University of California, Santa Barbara

Department of Electrical & Computer Engineering

ECE 147b: Digital Control

Lab 2: Linear Flexible Joint

Overview

The linear flexible joint system builds on the motor cart system used in the previous lab. Here, the motor cart is coupled to a second cart (which we will refer to as the load cart) via a spring. The second cart is equipped with an optical encoder, making position measurements possible. Below is a free-body diagram describing this system.

Modeling

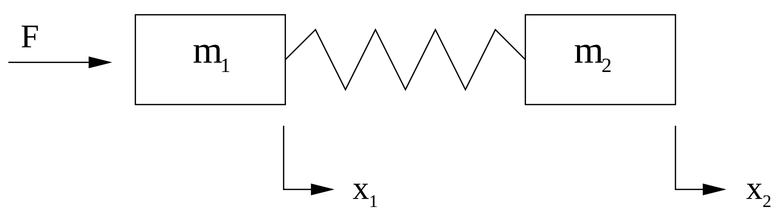

Figure 1: Free Body Diagram of Linear Flexible Joint

The variables for this system are: F (input force to the motor cart in Newtons), k (spring stiffness in Newtons/meter), and m_1 and m_2 (mass of the motor and load carts, respectively, in kilograms). Letting x_1 and x_2 be the position of the motor and load carts respectively, we can write the following equations describing the system behavior:

$$F - k(x_1 - x_2) = m_1\ddot{x}_1 \tag{1}$$
$$k(x_1 - x_2) = m_2\ddot{x}_2 \tag{2}$$

The conversion from force to input voltage is given by

$$F = \frac{K_m K_g}{R_m r}V - \frac{K_m^2 K_g^2}{R_m r^2}\dot{x}_1. \tag{3}$$

Taking Laplace transforms of Equations 1, 2, and 3 and substituting Equation 3 into Equation 1 yields:

$$\begin{aligned} X_2(s)(m_2s^2 + k) &= kX_1(s) \\ X_1(s)(m_1s^2 + \frac{K_m^2K_g^2}{R_mr^2}s + k) &= \frac{K_mK_g}{R_mr}V(s) + kX_2(s) \end{aligned}$$

By substitution, we can generate the three transfer functions $\frac{X_1(s)}{V(s)}$, $\frac{X_2(s)}{V(s)}$, and $\frac{X_2(s)}{X_1(s)}$. Two of the transfer functions, with numerical values substituted, are provided for ease of reference.

$$\frac{X_2(s)}{X_1(s)} = \frac{61.2}{s^2 + 61.2} \tag{4}$$

$$\frac{X_1(s)}{V(s)} = 2.97\frac{s^2 + 61.2}{s^4 + 13.24s^3 + 127.15s^2 + 810.37s} \tag{5}$$

We can now close loops around the position of either cart or perform a cascade control design.

Sample Design

Suppose we close a simple proportional gain loop around the system $P(s) = \frac{X_1(s)}{V(s)}$. By examining the root locus plot for $P(s)$, we pick a gain $K = 10$ to push the poles as far to the left as we can. This places the continuous-time closed-loop pole locations at $s = -3.29 \pm j9.33$ and $-3.33 \pm j3.60$. The zero-order hold plant equivalent, with sampling period $T_s = 0.05$s, is

$$P(z) = 2.964 \times 10^{-3}\frac{z^3 + 1.047z^2 + 0.4821z + 0.8016}{z^4 - 3.249z^3 + 4.086z^2 - 2.353z + 0.5158}.$$

With $K = 10$, the discrete-time closed-loop poles are at $z = 0.983 \pm j0.046$ and $0.984 \pm j0.018$, yielding a stable system.

