University of California, Santa Barbara

Department of Electrical & Computer Engineering

ECE 147b: Digital Control

Lab 4: Balancing the Seesaw

Overview

In this lab you will use the tools developed in the class to design and implement a stabilizing controller for the seesaw system shown in Figure 1.

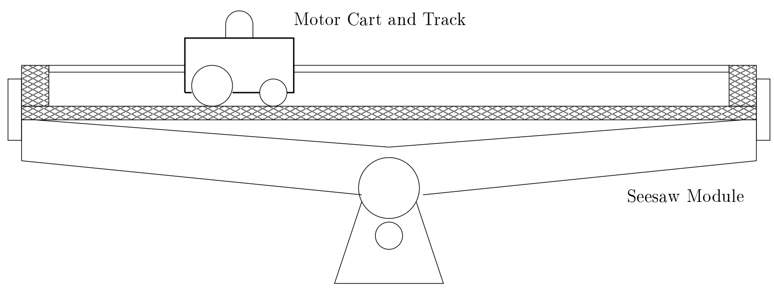

Figure 1: Seesaw System

Modeling

To develop the model of the seesaw system, we will use Euler-Lagrange Mechanics. Any good dynamics book should include a treatment of the Euler-Lagrange methodology.

First, we define the kinetic and potential energies for the two elements comprising our system (the motor cart and the seesaw). The potential and kinetic energies for the cart are (respectively):

$$\begin{aligned} U_c &= m_c g \left(h\cos(\theta) - p\sin(\theta) \right) \\ K_c &= \frac{1}{2} m_c \left((\dot{p} + h\dot{\theta})^2 + (p\dot{\theta})^2 \right) \ . \end{aligned}$$

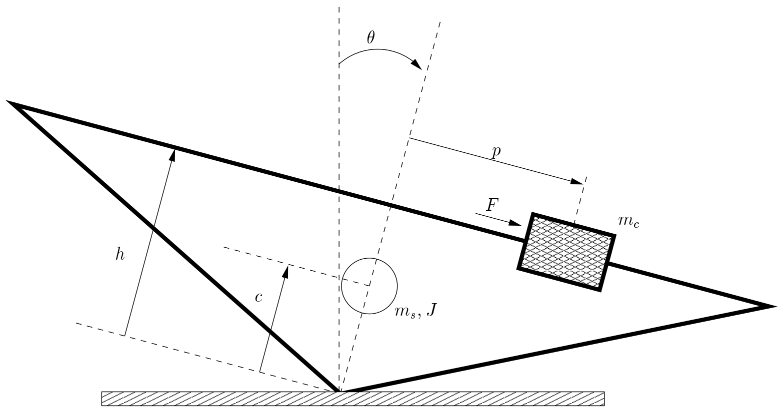

Figure 2: Seesaw System Diagram used for Euler-Lagrange Mechanics

The potential and kinetic energies for the seesaw are:

$$\begin{aligned} U_s &= m_s g c \cos(\theta) \\ K_s &= \frac{1}{2} J (\dot{\theta})^2 \end{aligned}$$

The Lagrangian is defined as the difference between the kinetic energy and the potential energy of the overall system. That is,

$$L = K - U = K_c + K_s - U_c - U_s \ . \tag{1}$$

The equations of motion for the system are then obtained from

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{p}} - \frac{\partial L}{\partial p} = F \tag{2}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \ . \tag{3}$$

Applying Equations 2 and 3 to Equation 1, we obtain

$$F = m_c \ddot{p} + m_c h \ddot{\theta} - m_c p (\dot{\theta})^2 - m_c g \sin(\theta) \tag{4}$$

$$\begin{aligned} 0 = {} & \ddot{\theta}(m_c h^2 + m_c p^2 + J) + m_c h \ddot{p} + 2 m_c p \dot{p} \dot{\theta} - m_c g h \sin(\theta) \\ & - m_c g p \cos(\theta) - m_s g c \sin(\theta) \ . \end{aligned} \tag{5}$$

Substituting Equation 5 into Equation 4 to eliminate $\ddot{\theta}$ and substituting Equation 4 into

Equation 5 to eliminate $\ddot{p}$ yields

$$\begin{aligned}\ddot{p} &= \frac{m_c h^2 + m_c p^2 + J}{(m_c p)^2 + m_c J}\left(F + m_c p(\dot{\theta})^2 + m_c g\sin(\theta)\right) \\ &\quad - \frac{m_c h}{(m_c p)^2 + m_c J}\left(m_s g c\sin(\theta) + m_c g p\cos(\theta) + m_c g h\sin(\theta) - 2m_c p\dot{p}\dot{\theta}\right) \qquad (6)\\ \ddot{\theta} &= \frac{1}{m_c p^2 + J}\left(m_s g c\sin(\theta) + m_c g p\cos(\theta) + m_c g h\sin(\theta) - 2m_c p\dot{p}\dot{\theta}\right) \\ &\quad - \frac{h}{m_c p^2 + J}\left(F + m_c p(\dot{\theta})^2 + m_c g\sin(\theta)\right)\ . \qquad (7)\end{aligned}$$

Equations 6 and 7 represent the full nonlinear model of the seesaw system.

Recall the relation between input voltage and force is

$$F = \frac{K_m K_g}{Rr}V - \frac{K_m^2 K_g^2}{Rr^2}\dot{p}\ . \qquad (8)$$

Equations 6, 7, and 8, coupled with the system parameters given in Table 1, are sufficient for you to design a stabilizing controller.

Parameter	**Symbol**	**Value**	**Units**
Motor Torque Constant	K_m	0.00767	$\frac{Nm}{Amp}$
Gearbox Ratio	K_g	3.7	N/A
Motor Armature Resistance	R	2.6	Ω
Motor Pinion Radius	r	0.00635	m
Cart Mass	m_c	0.455	kg
Seesaw and Track Mass	m_s	3.3	kg
Height of Track	h	0.14	m
Center of Gravity	c	0.058	m
Seesaw and Track Inertia	J	0.42	kg m^2

Table 1: System Parameters for Seesaw System

Prelab

Design a stabilizing controller for the seesaw system. You should treat this lab as you would a real world project. Your prelab should bear a strong resemblance to a project proposal (in other words, a typewritten prelab is highly recommended) and should include every element of your design. Be sure to discuss why you selected your particular design and to include complete simulations demonstrating that your controller works.

Experiment

Implement your designed controller. As mentioned in the prelab, treat this lab as a real world project. Your lab report must be thorough. Good luck.