ECE 158 Fall Quarter 2007
Gibson ***Due: 11/19/07***

Homework Assignment #5 Solutions

Problems are from the text:

1. Show that for a real sequence, the conjugate symmetric part is a real sequence and that the conjugate antisymmetric part is also a real sequence. See page 251 of the text.

2. Problem 5.61 (c)

3. Problem 5.62 (a)

Solutions:

1.

Problem # 1:

conjugate symmetric part:

$$x_{cs}[n] = \frac{1}{2}\left(x[n] + x^*[\langle -n \rangle_N]\right) \qquad : 0 \le n \le N-1$$

conjugate anti-symmetric part:

$$x_{ca}[n] = \frac{1}{2}\left(x[n] - x^*[\langle -n \rangle_N]\right) \qquad : 0 \le n \le N-1$$

$x[n]$ real : $x^*[\langle -n \rangle_N] = x[\langle -n \rangle_N]$

$$x_{cs}[n] = \frac{1}{2}\left(x[n] + x[\langle -n \rangle_N]\right) = \text{real} + \text{real} = \text{real}$$

$$x_{ca}[n] = \frac{1}{2}\left(x[n] - x[\langle -n \rangle_N]\right) = \text{real} - \text{real} = \text{real} \;//$$

2.

(c) Note that $\sum_{n=0}^{N-1}\cos\left(\frac{\pi k(2n+1)}{2N}\right)\cos\left(\frac{\pi m(2n+1)}{2N}\right)=\begin{cases} N, & \text{if } k=m=0, \\ N/2, & \text{if } k=m \text{ and } k\neq 0, \\ 0, & \text{otherwise.}\end{cases}$ Now,

$$g[n]g^*[n]=\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}\alpha[k]\alpha[m]G_{\text{DCT}}[k]G_{\text{DCT}}^*[m]\cos\left(\frac{\pi(2n+1)k}{2N}\right)\cos\left(\frac{\pi(2n+1)k}{2N}\right).$$

Thus,

$$\sum_{n=0}^{N-1}|g[n]|^2=\frac{1}{N^2}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}\alpha[k]\alpha[m]G_{\text{DCT}}[k]G_{\text{DCT}}^*[m]\cos\left(\frac{\pi(2n+1)k}{2N}\right)\cos\left(\frac{\pi(2n+1)k}{2N}\right).$$

Using the orthogonality property mentioned earlier we get

$$\sum_{n=0}^{N-1}|g[n]|^2=\frac{1}{2N}\sum_{k=0}^{N-1}\alpha[k]\left|G_{\text{DCT}}[k]\right|^2.$$

3.

5.62 (a) $\mathbf{H}_N=\begin{bmatrix}13 & 13 & 13 & 13\\ 17 & 7 & -7 & -17\\ 13 & -13 & -13 & 13\\ 7 & -17 & 17 & -7\end{bmatrix}$. The matrix $\mathbf{H}_N$ is orthogonal if $\mathbf{H}_N\mathbf{H}_N^T=c\mathbf{I}$

where $\mathbf{I}$ is the 4×4 identity matrix and c is a constant. Now,

$$\mathbf{H}_N\mathbf{H}_N^T=\begin{bmatrix}13 & 13 & 13 & 13\\ 17 & 7 & -7 & -17\\ 13 & -13 & -13 & 13\\ 7 & -17 & 17 & -7\end{bmatrix}\begin{bmatrix}13 & 17 & 13 & 7\\ 13 & 7 & -13 & -17\\ 13 & -7 & -13 & 17\\ 13 & -17 & 13 & -7\end{bmatrix}=\begin{bmatrix}676 & 0 & 0 & 0\\ 0 & 676 & 0 & 0\\ 0 & 0 & 676 & 0\\ 0 & 0 & 0 & 676\end{bmatrix}.$$

Hence, the matrix is orthogonal and all its rows have the same $\mathcal{L}_2$-norm.