Suppose, by looking at the step response, we decided that the rise time is to slow. We can add a lead compensator to improve the response. Using the compensator

$$C(s) = K_c\frac{s + 12}{s + 25}$$

we can again plot the root locus of $C(s)P(s)$ and select $K_c = 35$ to push the closed-loop poles to the left in the s-plane. The continuous-time closed-loop poles are at $s = -3.86 \pm j7.69$, $-4.88 \pm j5.07$, and -20.76. Discretizing this controller via the bilinear transformation yields

$$C(z) = 28\frac{z - 0.5386}{z - 0.2308}.$$

The discrete-time closed-loop pole locations are therefore located at $z = 0.703 \pm j0.338$, $0.818 \pm j0.216$, and 0.356. Again, all of the closed-loop system poles are stable. Comparing

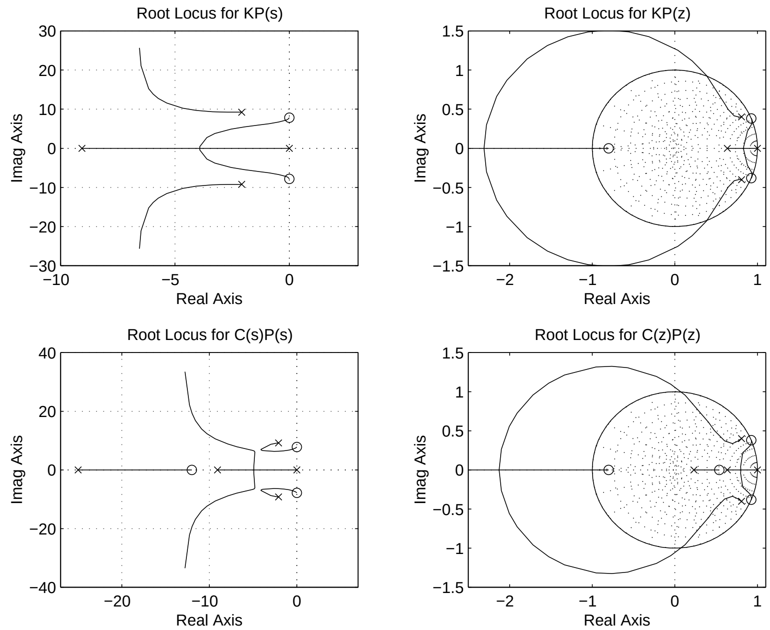

Figure 2: Root Loci for Continuous and Discrete Systems with Lead and Proportional Compensators

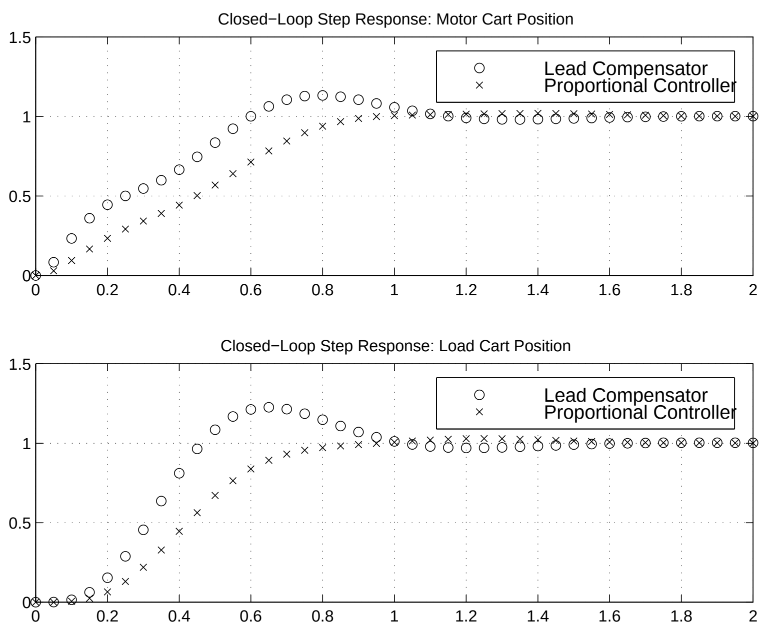

Figure 3: Step Responses for Proportional and Lead Compensators

the step response for the lead compensator and the proportional controller, we can see that the lead compensator decreases the rise time.

This is not a particularly good design, and you should be able to do better. Suggestions would be to vary the location of the pole and zero of the lead compensator, add lag compensation, or try something else. Note that the performance criteria are open-ended. Therefore, you should make sure your controller designs include a thorough discussion of why you selected a particular design.

