ECE 158 Fall 2008
Gibson

Homework #5 Solutions

11.16 $H_k(z) = \dfrac{1}{1 - W_N^{-k} z^{-1}}$. Hence, $Y(z) = \dfrac{X(z)}{1 - W_N^{-k} z^{-1}} = \dfrac{1 + z^{-N/2}}{1 - W_N^{-k} z^{-1}} = V(z) + z^{-N/2} V(z)$,

where $V(z) = \dfrac{1}{1 - W_N^{-k} z^{-1}}$. Or, in other words, $y[n] = v[n] + v[n - \frac{N}{2}]$.

Consider $k = 1$: Then $V(z) = \dfrac{1}{1 - W_N^{-1} z^{-1}}$. This implies,

$$v[n] = W_N^{-n}\mu[n] = \left\{1,\quad W_N^{-1},\quad W_N^{-2},\quad \cdots,\quad W_N^{-N/2},\quad W_N^{-(\frac{N}{2}+1)},\quad \cdots\right\}$$

$= \{1,\quad W_N^{-1},\quad W_N^{-2},\quad \cdots,\quad -1,\quad -W_N^{-1},\quad \cdots\}$ since $W_N^{-N/2} = -1$, $W_N^{-(\frac{N}{2}+1)} = -W_N^{-1}$, and son on. Thus, $v[n - \frac{N}{2}] = \{0,\quad 0,\quad 0,\quad \cdots,\quad 1,\quad W_N^{-1},\quad W_N^{-2},\quad \cdots\}$ Hence,

$$y[n] = v[n] + v[n - \tfrac{N}{2}] = \left\{1,\quad W_N^{-1},\quad W_N^{-2},\quad \cdots,\quad W_N^{-(\frac{N}{2}-1)},\quad 0,\quad 0,\quad 0,\quad \cdots\right\}.$$

Now, consider $k = \frac{N}{2}$: $V(z) = \dfrac{1}{1 - W_N^{-N/2} z^{-1}}$. This implies,

$$v[n] = W_N^{-(N/2)n}\mu[n] = \left\{1,\quad W_N^{-\frac{N}{2}},\quad W_N^{-N},\quad \cdots,\quad W_N^{-\frac{N}{2}\cdot\frac{N}{2}},\quad W_N^{-(\frac{N}{2}+1)\cdot\frac{N}{2}},\quad \cdots\right\}.$$

Thus, $v[n - \frac{N}{2}] = \{0,\quad 0,\quad 0,\quad \cdots,\quad 0,\quad 1,\quad W_N^{-N/2},\quad W_N^{-N},\quad \cdots\}$

Now, $W_N^{-\frac{N}{2}\cdot\frac{N}{2}} = (-1)^{N/2}$, $W_N^{-\frac{N}{2}} = -1$, $W_N^{-N} = 1$, $W_N^{-(\frac{N}{2}+1)\frac{N}{2}} = (-1)^{N/2}(-1)$, etc. Hence,

$$y[n] = \left\{1,\quad -1,\quad 1,\quad -1,\quad \cdots,\quad \underset{n=N/2}{1 + (-1)^{N/2}},\quad \underset{n=\frac{N}{2}+1}{1 + (-1)^{\frac{N}{2}+1}},\quad \cdots,\quad \cdots\right\}.$$ Therefore, if $N/2$ is even, $$y[n] = \left\{1,\quad -1,\quad 1,\quad -1,\quad \cdots,\quad \underset{n=N/2}{2},\quad \underset{n=\frac{N}{2}+1}{-2},\quad 2,\quad \cdots\right\},$$ and if $N/2$ is odd,

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$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}} = \sum_{n=0}^{3} \frac{1}{2}\, e^{-j\frac{2\pi nk}{8}}$$

$$X[0] = 4 \cdot \frac{1}{2} = 2$$

$$X[1] = \frac{1}{2}\left(1 + e^{-j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}\cdot 2} + e^{j\frac{\pi}{4}\cdot 3}\right) = \frac{1}{2} - 1.207j$$

$$X[2] = \frac{1}{2}\left(1 + e^{-j\frac{\pi}{4}\cdot 2} + e^{-j\frac{\pi}{4}\cdot 4} + e^{-j\frac{\pi}{4}\cdot 6}\right) = 0$$

$$X[3] = \frac{1}{2}\left(1 + e^{-j\frac{\pi}{4}\cdot 3} + e^{-j\frac{\pi}{4}\cdot 6} + e^{-j\frac{\pi}{4}9}\right) = \frac{1}{2} - .207j$$

$$X[4] = \frac{1}{2}\left(1 + e^{-j\frac{\pi}{4}\cdot 4} + e^{-j\frac{\pi}{4}\cdot 8} + e^{-j\frac{\pi}{4}\cdot 12}\right) = 0$$

$$X[5] = \frac{1}{2} + .207j$$

$$X[6] = 0$$

$$X[7] = \frac{1}{2} + 1.207j$$

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