

ECE 162A
HW 7
Due in Class Tuesday, Nov. 27

1. Distinguishable and Indistinguishable Particles: Counting

Consider a system with 5 distinct energy levels and four particles. In how many ways can the 4 particles be made to occupy the 5 energy levels if:

- (i) The particles are all distinguishable and any number can occupy any energy level?
- (ii) The particles are all distinguishable but only two can occupy any given level at the same time?
- (iii) The particles are all identical and any number can occupy any energy level?
- (iv) The particles are all identical but only two can occupy any given level at the same time?

By “distinguishable” we mean we can label them particle #1, particle #2 ...

2. Many-particle Systems: Symmetry

For this problem, build your solutions as linear combinations of terms like

$$\psi_A(1)\psi_B(2)\dots|\uparrow,\downarrow,\dots\rangle$$

where A and B are energy levels; $1, 2, \dots$ are particle labels, and

$|\uparrow,\downarrow,\dots\rangle$ means that particle #1 has spin up, particle #2 has spin down, ...

- (i) Consider a system of two *independent* electrons which occupy the same (spatial) energy level. Write down the form of the spatial, the spin and the overall wave-function. Show your reasoning at every step. Note that **electrons are Fermions**.
- (ii) Write down the form of the spatial, the spin and the overall wave-function for a system of two independent electrons which occupy two distinct spatial energy levels A and B . How many ways can this be done? Show each.
- (iii) **(Bonus)** Write down the form of the overall wave-function for a system of **three** independent electrons which occupy two distinct spatial energy levels A and B .

Hint: For (ii), please see Page 309, Eisberg and Resnick, to get started.

3. Exchange Energy

Consider a system of two electrons which occupy two distinct energy levels, namely **E_1 and E_2** .

- (i) Write down the two-particle Hamiltonian, ignoring electron-electron interaction.
- (ii) Write the spatial form of the solution exactly as you did for 2(ii). You must get two solutions for the spatial component. Normalize them.
- (iii) Determine the expectation value of the energy for each 2-particle wave-function you got in (ii).
- (iv) Now we will drop the assumption that the two electrons are independent, and add a coulomb repulsion term to the two-electron Hamiltonian. Write down the resultant two-particle Hamiltonian.
- (v) Assuming that the wave-functions will retain their form, find the expectation value of the energy $\langle E \rangle$ of the two-particle system with coulomb repulsion included. Simplify the expression as much as possible, knowing that **E_1 and E_2** are two of the energy eigen-values for each single-particle Hamiltonian. (Please don't try to evaluate the integrals involving the Coulomb term!)
- (vi) In the final form of the expression for $\langle E \rangle$, identify the classical Coulomb repulsion term and the exchange-interaction term. State which solution of 3(ii) will have the higher energy.