Cascade Control Design

Defining $P_1(s) = \frac{X_1(s)}{V(s)}$ and $P_2(s) = \frac{X_2(s)}{X_1(s)}$, we arrive at the following block diagram:

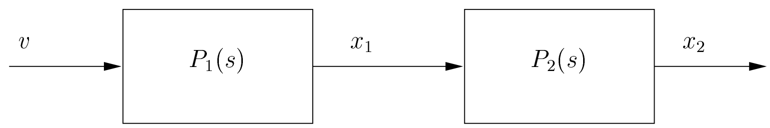

Figure 4: Series Model of Linear Flexible Joint

Cascade control may provide better performance than the above SISO designs. Figure 5 illustrates the cascade control configuration.

For simplicity, we can close the inner loop using a proportional controller. Having performed root locus analysis on $P_1(s)$ in the previous section, we select

$$C_2(s) = K_c = 10.$$

We can now examine the Bode plot of

$$P(s) = \frac{C_2(s)P_1(s)}{1 + C_2(s)P_1(s)} P_2(s)$$

which is shown in Figure 6. (Note that this corresponds to the system of Figure 5 with $C_1(s)$ removed.)

We notice that the low frequency gain is quite small. This will cause large steady-state errors. To improve this we can add the lag controller

$$C_1(s) = \frac{s + 0.5}{s + 0.01}.$$

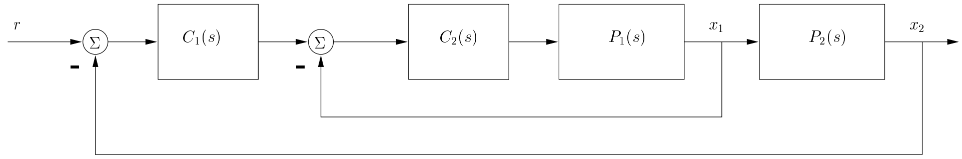

Figure 5: Cascade Control for Linear Flexible Joint

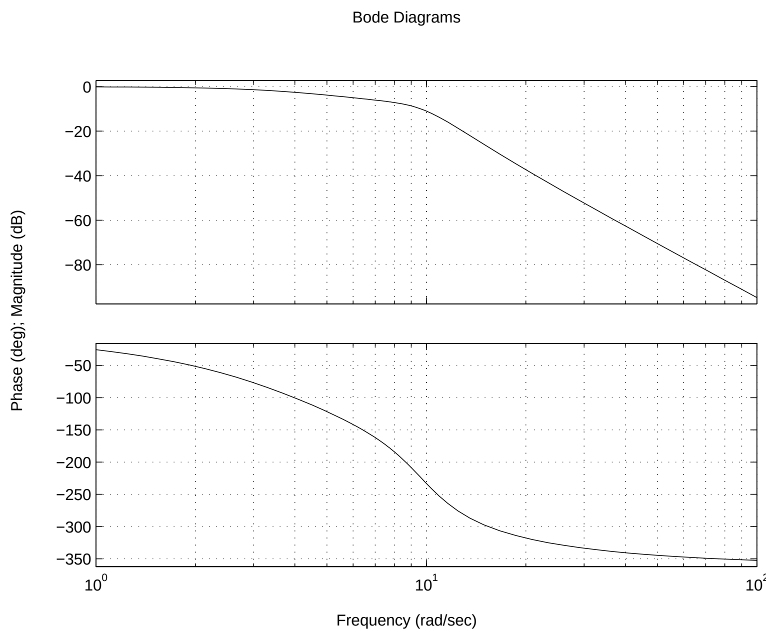

Figure 6: Closed Inner-Loop plus $P_2(s)$ Bode Plot

This corresponds to the Bode plot shown in Figure 7.

Examining the step response (Figure 8), we decide the response is too slow. To ameliorate this, we add the following lead controller

$$C(s) = 4\frac{s+8}{s+20}$$

so that our new $C_1(s)$ is given by

$$C_1(s) = 4\frac{(s+8)(s+0.5)}{(s+20)(s+0.01)}.$$

Note that we have reduced the rise time by almost half (Figure 9) at the expense of reducing the damping on the second cart. The Bode plot corresponding to the closed inner-loop system with the above $C_1(s)$ is displayed in Figure 10.

