ECE 147b: Digital Control

Lab 1: Digital Control Design

Overview

This lab will look at various means of transforming continuous-time designs to discrete-time designs. The plant we will attempt to control is the linear position servo. We will design multiple controllers for use in the unity feedback loop shown in Figure 1.
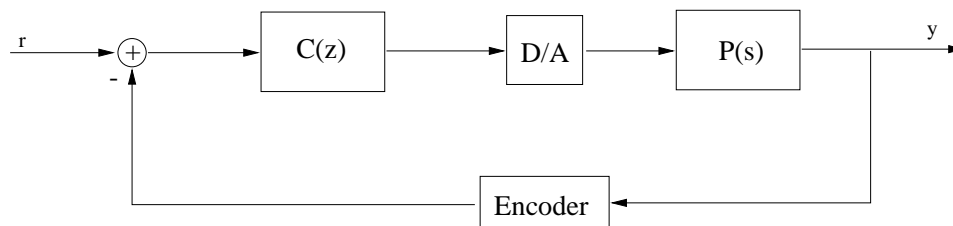

Figure 1: Closed-Loop System

Model Derivation

The DC motor equations are given by:

$$V = I_m R_m + K_m K_g \omega_g$$
$$= I_m R_m + K_m K_g \frac{\dot{x}}{r},$$

where V is the voltage applied to the motor, $I_m(\text{Amp})$ is the motor current, $K_m(\frac{V}{\text{rad/sec}})$ is the back EMF constant, K_g is the gear ratio in the motor gearbox, $\omega_g(\frac{\text{rad}}{\text{sec}})$ is the motor output angular velocity, $\dot{x}(\frac{\text{m}}{\text{sec}})$ is the cart velocity, and $r(\text{m})$ is the radius of the motor pinion that meshes with the track. The torque generated by the motor is:

$$\tau = K_m K_g I_m,$$

which exerts a force, F, on the cart via the pinion,

$$F = \frac{\tau}{r}.$$

This force results in an acceleration of the cart governed by Newton's second law:

$$F = m_c \ddot{x},$$

where m_c is the mass of the cart. Combining these two equations yields:

$$\frac{x}{V} = \frac{K_m K_g r}{s \left(m_c R_m r^2 s + K_m^2 K_g^2 \right)}.$$

The values of the coefficients are tabulated below. These have been taken from the manufacturer's manual and may not be exactly correct for your particular hardware.

	Parameter	Value	
K_m	Back EMF constant	0.00767	V/(rad/sec)
R_m	Motor resistance	2.6	Ohms
K_g	gear ratio	3.7	
N_p	Motor pinion teeth	24	
r	pinion radius	0.00635	m
m_c	cart mass	0.455	kg

Substituting the numerical values for the constants in the above equation, and defining $P(s) = x/V$, yields

$$P(s) = \frac{3.78}{s(s + 16.88)}. \tag{1}$$

Equation 1 describes the linear position servo which we will control in this lab.

Analog Design Approaches

We can design a controller by root locus or Bode methods. A suitable controller is given by

$$C_1(s) = \frac{100(s + 16.88)}{s + 30}.$$

This controller should place the poles of the closed-loop transfer function from the reference to the output at $-15.0 \pm j12.4$. Note that if no pole-zero cancellation is acchieved succesfully, we will have another pole. In the cases that you have more than two poles, assign the others

in stable locations. One possible location is -16.9 since the closed-loop transfer function from the disturbance to the output has a pole at that location. Check this. Substituting

$$s = \frac{1 - z^{-1}}{T}, \tag{2}$$

into the expression for $C_1(s)$ gives a discrete equivalent with the following pulse transfer function:

$$
\begin{aligned}
C_1(z) &= \frac{b_0 z + b_1}{z + a_1} \\
&= \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}.
\end{aligned}
$$

Equation 2 is known as the backward difference transformation. You may implement this filter in Simulink using the "Discrete Filter" block.

