ECE 158 Fall 2008
Gibson

Homework #2 Solutions

3.43 $x[n] = \alpha^n \mu[n], |\alpha| < 1.$ From Table 3.3, $\mathcal{F}\{x[n]\} = X(e^{j\omega}) = \frac{1}{1-\alpha e^{j\omega}}$. The total energy of $x[n]$ is $\mathcal{E}_x = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{1}{1-\alpha e^{-j\omega}}\right|^2 d\omega = \sum_{n=0}^{\infty}(\alpha^2)^n = \frac{1}{1-\alpha^2}\bigg|_{\alpha=1/2} = \frac{4}{3}$. To determine the 80% bandwidth of the signal, we set $\mathcal{E}_{x,80} = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\left|\frac{1}{1-\alpha e^{-j\omega}}\right|^2 d\omega = 0.8\,\mathcal{E}_x = 0.8\cdot\frac{4}{3}$ and solve for ω_c, i.e., set $\mathcal{E}_{x,80} = \left[\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\frac{1}{(1-\alpha\cos\omega)^2+\alpha^2\sin^2\omega}d\omega\right]_{\alpha=1/2} = \frac{3.2}{3}$.
A numerical solution of the above equation yields $\omega_c = 0.5081\pi$.

5.8 **(a)** $x_a[n] = \sin(2\pi n/N) = \frac{1}{2j}(e^{j2\pi n/N} - e^{-j2\pi n/N})$. Therefore,

$$X_a[k] = \frac{1}{2j}\sum_{n=0}^{N-1} e^{j2\pi n/N}e^{-j2\pi kn/N} - \frac{1}{2j}\sum_{n=0}^{N-1} e^{-j2\pi n/N}e^{-j2\pi kn/N}$$

$= \frac{1}{2j}\sum_{n=0}^{N-1} e^{-j2\pi(k-1)n/N} - \frac{1}{2j}\sum_{n=0}^{N-1} e^{-j2\pi(k+1)n/N}$. From Eq. (5.11), the first sum is equal to N when $k=1$ and 0 othereise. Likewise, from Eq. (5.11), the second sum is equal to N when $k = N-1$ and 0 otherwise. Therefore,

$$X_a[k] = \begin{cases} N/2j, & k = 1, \\ -N/2j, & k = N-1, \\ 0, & \text{otherwise.} \end{cases}$$

5.17 $Y[k] = \sum_{n=0}^{MN-1} y[n]W_{MN}^{nk} = \sum_{n=0}^{N-1} x[n]W_{MN}^{nk}$. Thus,

$$Y[kM] = \sum_{n=0}^{N-1} x[n]W_{MN}^{nkM} = \sum_{n=0}^{N-1} x[n]W_N^{nk} = X[k].$$

5.31 $\sum_{n=0}^{N-1} g[n]h^*[n] = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} G[k]W_N^{-nk}h^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} G[k]\sum_{n=0}^{N-1} h^*[n]W_N^{-nk}$

$= \frac{1}{N}\sum_{k=0}^{N-1} G[k]H^*[k].$

5.45 (a) $y_L[0] = g[0]h[0] = -4,$

$y_L[1] = g[0]h[1] + g[1]h[0] = 10,$

$y_L[2] = g[0]h[2] + g[1]h[1] + g[2]h[0] = -6,$

$y_L[3] = g[0]h[3] + g[1]h[2] + g[2]h[1] = 8,$

$y_L[4] = g[1]h[3] + g[2]h[2] = 7,$

$y_L[5] = g[2]h[3] = -3.$

5.50 (a)

$$g[n] = [2, -1, 3, 0]$$

$$h[n] = [-2, 4, 2, -1]$$

$$G[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1+j \\ 6 \\ -1-j \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} -2 \\ 4 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4-5j \\ -3 \\ -4+5j \end{bmatrix}$$

5.62

(b) $\mathbf{G}_N = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & -1 & 1 \\ 1 & -2 & 2 & -1 \end{bmatrix}$. Next, we observe

$\mathbf{G}_N\mathbf{G}_N^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & -1 & 1 \\ 1 & -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & -1 & -2 \\ 1 & -1 & -1 & -2 \\ 1 & -2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$ which shows that the rows of $\mathbf{G}_N$ are orthogonal but do not have the same $\mathcal{L}_2$-norms.