ECE 158 Final Solutions

Gibson, Fall 2008

1. $$X[2l] = \sum_{n=0}^{N-1} x[n]\, W_N^{2ln} \qquad l = 0, 1, \ldots \tfrac{N}{2}-1$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n]\, W_N^{2ln} + \sum_{n=\frac{N}{2}}^{N-1} x[n]\, W_N^{2ln}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n]\, e^{-j\frac{2\pi}{N}\cdot 2ln} + \sum_{n=\frac{N}{2}}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}\cdot 2ln}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n]\, e^{\frac{-j2\pi ln}{(\frac{N}{2})}} + \sum_{m=0}^{\frac{N}{2}-1} x[m+\tfrac{N}{2}]\, e^{-\frac{j2\pi}{(\frac{N}{2})} l(m+\frac{N}{2})}$$

$$(m = n - \tfrac{N}{2} \Rightarrow n = m + \tfrac{N}{2})$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n]\, e^{\frac{-j2\pi ln}{(\frac{N}{2})}} + \sum_{m=0}^{\frac{N}{2}-1} x[m+\tfrac{N}{2}]\, e^{\frac{j2\pi lm}{(\frac{N}{2})}}\, \overset{1}{\cancel{e^{-j2\pi l(\frac{N}{2}/\frac{N}{2})}}}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} (x[n] + x[n+\tfrac{N}{2}])\, e^{\frac{-j2\pi ln}{(\frac{N}{2})}} = \sum_{n=0}^{\frac{N}{2}-1} (x[n] + x[n+\tfrac{N}{2}])\, W_{\frac{N}{2}}^{ln} \quad ✓$$

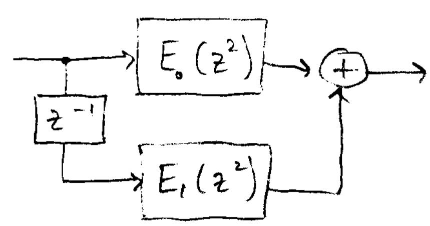
2. $$H(z) = 1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4}$$

$$H(z) = \sum_{m=0}^{L-1} z^{-m} E_m(z^L) \qquad E_m(z) = \sum_{n=0}^{\lfloor (N+1)/L \rfloor} h[Ln+m]\, z^{-n}, \quad 0 \le m \le L-1$$

$L=2$ $$H(z) = E_0(z^2) + z^{-1} E_1(z^2)$$

$$E_0(z) = 1 + 6z^{-1} + z^{-2}$$

$$E_1(z) = 4 + 4z^{-1}$$



A reduction in computational complexity is possible by realizing the FIR filters using the polyphase decomposition, particularly when doing sample rate conversion.

3.

$x[n] \rightarrow \boxed{\uparrow L} \xrightarrow{v_1[n]} \boxed{\downarrow M} \xrightarrow{y_1[n]}$ $\qquad$ $x[n] \rightarrow \boxed{\downarrow M} \xrightarrow{v_2[n]} \boxed{\uparrow L} \xrightarrow{y_2[n]}$

$$V_1(z) = X(z^L)$$

$$Y_1(z) = \frac{1}{M}\sum_{k=0}^{M-1} V_1(z^{1/M} W_M^{-k})$$

$$V_2(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{1/M} W_M^{-k})$$

$$Y_2(z) = V_2(z^L)$$

$$Y_1(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{L/M} W_M^{-kL})$$

$$Y_2(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{L/M} W_M^{-k})$$

$Y_1(z) = Y_2(z)$ iff M and L are <u>relatively</u> <u>prime</u>.

4. Condition for perfect reconstruction:

$$H_0(z)\,G_0(z) + H_1(z)\,G_1(z) = 2z^{-\ell}$$

$H_0(z) = 3z^{-2}$ $\quad H_1(z) = 2$ $\quad G_0(z) = 1.5z^{-1}$ $\quad G_1(z) = 4z^{-1}$

$$(3z^{-2})(1.5z^{-1}) + (2)(4z^{-1}) = 4.5z^{-3} + 8z^{-1} \neq 2z^{-\ell}$$

hence it is <u>not</u> a perfect reconstruction.