The final step is to check that the discrete-time design is adequate. The zero-order hold equivalent of $P_1(s)$ is derived in the previous section. The zero-order hold equivalent for $P_2(s)$ is given by

$$P_2(z) = 0.0755\frac{z+1}{z^2 - 1.849z + 1}.$$

The discretization for $C_2(s)$ is trivial. Applying the bilinear transformation to $C_1(s)$ yields

$$C_1(z) = 3.239\frac{z^2 - 1.642z + 0.650}{z^2 - 1.333z + 0.333}.$$

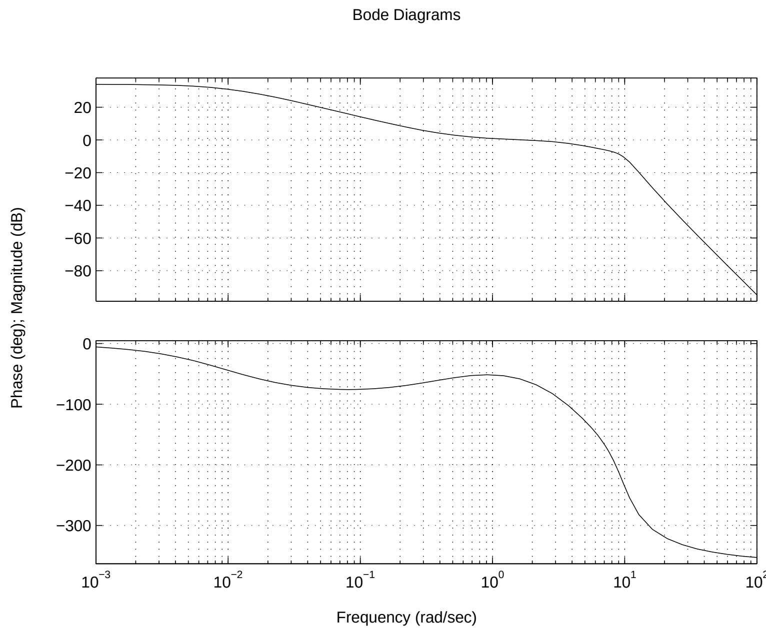

Figure 7: Bode Plot for Closed Inner-Loop plus Lag Controller

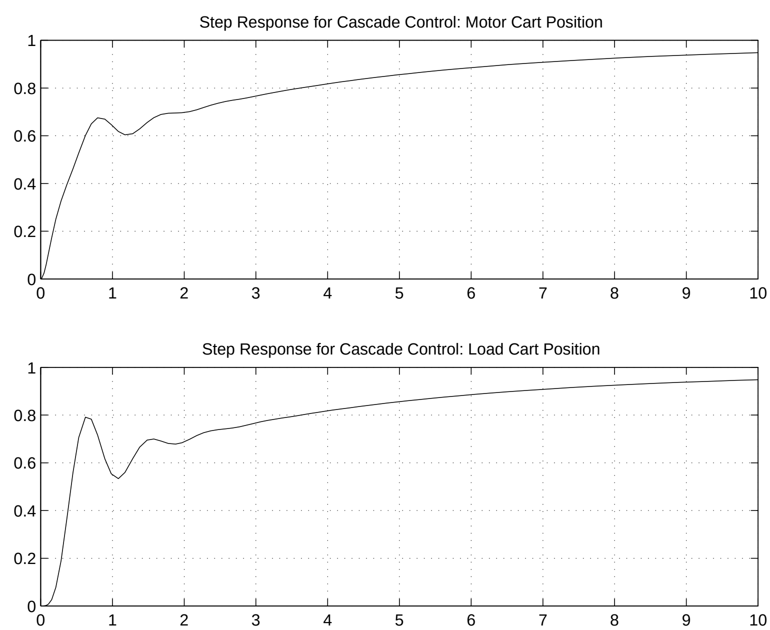

Figure 8: Step Response for Cascade Control with Lag Controller

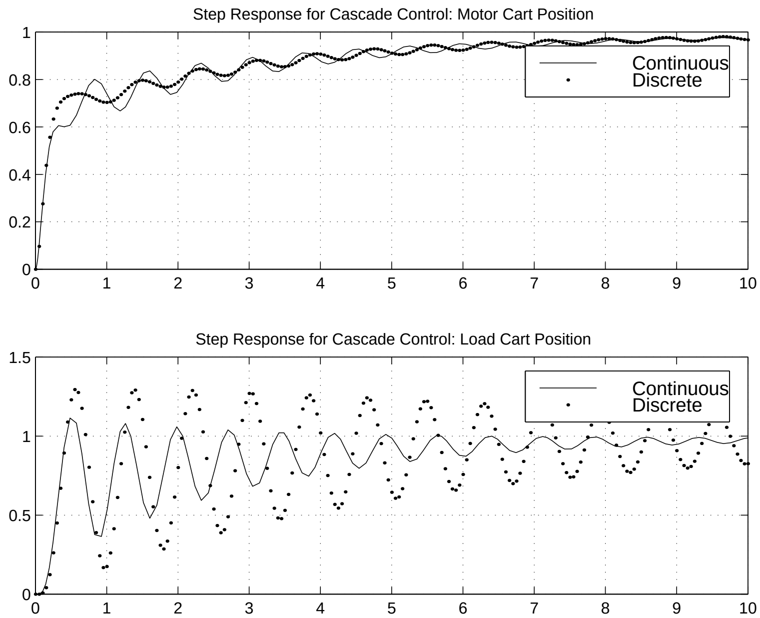

Figure 9: Step Response with Lead & Lag Controller

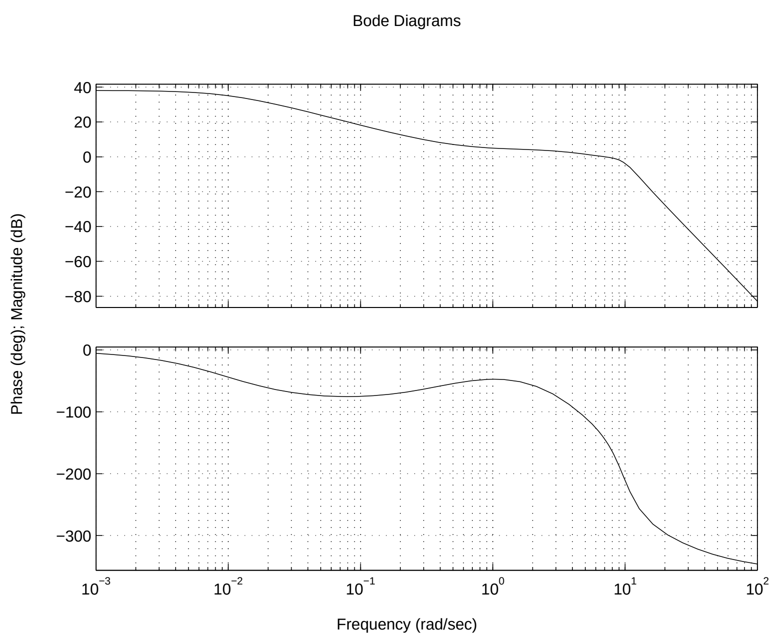

Figure 10: Bode Plot for Cascade Control with Lead & Lag Controller

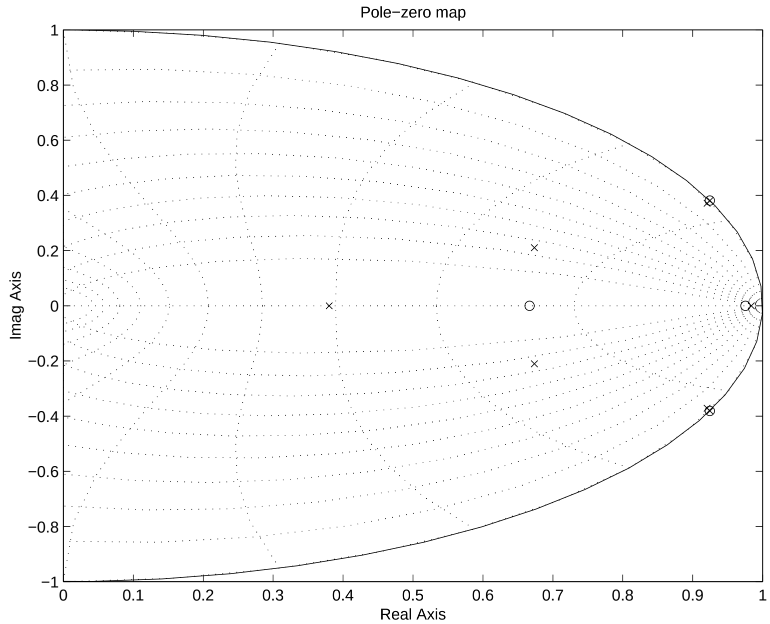

Figure 11: Poles & Zeros of Discrete-Time Cascade Control System

We can close both loops and check the pole locations to ensure that the system is still stable (see Figure 11). We then compare the step response of the continuous design to the discrete-time response (Figure 9). The response of the load cart in the discrete design is far from ideal, but it gives us a reasonable starting point in our design.

Again, you should be able to do better than this design. Suggestions here would again be to vary pole and zero locations of the controllers, or to try to design something besides a proportional controller for the inner-loop.

Aside: Frequently in a cascade control structure, the inner-loop will be closed with an analog controller while the outer-loop is implemented digitally. This would be one reason to use a simple proportional controller for the inner loop.

