

Final Exam

Instructions: Do all problems. Show all work. Problems are weighted as shown.

1. (14) Given the expression for the even indexed samples of the DFT, namely,

$$X[2l] = \sum_{n=0}^{N-1} x[n]W_N^{2ln}, l = 0, 1, \dots, \frac{N}{2} - 1., \text{ show that this can be rewritten as an } N/2\text{-point}$$

DFT as

$$X[2l] = \sum_{n=0}^{(N/2)-1} (x[n] + x[n + \frac{N}{2}])W_{N/2}^{ln}, l = 0, 1, \dots, \frac{N}{2} - 1.$$

2. (10) Given the filter $H(z) = 1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4}$, find a two branch polyphase decomposition and draw the block diagram. What are some advantages of polyphase filters?

3. (10) What is the condition for a factor-of- L up-sampler followed by a factor-of- M down-sampler to be interchangeable with a factor-of- M down-sampler followed by a factor-of- L up-sampler? Find the output of each in terms of the input and compare.

4. (10) A two-channel filter bank has the filters

$$H_0(z) = 3z^{-2}, H_1(z) = 2, G_0(z) = 1.5z^{-1}, G_1(z) = 4z^{-1}. \text{ Is this a perfect reconstruction filter bank?}$$

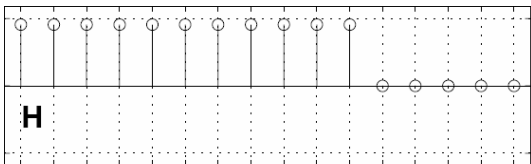
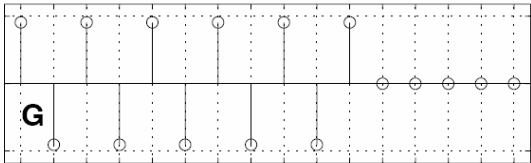
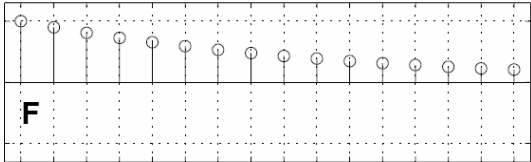
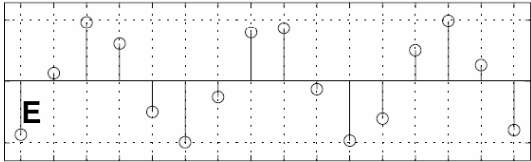
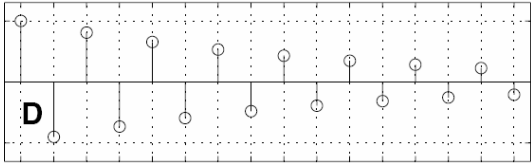
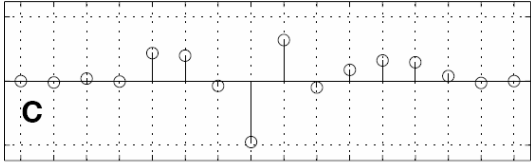
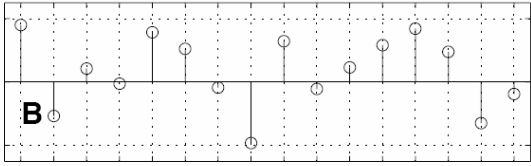
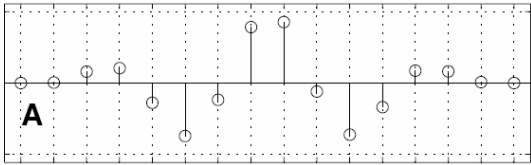
5. (20) An autoregressive model has the linear prediction coefficients given by $a_1 = 1.793, a_2 = -1.401, a_3 = 0.566, a_4 = -0.147$. Write a recursive expression for the filter output given the input is $x[n]$. Is this filter stable? Show why or why not.

6. (10) (a) What methods from the class are used in MP3 players? Be as specific as possible.

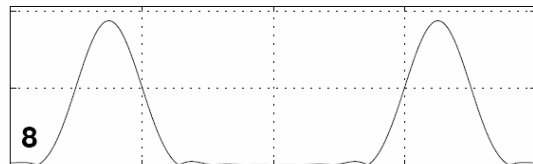
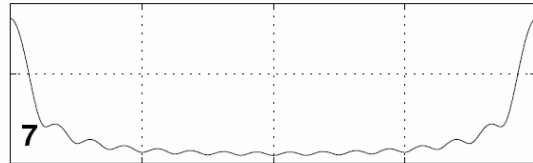
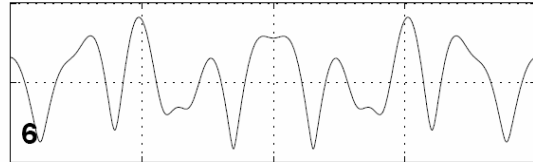
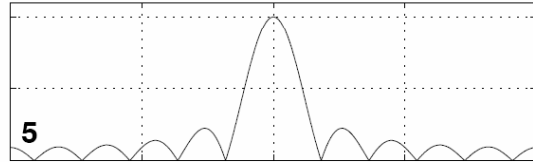
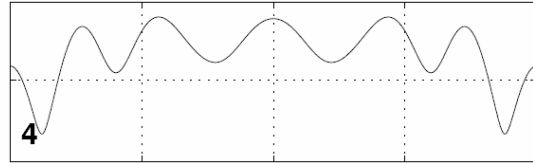
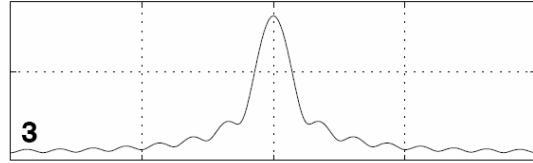
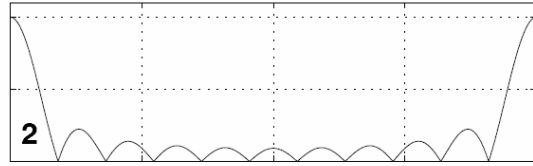
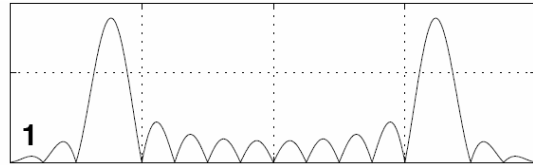
(b) What methods from the class are used in cell phones? Be as specific as possible.

7. (10) A very long sequence of samples $\{x[n]\}$ is multiplied by a window function $w[n] = 1, 0 \leq n \leq L - 1, w[n] = 0, \text{ otherwise}$. If $x[n] = \cos \omega_0 n$, write expressions for the product and describe what is happening to the input signal.

8. (16) In the figures below, match each time-domain signal on the left to its corresponding DTFT on the right by specifying the letter A through H that corresponds to the numbers 1 through 8.



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
n



0 pi 2pi
ω