HW6 Solutions. ①

10.1 The spectrum of baseband signal is

$$S_v(f) = \frac{1}{T} S_a(f) |X_{rc}(f)|^2 = \frac{1}{T} |X_{rc}(f)|^2$$

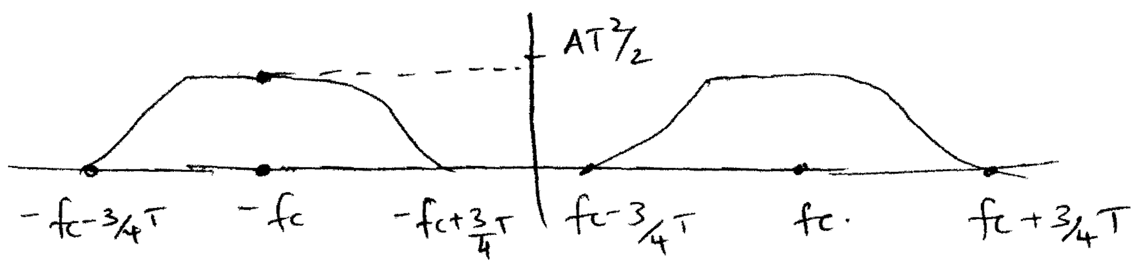
when $T = \frac{1}{2400}$ and

$$X_{rc}(f) = \begin{cases} T & 0 \le |f| \le \frac{1}{4T} \\ \frac{T}{2}\left(1 + \cos\left(2\pi T\left(|f| - \frac{1}{4T}\right)\right)\right) & \frac{1}{4T} \le |f| \le \frac{3}{4T} \\ 0 & \text{otherwise.} \end{cases}$$

If the carrier signal has the form $c(t) = A\cos 2\pi f_c t$

then spectrum of DSB-SC modulated signal,

$$S_v(f) = \frac{A}{2}\left[S_v(f - f_c) + S_v(f + f_c)\right]$$

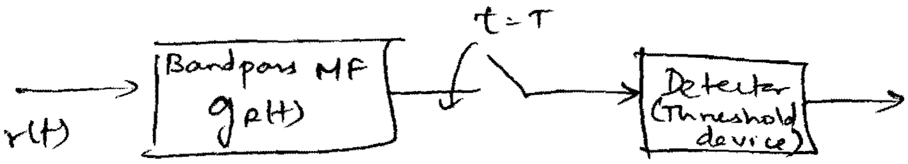


2) Assuming bandpass coherent demodulation using a matched filter, the received signal $r(t)$ is first passed through a linear filter with impulse response

$$g_R(t) = A X_{rc}(T - t)\cos(2\pi f_c(T - t))$$

The output of the matched filter is sampled at $t = T$ and the samples were passed to the detector. The detector is a simple threshold device that decides if a binary 1 or 0 was transmitted depending on the sign of the input samples.

The following figure shows block diagram of the optimal bandpass coherent demodulator.



10.5. The Bandwidth of channel is

$$W = 3000 - 600 = 2400 \text{ Hz}.$$

Symbol rate $R = \frac{2400}{2} = 1200$ symb/sec (QPSK)

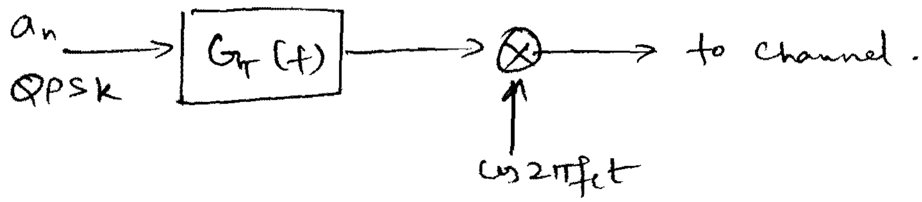
We use raised cosine ($\alpha = 1$) for spectral shaping.

$$X_{rc}(f) = T/2 \left[1 + \cos(\pi T |f|)\right] = \frac{1}{2400} \cos^2\left(\frac{\pi |f|}{2400}\right)$$

If the desired spectral characteristic is split evenly between the transmit filter $G_T(f)$ and receive filter $G_R(f)$

$$G_T(f) = G_R(f) = \sqrt{\frac{1}{1200}} \cos\left(\frac{\pi |f|}{2400}\right), \quad |f| < \frac{1}{T} = 1200$$

