ECE 158 Fall Quarter 2007
Gibson **12/15//07**

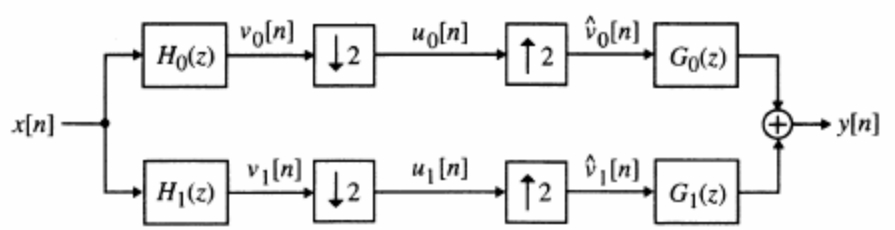
Final Exam

Instructions: Do all problems. Show all work. Problems are weighted as shown.

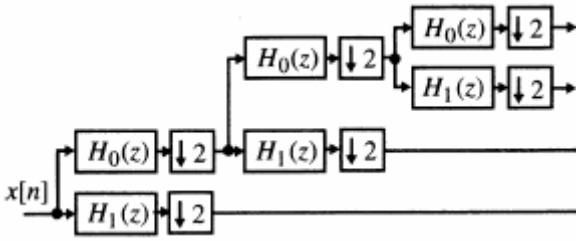
1. (10) Show that **(a)** $W_N^{kN/4} = (-j)^k$ **(b)** $W_N^{kN/2} = (-1)^k$ **(c)** $W_N^{3kN/4} = (j)^k$

2. (15) Given the N point DFT $X(k) = \sum_{n=0}^{N-1} x[n]W_N^{kn}$, break it into four $N/4$ point DFT's in preparation for a radix-4 decimation-in-frequency FFT algorithm. Simplify the expressions as much as you can.

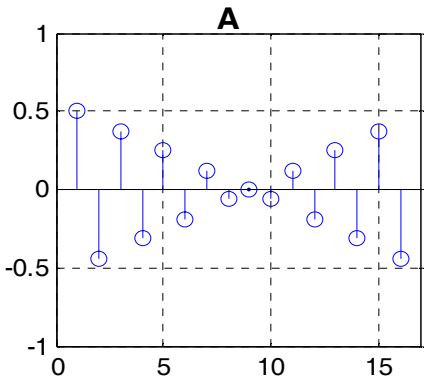
3. (15) For the two-channel QMF filter bank below,
(a) If the analysis filters are $H_0(z) = 3 + 4z^{-1}$ and $H_1(z) = 1 + 2z^{-1}$, find the synthesis filters that achieve perfect reconstruction.
(b) Show that your filters from (a) satisfy the perfect reconstruction and aliasing cancellation properties.

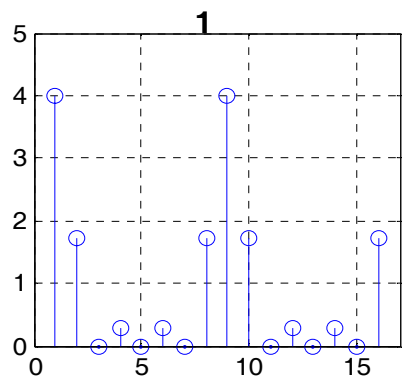


4. (20) If $H_0(z)$ and $H_1(z)$ are as given in Problem 4 (a),
(a) Find the transfer function from the input to the output for each branch in the filter bank below.
(b) What are the advantages and disadvantages of the given filter bank structure?



5. (20) In the figures below, match each time domain signal on the left to its corresponding DFT magnitude on the right by specifying the letter A through H that corresponds to numbers 1 through 8. Explain your reasoning for full credit.


