ECE 158 Fall 2008
Gibson

Homework #3 Solutions

5.25

(c) $X[\frac{N}{2}] = \sum_{n=0}^{N-1} x[n] W_N^{(N/2)n} = \sum_{n=0}^{N-1} (-1)^n x[n]$ which is real.

5.26 $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$.

(a) $X^*[k] = \sum_{n=0}^{N-1} x^*[n] W_N^{-nk}$. Replacing n by $N-n$ in the summation we obtain $X^*[k] = \sum_{n=0}^{N-1} x^*[N-n] W_N^{-(N-n)k} = \sum_{n=0}^{N-1} x^*[N-n] W_N^{nk}$. Thus, $\text{DFT}\{x^*[N-n]\} = \text{DFT}\{x^*[\langle -n \rangle_N\} = X^*[k]$.

5.27 Since for a real sequence, $x[n] = x^*[n]$, taking the DFT of both sides we get $X[k] = X^*[\langle -k \rangle_N]$. This implies $\text{Re}\{X[k]\} + j\,\text{Im}\{X[k]\} = \text{Re}\{X[\langle -k \rangle_N]\} - j\,\text{Im}\{X[\langle -k \rangle_N]\}$.
Comparing real and imaginary parts we get $\text{Re}\{X[k]\} = \text{Re}\{X[\langle -k \rangle_N]\}$ and $\text{Im}\{X[k]\} = -\text{Im}\{X[\langle -k \rangle_N]\}$.

5.45 **(a)** $y_L[0] = g[0]h[0] = -4,$

$y_L[1] = g[0]h[1] + g[1]h[0] = 10,$

$y_L[2] = g[0]h[2] + g[1]h[1] + g[2]h[0] = -6,$

$y_L[3] = g[0]h[3] + g[1]h[2] + g[2]h[1] = 8,$

$y_L[4] = g[1]h[3] + g[2]h[2] = 7,$

$y_L[5] = g[2]h[3] = -3.$

(b)
$$\begin{aligned}
y_C[0] &= g_e[0]h[0] + g_e[1]h[3] + g_e[2]h[2] + g_e[3]h[1] \\
&= g[0]h[0] + g[1]h[3] + g[2]h[2] = 3, \\
y_C[1] &= g_e[0]h[1] + g_e[1]h[0] + g_e[2]h[3] + g_e[3]h[2] \\
&= g[0]h[1] + g[1]h[0] + g[2]h[3] = 7, \\
y_C[2] &= g_e[0]h[2] + g_e[1]h[1] + g_e[2]h[0] + g_e[3]h[3] \\
&= g[0]h[2] + g[1]h[1] + g[2]h[0] = -6, \\
y_C[3] &= g_e[0]h[3] + g_e[1]h[2] + g_e[2]h[1] + g_e[3]h[0] \\
&= g[0]h[3] + g[1]h[2] + g[2]h[1] = 8.
\end{aligned}$$

(c)
$$\begin{bmatrix} G_e[0] \\ G_e[1] \\ G_e[2] \\ G_e[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1+j \\ 6 \\ -1-j \end{bmatrix},$$

$$\begin{bmatrix} H[0] \\ H[1] \\ H[2] \\ H[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4-j5 \\ -3 \\ -4+j5 \end{bmatrix}.$$

$$\begin{bmatrix} Y_C[0] \\ Y_C[1] \\ Y_C[2] \\ Y_C[3] \end{bmatrix} = \begin{bmatrix} G_e[0]H[0] \\ G_e[1]H[1] \\ G_e[2]H[2] \\ G_e[3]H[3] \end{bmatrix} = \begin{bmatrix} 12 \\ 9+j \\ -18 \\ 9-j \end{bmatrix}.$$ Therefore

$$\begin{bmatrix} y_C[0] \\ y_C[1] \\ y_C[2] \\ y_C[3] \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 12 \\ 9+j \\ -18 \\ 9-j \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -6 \\ 8 \end{bmatrix}.$$

