

**ECE 146B: Digital Communication Theory and Techniques**  
**Lab 3:  $M$ -ary Phase Shift Keying (PSK) and  $M$ -ary Quadrature Amplitude Modulation (QAM)**

**Lab Report Due: 5:00 p.m., Friday, May 22, 2009**  
(Place in the ECE 146B Homework Box on the 3rd Floor of Harold Frank Hall)

## 1 Objective

The objective of this lab is to implement the modulator and demodulator for  $M$ -ary phase-shift keying (PSK) and  $M$ -ary quadrature amplitude modulation (QAM), and to study the advantages of each method over a noisy channel.

## 2 Equipment

Matlab, Simulink, and the Communication Toolbox software are available on the ECI workstations.

## 3 Background and Preparation

### 3.1 $M$ -ary Pulse Amplitude Modulation

$M$ -ary pulse amplitude modulation (PAM) is generated by subdividing the binary input stream into blocks, each of size  $k$  bits, and mapping them to  $M$  symbols. Each symbol is represented by one of  $M = 2^k$  possible amplitude values. When the bit rate  $R_b$  is fixed, the symbol interval is computed according to  $T = k/R_b = kT_b$  where  $T_b$  is the bit interval. The signal waveforms for  $M$ -ary PAM can be represented by  $S_m(t) = A_m$ ,  $0 \leq t \leq T_b$ ,  $m = 1, \dots, M$ , where the amplitude values are equally spaced according to  $A_m = (2m - 1 - M)$ . Because the amplitudes are different, the PAM symbols have different energies, given by  $E_m = A_m^2$ .

- (1) Determine the average energy  $E_{\text{avg}}$  and the average power  $P_{\text{avg}}$  for the previously defined  $M$ -ary PAM signal using

$$E_{\text{avg}} = \frac{1}{M} \sum_{m=1}^M E_m \quad \text{and} \quad P_{\text{avg}} = \frac{E_{\text{avg}}}{T} \quad (1)$$

where  $T = T_b \log_2 M$ .

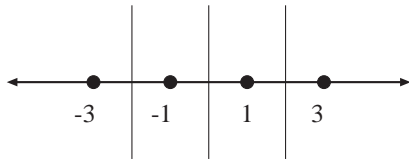


Figure 1: Decision boundaries for 4-ary PAM.

The average probability of error for  $M$ -ary PAM can be determined from the decision rule where the detector compares the demodulator output  $r$  with a set of  $M - 1$  thresholds that are placed at the midpoints between the amplitude levels as shown in Figure 1. It is clear that the decision is made in favor of the amplitude closest to  $r$ . If the  $m$ th amplitude level is transmitted, the demodulator output is given by  $r = (2m - 1 - M) + n$  where  $n$  is a Gaussian random variable with zero mean and variance  $N_o/2$ . The power spectral density of the additive white Gaussian noise (AWGN) in the channel is  $N_o/2$ .

- (2) Under the assumption that the PAM symbols are equally likely, derive the average probability of a symbol error  $P_M$  in terms of  $M$ ,  $P_{\text{avg}}$ , and  $T_b$ .
- (3) Use Matlab to plot  $P_M$  as a function of  $10\log_{10}(P_{\text{avg}}T_b/N_o)$  for  $M = 2, 4, 8$ , and  $16$ . What is the performance gain (in dB) for each factor of two increase in  $M$ ?
- (4) What are the advantages and disadvantages of  $M$ -ary PAM?

### 3.2 8-ary Phase Shift Keying (8-PSK)

In a bandpass modulator using 8-PSK, there are eight possible phases (usually equally spaced). The incoming bits are mapped to eight possible symbols (phases) by subdividing the bit stream into groups containing three bits each. A block diagram of an 8-PSK modulator is shown in Figure 2. The bits enter a serial-to-parallel (S/P) converter where they are passed to three parallel channels labeled I, Q, and C. The bit rate in each of the three channels is  $f_b/3$  where  $f_b$  is the input bit rate. The bits in the I- and Q-channels enter two separate 2-to-4 level converters. When these are combined with the C and  $-C$  channels, the 2-to-4 level converters yield two 4-ary PAM signals which are processed by a quadrature modulator for transmission across the channel.

- (5) Use Matlab to generate and plot the waveforms for 8-ary PSK to illustrate their constant-envelope property. The passband signals for  $M$ -ary PAM are given by

$$s_m(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 2\pi m/M), \quad m = 0, \dots, M - 1 \quad (2)$$

where  $E$  is the symbol energy and  $f_c$  is the carrier frequency.

### 3.3 16-ary Quadrature Amplitude Modulation (16-QAM)

In a bandpass modulator using 16-QAM, the incoming bit stream is subdivided into groups of four bits to yield 16 different symbols. A block diagram of a 16-QAM transmitter is shown in Figure 3.

