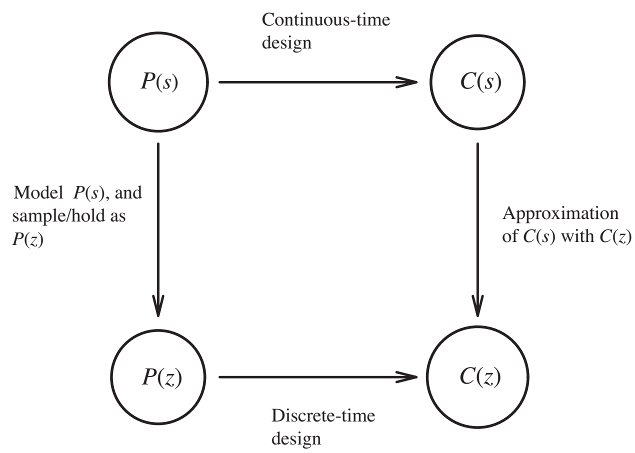
Calculating $C(z)$ to control $P(z)$



- ZOH equivalence gives $P(z)$.
- Design $C(z)$ for good closed-loop control of $P(z)$

Design methods for $P(z)$

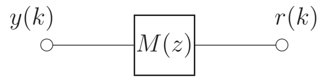
- Pole placement for $\dfrac{1}{1+P(z)C(z)}$
- Frequency response based designs (loopshaping)
- Root locus
- Tuning PID controllers

The above methods are analogous to the continuous time methods (except for the different interpretation between s-plane and z-plane).

The following does not have a continous time analogue.

- Finite settling time (deadbeat) control.

Ideal closed-loop response:



Consider the best possible closed-loop response:

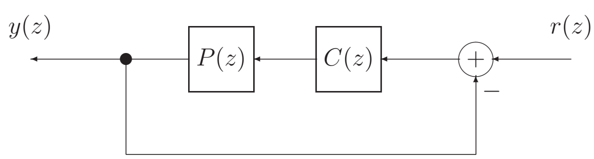
$$M(z) = z^{-1}$$

A step input, $r(z) = \dfrac{z}{z-1}$

Gives $y(z) = \dfrac{1}{z-1}$ (a delayed step output).

The error $(e(z) = r(z) - y(z))$ is simply a unit pulse: $e(z) = 1$.

Approach:



The closed-loop response is: $M(z) = \dfrac{P(z)C(z)}{1 + P(z)C(z)}$.

So:

- Choose $M(z)$, the desired closed-loop response.
- Solve the above to get $C(z)$.

Approach:

$M(z) = \dfrac{P(z)C(z)}{1 + P(z)C(z)}.$

So,

$$\begin{aligned} P(z)C(z) &= M(z) + M(z)P(z)C(z) \\ C(z)P(z)(1 - M(z)) &= M(z) \end{aligned}$$

Which gives,

$$C(z) = \frac{1}{P(z)} \frac{M(z)}{(1 - M(z))}$$

Ideal case: $M(z) = z^{-1}$

$$C(z) = \frac{1}{P(z)} \frac{z^{-1}}{(1 - z^{-1})}$$

Why can't we do this for continuous time systems ?

Example:

$$P(s) = \frac{1}{s^2} \quad \text{A double integrator: e.g. satellite force to position response}$$

ZOH equivalent

$$\begin{aligned} P(z) &= (1 - z^{-1})\mathcal{Z}\left\{\frac{P(s)}{s}\right\} \\ &= (1 - z^{-1})\mathcal{Z}\left\{\frac{1}{s^3}\right\} \\ &= (1 - z^{-1})\frac{T^2 z^{-1}(1 + z^{-1})}{2(1 - z^{-1})^3} \\ &= \frac{T^2}{2} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^2} \end{aligned}$$

Example:

$$P(s) = \frac{1}{s^2} \quad \text{A double integrator: e.g. satellite force to position response}$$

Ideal response: $M(z) = z^{-1}$

$$\begin{aligned} C(z) &= \frac{1}{P(z)} \frac{z^{-1}}{(1-z^{-1})} \quad = \quad \frac{2}{T^2} \frac{(1-z^{-1})^2}{z^{-1}(1+z^{-1})} \frac{z^{-1}}{(1-z^{-1})} \\ &= \frac{2}{T^2} \frac{(1-z^{-1})}{(1+z^{-1})} \quad = \quad \frac{2}{T^2} \frac{(z-1)}{(z+1)} \end{aligned}$$

