ECE 158 | Fall Quarter 2008
Gibson | **11/05/08**

Linear Prediction Example Problems

Problems

13.9–23. A block of speech data has autocorrelation terms as in Eq. (13.5–14) given by $R(0) = 1.0$, $R(1) = 0.866$, $R(2) = 0.554$, and $R(3) = 0.225$. Find the predictor coefficients $\{a_i,\ i = 1,\ 2,\ 3\}$.

13.9–24. Given the set of predictor coefficients $a_1 = 1.295$, $a_2 = -0.535$, $a_3 = 0.171$, and $a_4 = -0.233$, assume that $R(0) = 1$.

(a) Find the PARCOR coefficients $\{p_i,\ i = 1,\ 2,\ 3,\ 4\}$;

(b) Calculate the mean squared prediction error for the first through fourth order systems;

(c) Is the fourth order system (synthesizer) stable?

Solutions

13.9–23. With $R(0) = 1.0$, $R(1) = 0.866$, $R(2) = 0.554$, and $R(3) = 0.225$, we can use the recursion in Eqs. (13.5–15) through (13.5–19). Thus,

$$E^{(0)} = R(0) = 1.0,$$

$$p_1 = R(1)/R(0) = 0.866, \text{ so}$$

$$a_1^{(1)} = p_1 = 0.866, \text{ and}$$

$$E^{(1)} = (1 - (.866)^2)E^{(0)} = 0.25.$$

Next,

$$p_2 = \frac{R(2) - a_1^{(1)}R(1)}{E^{(1)}} = \frac{0.554 - (0.866)^2}{0.25} = -0.7838,$$

hence

$$a_2^{(2)} = -0.7838$$

and

$$a_1^{(2)} = a_1^{(1)} - p_1 a_1^{(1)} = 0.866 - (-.7838)(.866) = 1.545.$$

So

$$E^{(2)} = (1 - p_2^2)E^{(1)} = (1 - (-.7838)^2)(0.25) = 0.0964.$$

Continuing

$$p_3 = \frac{R(3) - a_1^{(2)}R(2) - a_2^{(2)}R(1)}{E^{(2)}} = 0.4969 = a_3^{(3)}.$$

$$a_1^{(3)} = a_1^{(2)} - p_3 a_2^{(2)} = 1.545 - (.4969)(-.7838) = 1.9345,$$

$$a_2^{(3)} = a_2^{(2)} - k_3 a_1^{(2)} = -0.7838 - (.4969)(1.545) = -1.552.$$

The desired coefficients are thus

$$a_1 = 1.9345,\ a_2 = -1.552,\ a_3 = 0.4969.$$

13.9–24. $a_1 = 1.295,\ a_2 = -0.535,\ a_3 = 0.171,\ a_4 = -0.233$, and $R(0) = 1$.

(a) Using Eqs. (13.5–20) and (13.5–21),

$$p_4 = a_4^{(4)} = -0.233$$

$$a_1^{(3)} = \frac{a_1^{(4)} + a_4^{(4)}a_3^{(4)}}{1 - p_4^2}$$

$$= \frac{1.295 + (-.233)(.171)}{0.9457}$$

$$= 1.327$$

$$a_2^{(3)} = \frac{a_2^{(4)} + a_4^{(4)}a_2^{(4)}}{1 - p_4^2} = -0.434$$

$$a_3^{(3)} = \frac{a_3^{(4)} + a_4^{(4)}a_1^{(4)}}{1 - p_4^2} = -0.138 = p_3$$

$$a_1^{(2)} = \frac{a_1^{(3)} + a_3^{(3)}a_2^{(3)}}{1 - p_3^2} = \frac{1.327 + (-.138)(-.434)}{0.981}$$

$$= 1.414$$

$$a_2^{(2)} = \frac{a_2^{(3)} + a_3^{(3)}a_1^{(3)}}{1 - p_3^2} = -0.629 = p_2$$

$$a_1^{(1)} = \frac{a_1^{(2)} + a_2^{(2)}a_1^{(2)}}{1 - p_2^2} = \frac{1.414(.371)}{0.604}$$

$$= 0.8685 = p_1.$$

(b)

$$E^{(0)} = R(0) = 1$$

$$E^{(1)} = 1 - p_1^2 = 0.2457$$

$$E^{(2)} = \prod_{i=1}^{2}(1 - p_i^2) = 0.1484$$

$$E^{(3)} = \prod_{i=1}^{3}(1 - p_i^2) = 0.1456$$

$$E^{(4)} = \prod_{i=1}^{4}(1 - p_i^2) = 0.1377.$$

(c) Yes, since $|p_i| < 1$ for $i = 1,\ 2,\ 3$, and 4.