A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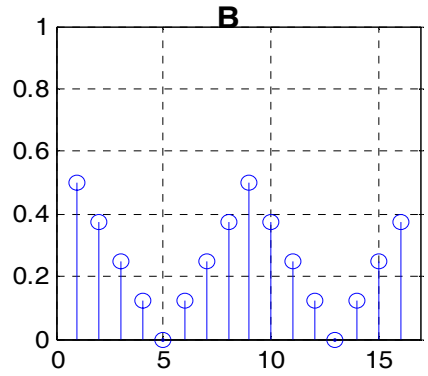
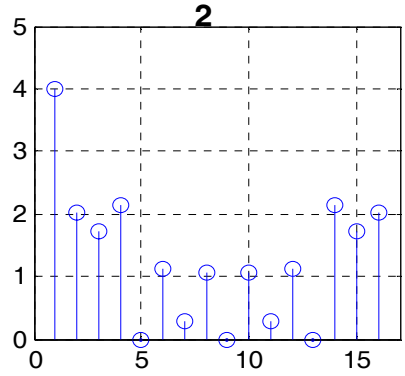

B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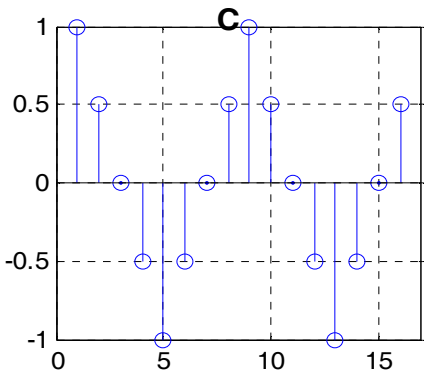
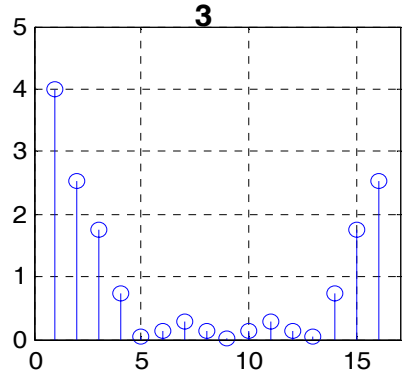