Furthermore, a good design will have an inner-loop which is faster than the outer-loop (again, this lends itself to an analog design for the inner-loop). Intuitively, the inner-loop needs to be at least as fast as the outer-loop and it is standard practice to have the inner-loop two to ten times faster than the outer-loop. We will not explore these issues in this lab.

Experimental Procedure

Prelab: Week 1

It is highly recommended that you use Matlab to design your controllers. You should turn in a printout of your m-file as well as writing out the controllers and the closed-loop pole locations. Be sure to justify your controller design. In particular, simulate the system with the controller you design and include Bode, Nyquist, or root locus plots you used to arrive at your design. Use a sampling period of $T_s = 0.05$ for your calculations.

1. Suppose that we didn't know much about our system or that we were too lazy to model it in its entirety. Using the transfer function of the motor cart by itself, namely

   $$P(s) = \frac{3.85}{s(s+17.2)}$$

   design a controller to give the closed-loop system good disturbance rejection properties. Hint: One way to do this is to try to reduce the steady-state error.

2. Digitize this controller using a method of your choice. Using the plant zero-order hold equivalent, check that the closed-loop poles of the discrete-time system are stable.

3. Suppose that we're unhappy with the response obtained above, so we put forth the effort to obtain a better model of the plant. Assuming that only the motor cart position is available for measurement, design a controller which gives a good closed-loop response. This corresponds to designing for the plant given by Equation 5.

