t_0)]$$

3) At $t_0 = T$

$$z(kT) = R_{ss}((k-1)T) + \alpha_{\frac{1}{2}} [R_{ss}((k-2)T) + R_{ss}(kT)]$$

$$\therefore \text{ISI is } \alpha_{\frac{1}{2}} R_{ss}((k-2)T) + \alpha_{\frac{1}{2}} R_{ss}(kT)$$

As Matched filter brought a delay of T units.

9.5.

The pulse $x(t)$ having raised cosine spectrum is

$$x(t) = \frac{\text{sinc}(t/T) \cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$

$$\text{sinc}(t/T) = 1 \text{ if } t=0$$

and 0 when $t = nT$.

and,

$$g(t) = \frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2} = \begin{cases} 1, & t=0 \\ \text{bounded} & t \neq 0 \end{cases}$$

The function $g(t)$ needs to be checked for only those values of t such that $4\alpha^2 t^2/T^2 = 1$

$$\text{or } \alpha t = T/2.$$

$$\lim_{t \rightarrow T/2\alpha} \frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2} = x \lim_{x \rightarrow 1} \frac{\cos(\pi x/2)}{1-x}$$

By L'Hospital rule

$$= \lim_{x \rightarrow 1} \frac{\cos \pi x}{1-x} = \lim_{x \rightarrow 1} \frac{\pi/2 \cdot \sin(\pi x/2)}{1-x} = \pi/2 < \infty$$

Hence,

$$x(nT) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

$\Rightarrow x(t)$ is Nyquist

(3)

9.8.

1) Channel bandwidth = 1400 Hz

$$\therefore R_{\max} = 2800$$

If an M-ary PAM modulation is used for transmission, then in order to achieve bit rate of 9600 bps, with maximum symbol rate of R_{\max} , the minimum size of constellation is $M = 2^k = 16$, In this case the symbol rate is

$$R = \frac{9600}{k} = \underline{2400 \text{ sym/sec}}$$

$$\text{and symbol interval} = \frac{1}{R} = \frac{1}{2400} \text{ sec}$$

2) To find excess bandwidth,

$$1200(1+\alpha) = 1400$$

$$\boxed{\alpha = 0.166.}$$