ECE 158 Fall 2008
Gibson

Homework #6 Solutions

13.4 $c[n] = \frac{1}{M}\sum_{k=0}^{M-1} W_M^{kn} = \frac{1}{M}\left(\frac{1-W_M^{nM}}{1-W_M^{n}}\right)$. Hence, if $n \neq rM$, $c[n] = \frac{1}{M}\left(\frac{1-1}{1-W_M^{n}}\right) = 0$. On the other hand, if $n = rM$, then $c[n] = \frac{1}{M}\sum_{k=0}^{M-1} W_M^{kn} = \frac{1}{M}\sum_{k=0}^{M-1} W_M^{krM} = \frac{1}{M}\sum_{k=0}^{M-1} 1 = \frac{M}{M} = 1.$

2.

a.)
$$H_0(z)G_0(z) + H_1(z)G_1(z) = 2z^{-l}$$
$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0.$$
$$\hookrightarrow \quad G_1(z) = -\frac{H_0(-z)}{H_1(-z)}G_0(z) = -\frac{3-4z^{-1}}{1-2z^{-1}}G_0(z)$$

$$H_0(z)G_0(z) + H_1(z)G_1(z)$$
$$= H_0(z)G_0(z) + H_1(z)\left(-\frac{3-4z^{-1}}{1-2z^{-1}}\right)G_0(z)$$
$$= \left[3+4z^{-1} - \frac{3-4z^{-1}}{1-2z^{-1}}(1+2z^{-1})\right]G_0(z)$$
$$= \left[\frac{3-2z^{-1}-8z^{-2}-(3+2z^{-1}-8z^{-2})}{1-2z^{-1}}\right]G_0(z)$$
$$= \frac{-4z^{-1}}{1-2z^{-1}}G_0(z) = 2z^{-l}$$

$$G_0(z) = \tfrac{1}{2}(2z^{-1}-1)z^{-l}$$
$$G_1(z) = -\tfrac{1}{2}(2z^{-1}-1)z^{-l}\,\frac{3-3z^{-1}}{1-2z^{-1}}$$
$$= \tfrac{1}{2}(3-4z^{-1})z^{-l}$$

b.) Show
$$H_0(z)G_0(z) + H_1(z)G_1(z) = 2z^{-l}$$
$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$$

$$(3+4z^{-1})\cdot\frac{1}{2}(2z^{-1}-1)z^{-l} + (1+2z^{-1})\cdot\frac{1}{2}(3-4z^{-1})z^{-l}$$

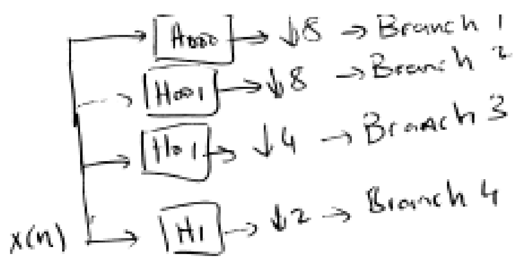
$$= \frac{1}{2}z^{-l}\left[(3+4z^{-1})(2z^{-1}-1) + (1+2z^{-1})(3-4z^{-1})\right]$$

$$= \frac{1}{2}z^{-l}\left[6z^{-1} \cancel{-3} + \cancel{8z^{-(l+1)}} - 4z^{-1} \cancel{+3} - 4z^{-1} + 6z^{-1} \cancel{-8z^{-(l+1)}}\right]$$

$$= \frac{1}{2}z^{-l}\cdot(12z^{-1} - 8z^{-1})$$

$$= 2z^{-l}z^{-1} \sim \text{Delay}$$

3.



$$H_1(z) = 1+2z^{-1}$$

$$H_{01}(z) = H_0(z).\,H_1(z^2) = (3+4z^{-1})(1+2z^{-2}) = 3+4z^{-1}+6z^{-2}+8z^{-3}$$

$$H_{001}(z) = H_0(z).H_0(z^2).H_1(z^4)$$

$$= 9+12z^{-1}+12z^{-2}+16z^{-3}+18z^{-4}+24z^{-5}+24z^{-6}+32z^{-7}$$

$$H_{000}(z) = H_0(z).\,H_0(z^2).\,H_0(z^4)$$

$$= (3+4z^{-1})(3+4z^{-2})(3+4z^{-4})$$

$$= 27+36z^{-1}+36z^{-2}+48z^{-3}+36z^{-4}+48z^{-5}+48z^{-6}+64z^{-7}$$

Transfer function of branch 1, $T_1(z)$.

$$T_1(z) = \frac{1}{8}\sum_{k=0}^{7} H_{00}\left(z^{1/8}.\,W_8^{-k}\right)$$

$$T_2(z) = \frac{1}{8}\sum_{k=0}^{7} H_{001}\left(z^{1/8}.\,W_8^{-k}\right)$$

$$T_3(z) = \frac{1}{4}\sum_{k=0}^{3} H_{01}\left(z^{1/4}\,W_4^{-k}\right)$$

$$T_4(z) = \frac{1}{2}\sum_{k=0}^{1} H_1\left(z^{1/2}\,W_2^{-k}\right)$$