(d) $g_e[n] = [2, \quad -1, \quad 3, \quad 0, \quad 0, \quad 0]$, $h_e[n] = [-2, \quad 4, \quad 2, \quad -1, \quad 0, \quad 0]$.

$$\begin{aligned}
y_C[0] &= g_e[0]h_e[0] + g_e[1]h_e[5] + g_e[2]h_e[4] + g_e[3]h_e[3] + g_e[4]h_e[2] + g_e[5]h_e[1] \\
&= g[0]h[0] = -4 = y_L[0], \\
y_C[1] &= g_e[0]h_e[1] + g_e[1]h_e[0] + g_e[2]h_e[5] + g_e[3]h_e[6] + g_e[4]h_e[3] + g_e[5]h_e[2] \\
&= g[0]h[1] + g[1]h[0] = 10 = y_L[1], \\
y_C[2] &= g_e[0]h_e[2] + g_e[1]h_e[1] + g_e[2]h_e[0] + g_e[3]h_e[5] + g_e[4]h_e[4] + g_e[5]h_e[3] \\
&= g[0]h[2] + g[1]h[1] + g[2]h[0] = -6 = y_L[2], \\
y_C[3] &= g_e[0]h_e[3] + g_e[1]h_e[2] + g_e[2]h_e[1] + g_e[3]h_e[0] + g_e[4]h_e[5] + g_e[5]h_e[4] \\
&= g[0]h[3] + g[1]h[2] + g[2]h[1] = 8 = y_L[3], \\
y_C[4] &= g_e[0]h_e[4] + g_e[1]h_e[3] + g_e[2]h_e[2] + g_e[3]h_e[1] + g_e[4]h_e[0] + g_e[5]h_e[5] \\
&= g[1]h[3] + g[2]h[2] = 7 = y_L[4], \\
y_C[5] &= g_e[0]h_e[5] + g_e[1]h_e[4] + g_e[2]h_e[3] + g_e[3]h_e[2] + g_e[4]h_e[1] + g_e[5]h_e[0] \\
&= g[2]h[3] = -3 = y_L[5].
\end{aligned}$$

5.50 $v[n] = g[n] + jh[n] = [2-j2, \quad -1+j4, \quad 3+j2, \quad -j]$. Therefore,

$$\begin{bmatrix} V[0] \\ V[1] \\ V[2] \\ V[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2-j2 \\ -1+j4 \\ 3+j2 \\ -j \end{bmatrix} = \begin{bmatrix} 4+j3 \\ 4-j3 \\ 6-j3 \\ -6-j5 \end{bmatrix}, \text{ i.e.,}$$

$$V[k] = [4+j3, \quad 4-j3, \quad 6-j3, \quad -6-j5].$$

Thus, $V^*[\langle -k\rangle_4] = [4+j3, \quad -6+j5, \quad 6+j3, \quad 4+j3]$. Therefore,

$G[k] = \frac{1}{2}(V[k] + V^*[\langle -k\rangle_4]) = [4, \quad -1+j, \quad 6, \quad -1-j]$ and

$H[k] = \frac{1}{2j}(V[k] - V^*[\langle -k\rangle_4]) = [3, \quad -4-j5, \quad -3, \quad -4+j5]$.

5.58 **(a)** $y[n] = \begin{cases} x[n/L], & n = 0, L, 2L, \ldots, (N-1)L, \\ 0, & \text{elsewhere.} \end{cases}$

$Y[k] = \sum_{n=0}^{NL-1} y[n] W_{NL}^{nk} = \sum_{n=0}^{N-1} x[n] W_{NL}^{nLk} = \sum_{n=0}^{N-1} x[n] W_N^{nk}$. For $k \geq N$, let $k = k_o + rN$

where $k_o = \langle k\rangle_N$. Then, $Y[k] = Y[k_o + rN] = \sum_{n=0}^{N-1} x[n] W_N^{n(k_o + rN)} = \sum_{n=0}^{N-1} x[n] W_N^{nk_o}$

$= X[k_o] = X[\langle k\rangle_N]$.