Homework #4 Solutions

6.37 $H(z) = H_1(z)H_2(z) + H_3(z) = (1.2 + 3.3z^{-1} + 0.7z^{-2})(-4.1 - 2.5z^{-1} + 0.9z^{-2})$
$+ 2.3 + 4.3z^{-1} + 0.8z^{-2} = -2.62 - 12.23z^{-1} - 9.24z^{-2} + 1.22z^{-3} + 0.63z^{-4}$.

6.38 (a) $(1 - 0.1z^{-1} + 0.14z^{-2} + 0.49z^{-3})Y(z) = (5 + 9.5z^{-1} + 1.4z^{-2} - 24z^{-3})X(z) \Rightarrow$

$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{5 + 9.5z^{-1} + 1.4z^{-2} - 24z^{-3}}{1 - 0.1z^{-1} + 0.14z^{-2} + 0.49z^{-3}}$. Using Program 6_1.m we factorize $H(z)$ and develop is pole-zero plot shown below:

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Numerator factors
1.00000000000000   3.10000000000000   4.00000000000000
1.00000000000000  -1.20000000000000                  0

Denominator factors
1.00000000000000  -0.80000000000000   0.70000000000000
1.00000000000000   0.70000000000000                  0

Gain constant
  5
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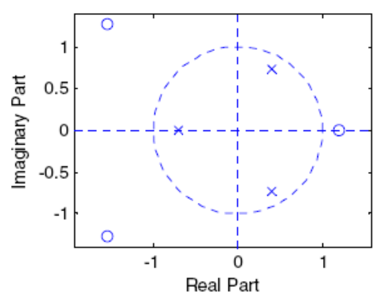
The factored form of $H(z)$ is thus $H(z) = \dfrac{5(1 + 3.1z^{-1} + 4z^{-2})(1 - 1.2z^{-1})}{(1 - 0.81z^{-1} + 0.7z^{-2})(1 + 0.7z^{-1})}$

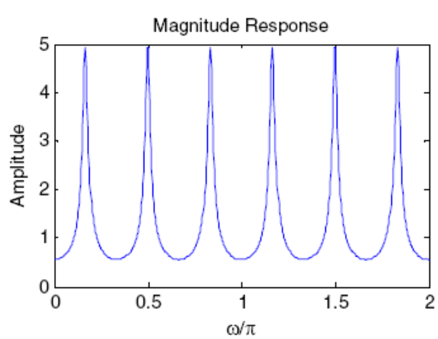


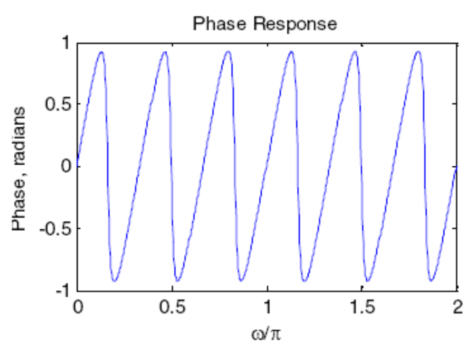
As all poles are inside the unit circle, $H(z)$ is BIBO stable.

6.43 (a) Taking the –transform of both sides of the difference equation we get
$Y(z) = 0.2z^{-1}Y(z) + 0.08z^{-2}Y(z) + 2X(z)$. Hence, $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{2}{1 - 0.2z^{-1} - 0.08z^{-2}}$.

6.47 $G(e^{j\omega}) = \sum_{n=0}^{M-1} \alpha^n e^{-j\omega n} = \frac{1-\alpha^M e^{-j\omega M}}{1-\alpha e^{-j\omega}}$. Note that $G(e^{j\omega}) = H(e^{j\omega})$ for $\alpha^n = \frac{1}{M}, 0 \le n \le M-1$. Now, $G(e^{j0}) = \frac{1-\alpha^M}{1-\alpha}$. Hence, to make the dc value of the magnitude response equal to unity, the impulse response should be multiplied by a constant $K = \left|(1-\alpha)/(1-\alpha^M)\right|$.

6.48 $Y(e^{j\omega}) = X(e^{j\omega}) + \alpha e^{-j\omega R} Y(e^{j\omega})$. Hence, $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1-\alpha e^{-j\omega R}}$. Maximum value of $\left|H(e^{j\omega})\right|$ is $\frac{1}{1-|\alpha|}$ and the minimum value is $\frac{1}{1+|\alpha|}$. There are R peaks and dips in the range $0 \le \omega \le 2\pi$. The locations of the peaks and dips are given by $1-\alpha e^{-j\omega R} = 1 \pm |\alpha|$ or, $e^{-j\omega R} = \pm\frac{|\alpha|}{\alpha}$. The locations of the peaks are given by $\omega = \omega_k = \frac{2\pi k}{R}$ and the locations of the dips are given by $\omega = \omega_k = \frac{(2\pi+1)k}{R}$, $0 \le k \le R-1$. Plots of the magnitude and phase responses of $H(e^{j\omega})$ for $\alpha = 0.8$ and $R = 6$ are shown below:





6.54 $\sum_{n=0}^{K} \left|h[n]\right|^2 = 0.95 \sum_{n=0}^{\infty} \left|h[n]\right|^2$. Since $H(z) = 1/(1-\beta z^{-1})$, $h[n] = (\beta)^n \mu[n]$. Thus, $\frac{1-|\beta|^{2K}}{1-|\beta|^2} = \frac{0.95}{1-|\beta|^2}$. Solving this equation for we get $K = 0.5\frac{\log(0.05)}{\log(|\alpha|)}$.