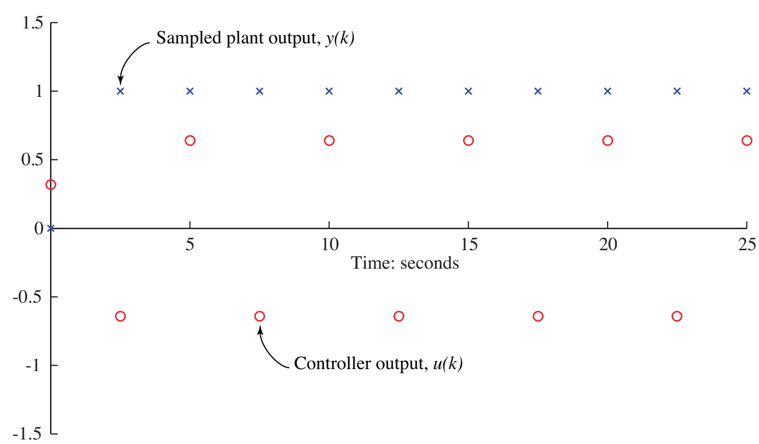
In the time domain:

$$\begin{aligned} u(z) &= \frac{2}{T^2} \frac{(1-z^{-1})}{(1+z^{-1})} e(z) \\ u(z) T^2 (1+z^{-1}) &= 2(1-z^{-1}) e(z) \\ T^2 u(k) + T^2 u(k-1) &= 2e(k) - 2e(k-1) \\ u(k) &= -u(k-1) + \frac{2}{T^2} e(k) - \frac{2}{T^2} e(k-1) \end{aligned}$$

This is causal.

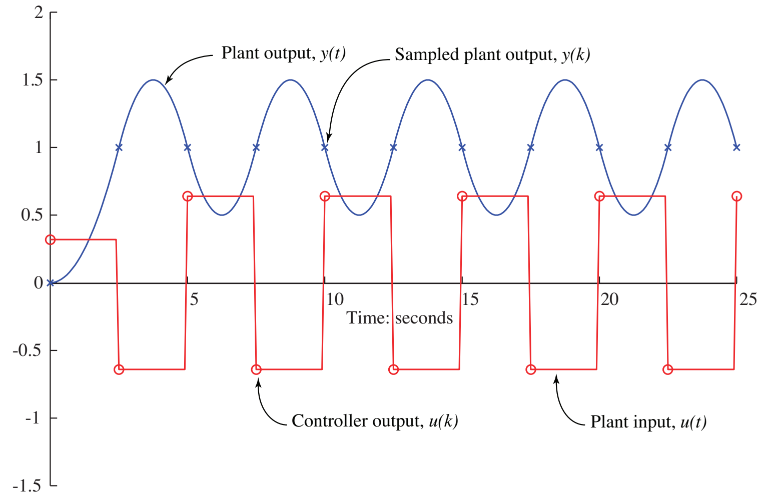
Deadbeat control example:

$$P(s) = \frac{1}{s^2} \qquad \text{desired response:} \quad M(z) = z^{-1}$$



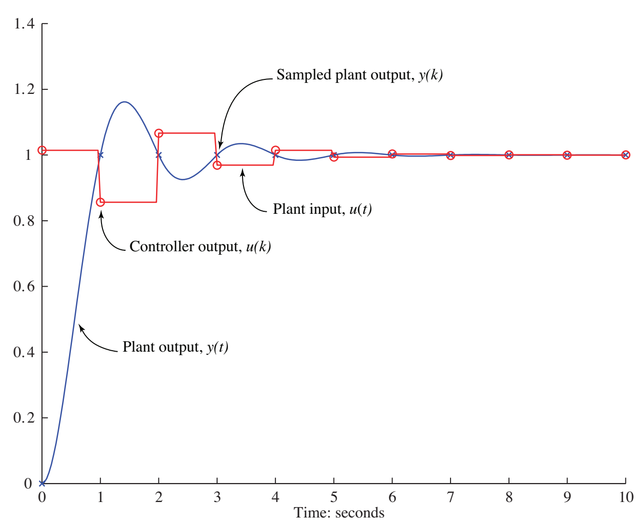
Intersample behavior

$$P(s) = \frac{1}{s^2} \qquad \text{desired response:} \quad M(z) = z^{-1}$$



Sample rate selection

$$P(s) = \frac{5}{(s^2 + 2s + 5)}, \qquad \text{desired response:} \quad M(z) = z^{-1}$$

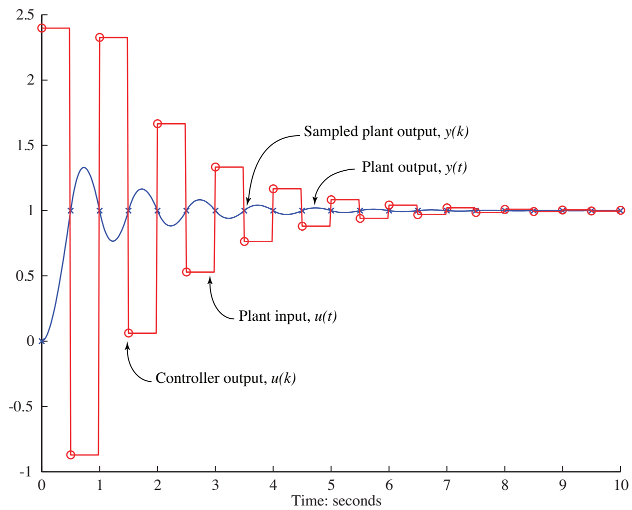


Sample period:

