ECE 158 Fall Quarter 2007
Gibson **11/05/07**

Mid-Term Exam

Instructions: Do all problems. Show all work. Problems are weighted as shown.

1. (15) (a) Given the 8-point sequence

$x[n] = \{1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\}$ for $n = 0,1,2,3,....,7$ Find the DFT $X[k]$.

(b) Find the DFT of the 8 point sequence

$y[n] = \{0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\}$, $n = 1,2,3,....,7$

2. (15) Does the 4 by 4 matrix below represent an orthogonal transform?

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & -1 & 1 \\ 1 & -2 & 2 & -1 \end{bmatrix}$$

3. (15) Given an analog signal $x(t)$ with continuous time Fourier transform as shown in Fig. 1.4 (a) on page 8 of the text:
(a) Describe the steps in obtaining uniform samples of this signal for processing in a DSP, and sketch the spectrum of the sampled signal if there is no aliasing.
(b) What causes aliasing and what happens to the spectrum of the sampled signal?

4. (15) A filter with impulse response $h[n] = \{2\ \ 1\ \ 2\ \ 1\}$ has the input $x[n] = \{1\ \ 2\ \ 3\ \ 4\}$. Find the circular convolution of $h[n]$ and $x[n]$.

5. (15) Given that $H[k] = \{\ 6\ \ 0\ \ 2\ \ 0\ \}$ and $X[k] = \{\ 10\ \ -2+j2\ \ -2\ \ -2-j2\ \}$ for the $h[n]$ and $x[n]$ sequences in Problem 4, find $H[k]X[k]$ and take the IDFT using the 4x4 IDFT matrix. How does this result relate to the answer to Problem 4?

6. (10) A system has an impulse response $h[n]$ of length 4 and an input sequence $x[n]$ of length 5. Describe how to calculate the linear convolution of $h[n]$ and $x[n]$ in terms of a circular convolution.

7. (15) Let $X[k]$ denote the N-point DFT of the length-N sequence $x[n]$, where N is even. We define two length-$N/2$ sequences given by

$$g[n] = \frac{1}{2}(x[2n] + x[2n+1]) \quad \text{and} \quad h[n] = \frac{1}{2}(x[2n] - x[2n+1]) \quad \text{for } 0 \le n \le \frac{N}{2} - 1$$

If $G[k]$ and $H[k]$ denote the $\frac{N}{2}$ point DFTs of $g[n]$ and $h[n]$, respectively, determine an expression for the N-point DFT $X[k]$ in terms of $G[k]$ and $H[k]$.