5. $$Y[n] = \sum_{i=1}^{4} a_i Y[n-i] + G X[n]$$

$$= 1.793\, Y[n-1] - 1.401\, Y[n-2] + .566\, Y[n-3] - .147\, Y[n-4] + G X[n]$$

PARCOR Coeff:

$$P_4 = a_4^{(4)} = -.147$$

$$a_1^{(3)} = \frac{a_1^{(4)} + a_4^{(4)} a_3^{(4)}}{1 - P_4^2} = 1.748$$

$$a_2^{(3)} = \frac{a_2^{(4)} + a_4^{(4)} a_2^{(4)}}{1 - P_4^2} = -1.221$$

$$a_3^{(3)} = \frac{a_3^{(4)} + a_4^{(4)} a_1^{(4)}}{1 - P_4^2} = .309 = P_3$$

$$a_1^{(2)} = \frac{a_1^{(3)} + a_3^{(3)} a_2^{(3)}}{1 - P_3^2} = 1.515$$

$$a_2^{(2)} = \frac{a_2^{(3)} + a_3^{(3)} a_1^{(3)}}{1 - P_2^2} = -.753 = P_2$$

$$a_1^{(1)} = \frac{a_1^{(2)} + a_2^{(2)} a_1^{(2)}}{1 - P_2^2} = .864 = P_1$$

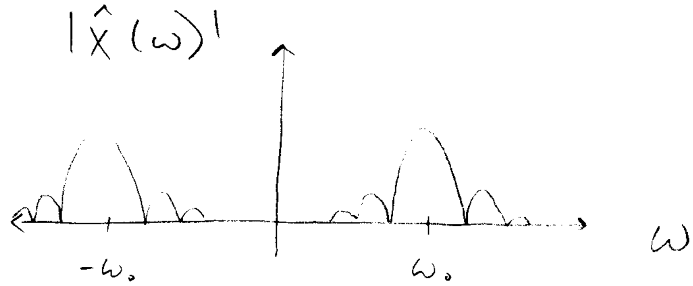
The filter is <u>stable</u> since $|P_i| < 1$ for $i = 1, 2, 3$ and 4

6. a) The FFT is used in MP3 for perceptual modeling. The DCT is used for compact representation and coding. Windowing is also used to process the signals.

b) Linear predictive coding (LPC) is used to encode and decode speech signals for very low data rate transmission on cell phones.

7. $$\hat{x}[n] = \begin{cases} \cos \omega_o n, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$$

$x[n]$ is multiplied by a rectangular window in the time domain. If the length of the window, L, does not capture an integer number of periods of $\cos \omega_o n$, then this will result in two sinc functions in the DFT, as multiplication by a rect in the time domain is convolution by a sinc in the frequency domain. This effect is known as spectral leakage.



8. H is a rectangular pulse, which means a sinc in the frequency domain $\rightarrow$ $\boxed{H2}$

G is H multiplied by 1, -1, 1, -1 ..., or the highest frequency possible in the sampled signal $\left(\frac{F_s}{2}\right)$. Hence in the frequency domain there will be a sinc shifted by π $\rightarrow$ $\boxed{G5}$.

F is a decaying exponential, or a tapered rectangular pulse, so its frequency content will look like a sinc, but with smoother roll off $\rightarrow$ $\boxed{F7}$

D is again F modulated by π $\rightarrow$ $\boxed{D3}$

E appears periodic, like a sine wave. It is windowed by a rect, which means a sinc shifted by the frequency of the wave $\rightarrow$ $\boxed{E1}$

A is similar to E but windowed by a Hann window. There will be a peak at the same frequency in 1, but with a wider mainlobe and attenuated sidelobes $\rightarrow$ $\boxed{A8}$

B and C appear random. C is B windowed by a Hann. Since a rect has the narrowest mainlobe $\rightarrow$ $\boxed{B6}$ and $\boxed{C4}$