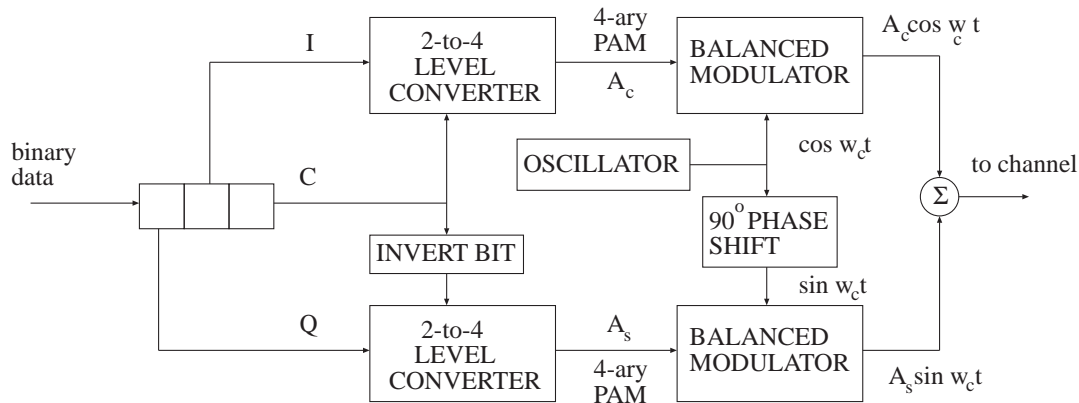


Figure 2: 8-PSK modulator.

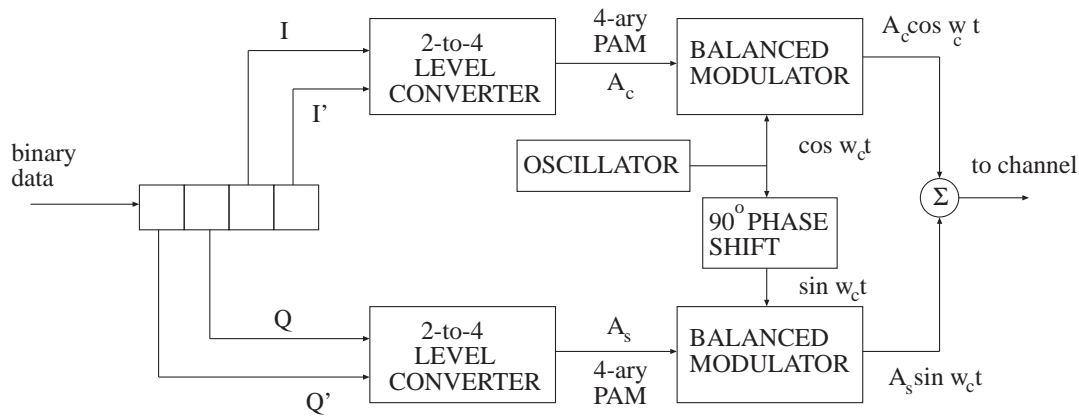


Figure 3: 16-QAM modulator.

Observe that the input data are divided into four channels labeled I, I', Q, and Q', each with a bit rate of  $f_b/4$ . Four bits are serially clocked into an S/P converter which passes them to two 2-to-4 level converters that generate the 4-ary PAM signals. The PAM signals are then processed by a quadrature modulator for transmission across the channel.

- (6) Use Matlab to generate and plot the waveforms for rectangular 16-QAM. The passband signals can be represented by

$$s_{mn}(t) = A_m \cos(2\pi f_c t + \theta_n), \quad m = 1, \dots, M_1, \quad n = 1, \dots, M_2. \quad (3)$$

For this case,  $M_1 = M_2 = 4$ .

I	C	Output	Q	C	Output
0	0	-0.541	0	0	-0.541
0	1	-1.307	0	1	-1.307
1	0	0.541	1	0	0.541
1	1	1.307	1	1	1.307

Table 1: Output Levels for the I- and Q-Channels of 8-PSK

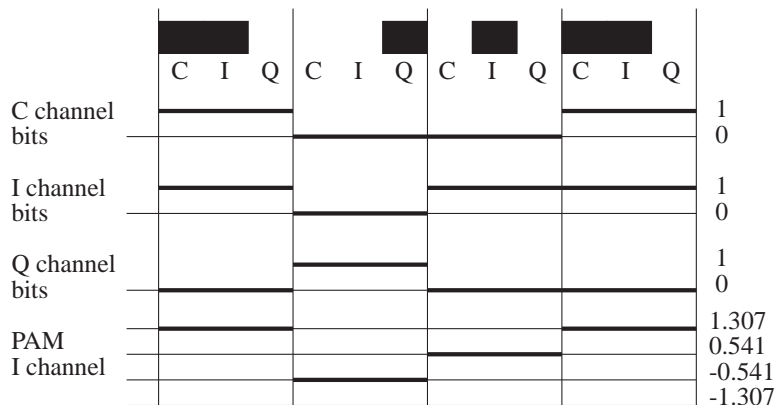


Figure 4: PAM I-channel output levels for an example bit stream.