We will also consider another continuous design which attempts to account for the zero order hold in the continuous-time design procedure. To do this, consider $P(s)$ to be given by

$$P(s) = \frac{1}{(1 + 0.5Ts)} \frac{3.78}{s(s + 16.88)}.$$

The additional pole provides additional phase and roll-off as an approximation to the effect of the zero order hold. Now we would like to design a controller of the form

$$C_2(s) = \frac{B_0 s + B_1}{s + A_1},$$

to give a suitable closed-loop system. To make a fair comparison, we should place the closed-loop poles in approximately the same locations as in the design of $C_1(s)$. In other words, near $-15.0 \pm j12.4$ and -16.9 if it is applicable. Note that in this design we will have four poles. Check that the fourth pole is still stable. We will again use a backward difference transformation (Equation 2) to discretize $C_2(s)$ and label the result $C_2(z)$.

We will consider two other methods of transforming $C_2(s)$ into a discrete-time controller. One of these methods is called the bilinear (or trapezoidal) transformation, given by substituting

$$s = \frac{2}{T} \frac{(z - 1)}{(z + 1)}, \tag{3}$$

in $C_2(s)$. We label the resulting controller $C_3(z)$.

The second transformation method is known as zero-pole matching equivalents[1]. We noted before that poles in the s-plane translate to poles in the z-plane via

$$z_p = e^{s_p T},$$

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[1] A step-by-step method of calculating zero-pole matching equvalents is given on p. 200 of Franklin & Powell (3rd edition).

where s_p is an s-domain pole and z_p is the corresponding z-domain pole. The idea here is that we also apply this transformation to the zeros of the system. In other words,

$$z_0 = e^{s_0 T},$$

where s_0 and z_0 are, respectively, the s-domain and z-domain zeros. The controller $C_2(s)$ has only one pole and one zero. Construct the pole-zero equivalent of $C_2(s)$ and denote this controller by $C_4(z)$. In your report you should derive an exact discrete model of the plant with a zero order hold and check the closed-loop z-domain pole locations for each controller.

Direct Digital Design

You will experimentally examine the direct digital design methods described below during the second week of this lab. Recall that the plant is given by

$$P(s) = \frac{3.78}{s(s + 16.88)}.$$

The zero-order hold equivalent of the plant is

$$P(z) = \frac{3.78}{16.88^2} \frac{\left((16.88T + e^{-16.88T} - 1) \, z - ((16.88T + 1)e^{-16.88T} - 1) \right)}{(z - 1) \, (z - e^{-16.88T})},$$

or, equivalently,

$$P(z) = \frac{\alpha(z - \beta)}{(z - 1) \, (z - e^{-16.88T})},$$

where,

$$\alpha = \frac{3.88 \left(16.88T + e^{-16.88T} - 1 \right)}{16.88^2}, \qquad \text{and} \quad \beta = \frac{\left((16.88T + 1)e^{-16.88T} - 1 \right)}{(16.88T + e^{-16.88T} - 1)}.$$

You should check this. Using the above variables, and the controller

$$C_i(z) = \frac{b_0 z + b_1}{z + a_1},$$

the closed-loop characteristic equation is

$$z^3 + z^2(a_1 - 1 - e^{-16.88T} + \alpha b_0) \quad + \quad z(\alpha(b_1 - \beta b_0) - a_1(1 + e^{-16.88T}) + e^{-16.88T})$$
$$+ \quad (a_1 e^{-16.88T} - \alpha \beta b_1) = 0.$$

Your report should include the above derivation with intermediate steps. Note that, although this equation is quite messy, we can choose a_1, b_0, and b_1 to arbitrarily place the poles of the closed-loop system. In the continuous-time design procedure, the two oscillatory poles of the

continuous-time closed-loop were at $s_0 = -15.0 \pm j12.4$. In the digital design, we will try placing these poles at the z-domain locations given by

$$z_0 = e^{s_0 T}.$$

The third pole will be placed at zero, giving a discrete-time controller which we will denote by $C_5(z)$. This choice of zero for the third pole location is somewhat arbitrary. Another controller, $C_6(z)$, will be designed with the third pole somewhere else.

