## Counter Design

Design a 3-bit counter that counts in binary or in Gray code, depending on the values of a "mode control input". This synchronous 3 -bit counter has a mode control input m . As long as m $=0$, the counter steps through the binary sequence $000,001,010,011,100,101,110,111$ and repeats this sequence. As long as $m=1$, the counter advances through the Gray code sequence $000,001,011,010,110,111,101,100$ and repeats this sequence. However, it is possible that the mode input changes at any time in any of these sequences.

For example, assume that the mode input is 0 for the first two clock cycles but changes to 1 in the third clock cycle and stays at 1 in the fourth clock cycle. (That is, $m$ goes through the sequence $0,0,1,1$.) Then, the output will be $000,001,010,110,111$. In this example, 000 is the initial state. The first two state transitions ( $000->001->010$ ) occurs in the binary counting mode. Then, because $m=1$ from the third clock cycle onwards, the state goes from $010->110->->111$, as indicated in the Gray code sequence.

In addition to the mode control input, there is a reset input. This synchronous counter should go to the 000 state if asserted.
a, State the inputs and outputs:

```
input m, reset, clock;
output [2:0] count;
reg [2:0] state;
```

b, Implement the counter as a Moore machine and draw the state diagram

c, Implement this counter as a Verilog module.

```
module GrayCounter(Clock, m, reset, count);
    input Clock, m, reset;
    output [2:0] count;
    reg [2:0] state;
    parameter S0 = 3'b000;
    parameter S1 = 3'b001;
    parameter S2 = 3'b010;
    parameter S3 = 3'b011;
    parameter S4 = 3'b100;
    parameter S5 = 3'b101;
    parameter S6 = 3'b110;
    parameter S7 = 3'b111;
    assign count = state;
    always @ (posedge Clock)
        if (reset)
            state<=S0;
        else
        begin
            case (state)
                            S0 : state <= S1;
                            S1 : state <= m? S3:S2;
                            S2 : state <= m? S6:S3;
                            S3 : state <= m? S2:S4;
                            S4 : state <= m? S0:S5;
                            S5 : state <= m? S4:S6;
                    S6 : state <= S7;
                    S7 : state <= m? S5:S0;
            endcase
        end
endmodule
```


## State Reduction

Reduce the following state table using an implication chart.

| PS | NS, S |  |
| :---: | :---: | :---: |
|  | $\mathrm{x}=0$ | $\mathrm{x}=1$ |
| A | F, 0 | $\mathrm{B}, 1$ |
| B | $\mathrm{G}, 0$ | $\mathrm{~A}, 1$ |
| C | B, 0 | C, 1 |
| D | C, 0 | B, 1 |
| E | D, 0 | A, 1 |
| F | E, 1 | F, 1 |
| G | E, 1 | G, 1 |

Pass I:

| B | F-G |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | B-F | B-G | B-C |  |  |  |
|  | C-B | A-C |  |  |  |  |
| D | C-F | C-G |  |  |  |  |
|  |  | A-B |  |  |  |  |
| E | D-F | D-G | B-D | C-D |  |  |
|  | A-B |  | A-C | A-B |  |  |
| F | X | X | X | X | X |  |
| G | X | X | X | X | X | $\checkmark$ |
|  | A | B | C | D | E | F |

Elimination of differing outputs: AF, AG, BF, BG, CF, CG, DF, DG, EF, EG.
Equivalent: FG

## Pass II:

| B | F-G |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | X | X |  |  |  |  |
| D | X | X | X |  |  |  |
| E | X | X | X | X |  |  |
| F | X | X | X | X | X |  |
| G | X | X | X | X | X | $\checkmark$ |
|  | A | B | C | D | E | F |

Elimination: $\mathrm{AC}, \mathrm{AD}, \mathrm{AE}, \mathrm{BC}, \mathrm{BD}, \mathrm{BE}, \mathrm{CD}, \mathrm{CE}$.
AB are equivalent.
Reduced to 5 states: AB, C, D, E, FG.

Write the Verilog code for a 4-bit up counter.

```
module fourbit_upcount(Reset, Clock, E, Q);
    input Reset, Clock, E;
    output [3:0] Q;
    reg [3:0] Q;
    always @ (posedge Clock)
        if (!Reset)
            Q <= 0;
            else if (E)
                Q <= Q+1;
endmodule
```