$T = 1.0$ seconds

Sample rate selection

$$P(s) = \frac{5}{(s^2 + 2s + 5)}, \qquad \text{desired response:} \quad M(z) = z^{-1}$$

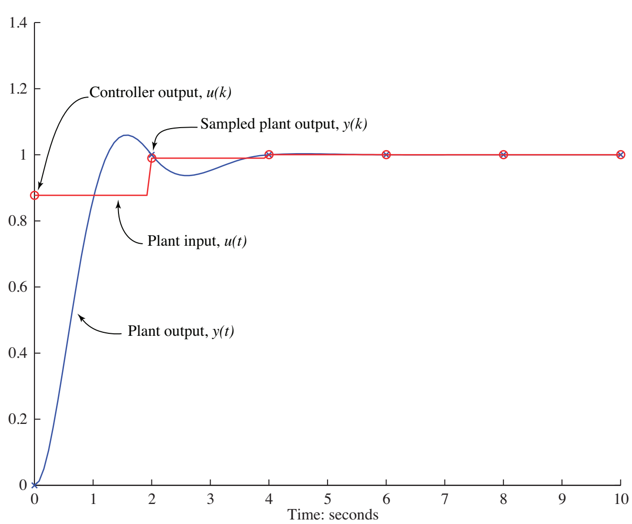


Sample period:

$T = 0.5$ seconds

Sample rate selection

$$P(s) = \frac{5}{(s^2 + 2s + 5)}, \qquad \text{desired response:} \quad M(z) = z^{-1}$$



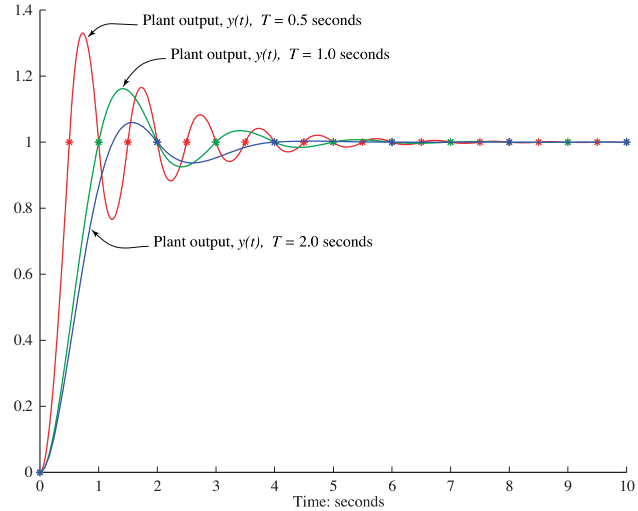
Sample period:

$T = 2.0$ seconds

Sample rate selection comparison

$$P(s) = \frac{5}{(s^2 + 2s + 5)}, \qquad \text{desired response:} \quad M(z) = z^{-1}$$

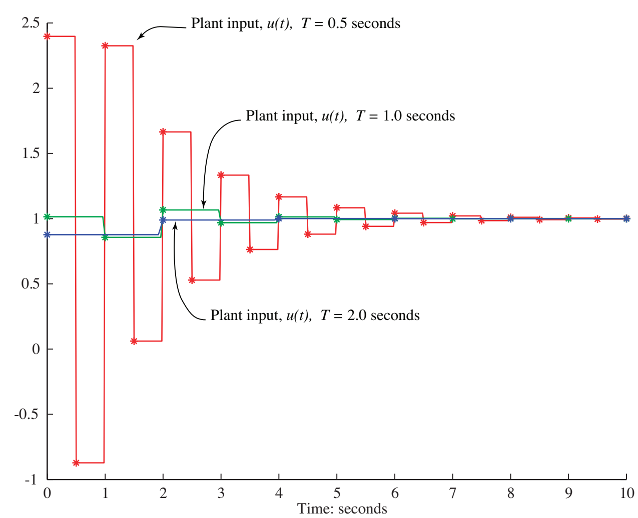
Plant output:



Sample rate selection comparison

$$P(s) = \frac{5}{(s^2 + 2s + 5)}, \qquad \text{desired response:} \quad M(z) = z^{-1}$$

Plant input:



Potential problems:

1. An aggressive design can lead to poor intersample behavior.
2. Making the sampling faster, without modifying the closed-loop requirement (e.g. $M(z) = z^{-1}$), makes the design more aggressive.
3. The calculated controller may not be causal if the specified response, $M(z)$, is "faster" than the system can respond.
4. $P(z)C(z)$ could have an unstable pole/zero cancellation. Note that

   $$C(z) = \frac{1}{P(z)} \frac{M(z)}{(1 - M(z))},$$

   and if $P(z)$ has a zero outside the unit circle, the $C(z)$ will have a pole outside the unit circle.

   Be aware that the zeros of $P(z)$ will change as a the sampling period changes.