The final controller, $C_7(z)$, will place all the poles at zero. In other words, we want the characteristic equation to be

$$z^3 = 0$$

This is known as a deadbeat controller.

Laboratory Procedure: Week 1

Prelab

The Prelab essentially involves doing all of the design calculations for $C_1(z)$ through $C_4(z)$. It is essential that you do these before the lab in order to finish in the available time.

You will calculate (and implement in lab) each controller for three sample period choices: $T = 0.001$ sec, $T = 0.005$ sec, and $T = 0.01$ sec. You should have twelve controllers calculated before coming to lab. You should also predict whether or not each of the controllers will stable, and give a brief description of their expected step response (i.e, fast, slow, overdamped, oscillatory, ...).

1. Calculate $C_1(z)$ for each of the three sample periods.

2. Design $C_2(s)$ for the same three sample periods as above. Hint: Work this out as a function of T and then just substitute in the required sampling period values.

3. Calculate the coefficients of $C_2(z)$ for each of the three sample periods.

4. Calculate the coefficients required to implement $C_3(z)$ for the three sample periods.

5. Calculate $C_4(z)$ for the three sample periods.

Experiment

Make the following connections to set up the linear position servo:

- Connect analog output channel zero to the Universal Power Module "From D/A" jack.

- Connect the "To Load" jack on the Universal Power Module to the cart motor.

- Connect the encoder for cart position to encoder input channel zero. (You can tell which encoder corresponds to cart position and which encoder corresponds to pendulum angle by examining the wires coming from the connectors.)

Set the sampling period for the MultiQ-3 board to $T_s = 0.001$ sec. The encoder has 4096 counts per revolution and the gear circumference is 0.093043 m. Therefore, one revolution corresponds to a distance of 0.093043 m. The calibration constant is then

$$\frac{0.093043 \text{ (m)}}{4096 \text{ (counts)}} = 0.0000227 \text{ (m/count)}.$$

We want to generate a step response for each of the four controllers. You should record the output of the controller, $C_i(z)$, as well as the cart position (which is the output of the plant).

Note that you will be testing twelve different digital controllers. However, the experimental setup is the same for each. All that is required is that you create multiple Simulink diagrams, or simply change values inside a single Simulink diagram. If you take the time to generate your Simulink diagrams before coming to lab, this lab will go very fast.

Laboratory Procedure: Week 2

Prelab

Again, calculate the controllers for each of the sample periods: $T = 0.001$ sec, $T = 0.005$ sec and $T = 0.01$ sec. As before, predict the stability and nature of the response for each controller.

1. Calculate the coefficients for $C_5(z)$.

2. Select a different location for the third pole and calculate the coefficients for $C_6(z)$. Make sure that you select a stable location. You might try $s_0 = -16.9$, which approximates our continuous-time design.

3. Calculate the coefficients for $C_7(z)$.

Experiment

You have nine controllers to test during this lab session. The experimental setup is identical to that of last week. As you would expect, if you take the time to generate your Simulink diagrams before coming to lab, you should finish early.

Record a step response for each controller. If you think other responses would be interesting, record them also.

Pay particular attention to the deadbeat controller ($C_7(z)$). Specifically, compare the actual analog output (cart position) with the digital samples of the output. To do this, insert a zero-order hold block after the calibration constant. Set the sampling period to be the same as that in the discrete transfer function block and examine the signals both before and after the zero-order hold. Be sure to use a sufficiently small time-scale when viewing the graph so that you can clearly see what is happening between sampling times.

Laboratory report

The laboratory report is due one week from the date of the last session. Do not reproduce every step response on a separate graph. Uses plots with multiple, labelled, lines so that the reader can see at a glance the comparison you wish to make. Style will count for a significant part of the grade. So think carefully about what it is you wish to say, and write it succinctly. Readers are not impressed by volumes of repetitive data. Where necessary reference material in textbooks or other sources.