C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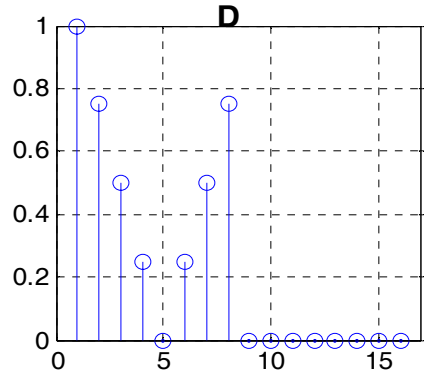
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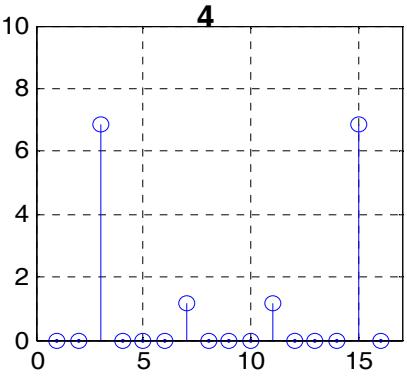
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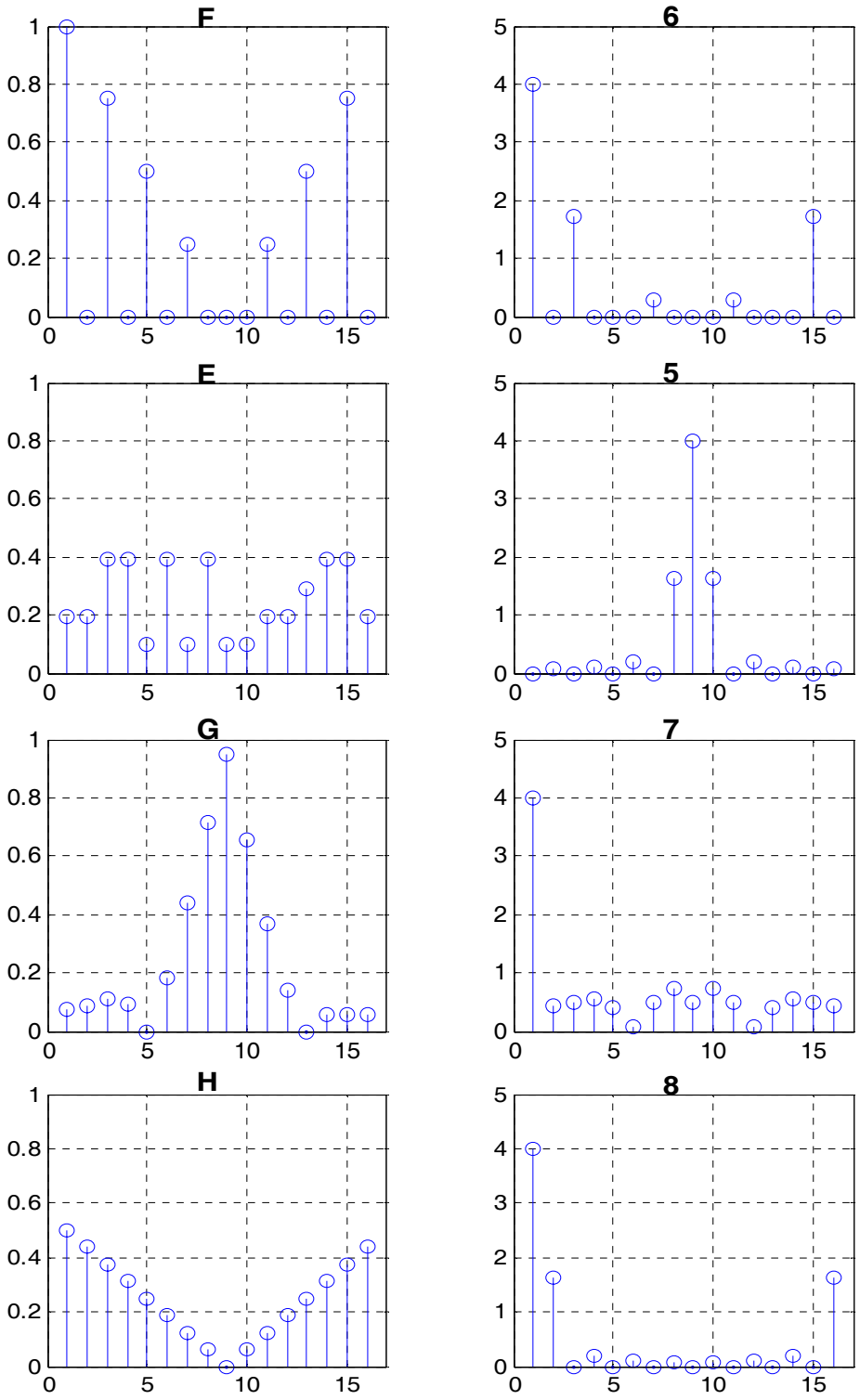

6. (20) You have a signal of $x[n] = \sin(2\pi n/100)$ $n = 0,1,2,...,200$. You get the first 64 samples of the signal and take the 64 point fft (You can only use size K ffts, where $K = 2^m$, m integer). You want to see the two impulses in the frequency domain.

a) What will your fft look like? Two impulses? Why?

b) One friend of yours suggested to window the 64 point signal with the 64 point Hamming window to see the impulses. Will it work? Why?

c) Another friend of yours suggested doubling the samples you take. Will it work? Why?

d) What can you do to see the two impulses?

e) What is the technical name of the effect in part a). (Hint: covered in one of the labs)