4. Again, discretize the controller and check the location of the closed-loop poles using the zero-order hold equivalent plant.

5. Suppose that rather than having the position of the motor cart as a measurement that we instead have the position of the load cart as our measurement. Design a controller for this system which gives a good closed-loop response. This corresponds to designing for the plant $\frac{X_2(s)}{V(s)}$.

6. Discretize the controller and check that the closed-loop poles of the discrete-time system are stable.

Experiment: Week 1

Implement and test the three controllers designed in the Prelab using a sampling period of $T_s = 0.05$. You should at least obtain step responses for each controller. If you think comparisons between other responses would be interesting, obtain those as well. Does the overall system performance change based on which position measurement you use as the output? If so, which one gives better performance? Can you justify this answer intuitively?

Prelab: Week 2

Perform your own design for the cascade control depicted in Figure 5. Again be sure to thoroughly justify your design. You may wish to design more than one set of controllers (i.e., multiple $C_1(s)$ and $C_2(s)$ pairs) and compare them.

Experiment: Week 2

Implement your cascade control design(s). Does your cascade control design give better performance than your controllers from week 1? How do they compare? Your report should include comparisons of all your designed controllers with plots of step responses (both experimental and simulated), as well as Bode, Nyquist, or root locus plots used to arrive at the various designs.