The block diagram of transmitter



2) If the bit rate is 4800 bps

$$R = 2400 \text{ sym/sec.}$$

In order to satisfy Nyquist criterion, the signal pulse used for spectral shaping should have the spectrum

$$X(f) = T\,\Pi\left(\frac{f}{W}\right)$$

Thus, freq. response of transmit filter $G_T(f) = \sqrt{T}\,\Pi\left(\frac{f}{W}\right)$

10.7

Bandwidth $W = 3300 - 300 = 3000\text{ Hz}$

In order to transmit 9600 bps, with $R = 2400$ sym/sec,

The amount of information per bit

$$k = \frac{9600}{2400} = 4.$$

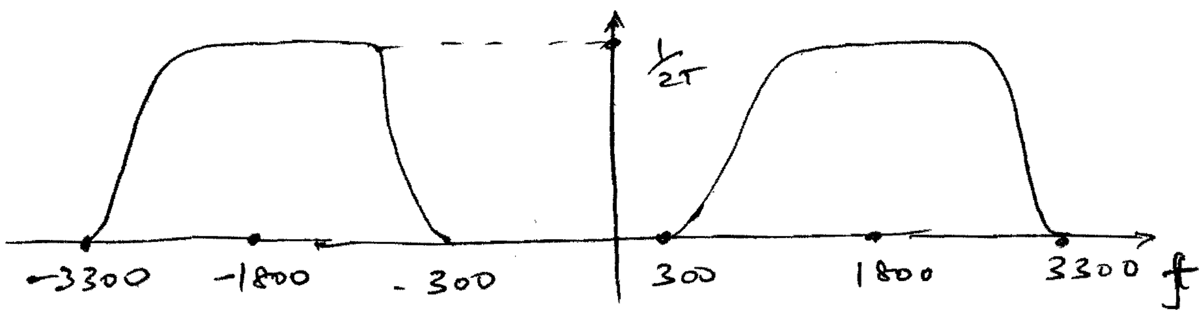
Hence, a $2^4 = 16$QAM signal constellation is needed.

The carrier frequency $f_c = 1800\text{ Hz}$, which is mid-frequency the frequency band of bandpass channel. If the pulse is raised cosine with roll off factor α,

$$R = 1200(1+\alpha)$$

$$\Rightarrow \quad \underline{\alpha = 0.25}$$

The spectrum sketch of transmitted pulse is shown bellow



10.9

First constellation

$$P_a = \frac{1}{8}\left(4\times(2A)^2 + 4\times(2\sqrt{2}A)^2\right) = \underline{6A^2}$$

Second constellation.

$$P_b = \frac{1}{8}\left(4\times(\sqrt{7}A)^2 + 2\times(\sqrt{3}A)^2 + 2A^2\right)$$

$$= \underline{\tfrac{9}{2}A^2}$$

$P_b < P_a$, second constellation is more power efficient.

10.13 The number of bits / symbol

$$k = \frac{4800}{R} = \frac{4800}{2400} = 2.$$

Thus 4 Q-AM is used. ~~Pe~~ Prob. of error for M-ary QAM

$$P_M = 1 - \left(1 - 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left[\sqrt{\frac{3kE_b}{(M-1)N_o}}\right]\right)^2 \longrightarrow (1)$$

with $P_M = 10^{-5}$, $k = 2$, we obtain

$$Q\left(\sqrt{\frac{2E_b}{N_o}}\right) = 5 \times 10^{-6} \Rightarrow \frac{E_b}{N_o} = 9.7682.$$

(2) If bit rate $= 9600$ bps.

$\underline{k = 4}$, we have 16-QAM constellation.

~~P_M~~

Use formula (1) for $k = 4$ and $P_M = 10^{-5}$, we get

$$\frac{E_b}{N_o} = 25.36$$

(3) If bit rate $= 19200$ bps

$\underline{\underline{k = 8}}$

$$\frac{E_b}{N_o} = 659.89$$

k	2	4	8
SNR(dB)	9.89	14.04	28.19

It is observed, there is an increase in transmitted power of approx. 3dB per additional bit. per symbol