## 4 Procedure

### 4.1 8-PSK

Assume that the bit interval is  $T_b = 1$  in your simulations. Examine the serial-to-parallel (S/P) converter block. The 2-to-4 level converters for the I- and Q-channels are given in Table 1.

The output levels for the 4-ary PAM signal generators should look similar to the example in Figure 4.

- (7) Sketch the corresponding PAM Q-channel for the example in Figure 4.
- (8) Design the 2-to-4 level converter for the I- and Q-channels, as well as the balanced modulator using  $\cos 2\pi 5t$  and  $\sin 2\pi t$  for the carriers. Verify your design by examining the modulator output for a test signal input.
- (9) Design and sketch a block diagram of a coherent demodulator for the 8-PSK signal.
- (10) If the modulated waveform is  $A_c \cos 2\pi 5t + A_s \sin 2\pi 5t$ , find the in-phase and quadrature components of your demodulator design (i.e., at the matched filter outputs and before the detectors).

I	I'	Output	Q	Q'	Output
0	0	-3	0	0	-3
0	1	-1	0	1	-1
1	0	1	1	0	1
1	1	3	1	1	3

Table 2: Output Levels for the I- and Q-Channels of 16-QAM

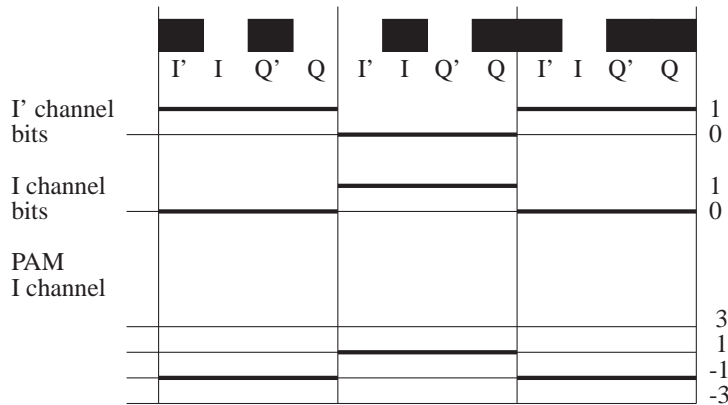


Figure 5: QAM I-channel output levels for an example bit stream.

- (11) Determine the optimum detector for the in-phase and quadrature components, and design it using Simulink.

## 4.2 16-QAM

In this part of the lab, you will design and implement a modulator and a demodulator for 16-QAM. The 2-to-4 level converters for the I- and Q-channels are given in Table 2.

The output levels for the 4-ary PAM signal generators should look similar to the example in Figure 5.

- (12) Sketch the corresponding QAM Q-channel for the example in Figure 5.
- (13) Design the 2-to-4 level converter for the I- and Q-channels, as well as the balanced modulator using  $\cos 2\pi 5t$  and  $\sin 2\pi t$  for the carriers. Verify your design by examining the modulator output for a test signal input.
- (14) Design and sketch a block diagram of a coherent demodulator for the 16-QAM signal.
- (15) If the modulated waveform is  $A_c \cos 2\pi 5t + A_s \sin 2\pi 5t$ , find the in-phase and quadrature components of your demodulator design (i.e., at the matched filter outputs and before the detectors).

- (16) Determine the optimum detector for the in-phase and quadrature components, and design it using Simulink.

### 4.3 16-QAM in Additive White Gaussian Noise

Modify the previous simulation by including zero-mean additive white Gaussian noise  $w(t)$  with power spectral density  $N_o/2$ .

- (17) Derive the variances of the sampled noise at the output of the matched filter.
- (18) For signal-to-noise ratios (SNRs) of 2, 4, 6, 8, 10, and 12 dB, determine the symbol error probabilities for the I- and Q-channels.
- (19) Plot the corresponding bit error probabilities.
- (20) Comparing with  $M$ -ary PAM, what is the major advantage of  $M$ -ary QAM?

### 4.4 16-QAM in a Fading Channel (Optional)

The Rayleigh probability density function (pdf) has the general form  $f_N(n) = (n/c^2)e^{-n^2/2c^2}u(n)$  where  $u(n)$  is the unit-step function.

- (21) Find the constant  $c$  such that  $E[N^2] = 1$ .
- (22) If the additive noise  $n(t)$  has a Rayleigh pdf, what can you determine about the performance at the detector output compared to AWGN?

## 5 Lab Report

Please include the following in your lab report.

1. Include printouts of all block diagrams that you designed.
2. Answer all questions (in the Preparation and Procedure sections) and back them up with results, analysis, theory, and any computations.
3. Write a paragraph about questions and confusions that you experienced in this lab.