The Quantum Description of Electronic Materials
ECE/Mat 162A, Fall 2007
Homework 1
Due Tuesday, Oct 9th 2007, in Class

1. Wave-Particle Duality
The purpose of this question is to get acquainted with some basic formulae that will be used frequently (often implicitly) in this class, as well as to get a feel for the numbers involved.

(a) For violet light, wavelength $\lambda \sim 120$ nm. Calculate the momentum 'p' of a single photon of violet light. Calculate the energy 'E' per quantum of violet light. Express your answer in eV.

(b) Photons are massless. But if $p = m_p c$ hypothetically, what would the mass of the photon m_p be?

(c) Electrons have a rest mass of $9.1X10^{-31}$ kg. For an electron of wavelength 3 Å, (a typical number for interatomic distance or bond length in a crystal), calculate the energy and momentum.

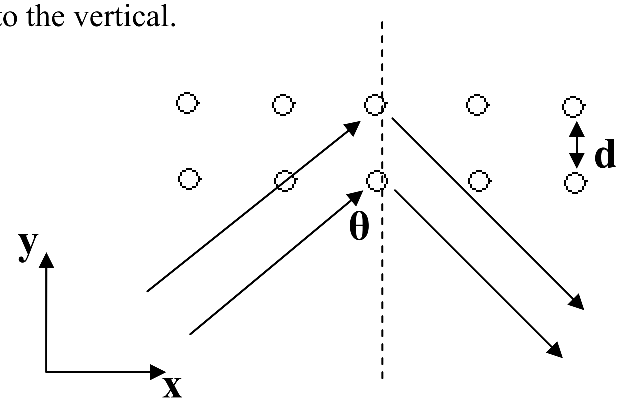
2. The Photoelectric Effect
We are familiar with light as a wave. The photoelectric effect demonstrates the particulate nature of light, which is quantized as photons.

A beam of light of intensity $1.6X10^{-12}$ Watt (10^7 $eV.s^{-1}$) impinges on a metal surface, the work-function of the metal being 5 eV. Assuming that all emitted electrons are instantly swept away by a collector, find the photoelectric current for light of frequency (a) $2.5X10^{15}$ Hz. (b) $1.21X10^{15}$ Hz. and (c) $2.4X10^{14}$ Hz. Assume the same intensity 10^7 $eV.s^{-1}$ in each case.

3. Diffraction of Electrons in a Crystal

We are familiar with the electron as a particle. The wave nature of the electron becomes apparent when we are working with length scales of the order of the electronic wavelength. Inter-atomic distances in crystals are of this order, and indeed we see the signatures of wave-like behavior- interference and diffraction- in electrons bound to a crystal.

Consider the crystal shown in Fig. 1: only two (adjacent) planes of atoms, distance 'd' apart, are shown. Let the x and y axis be as shown in the figure. The incident wave is at an angle θ to the vertical.



We show two waves on the same wave-front following different paths on reflection from two different atomic planes.

(a) From the geometry, write the expression for the path difference between the two paths shown in the figure.

(b) Write the condition for constructive interference of the two reflected waves, assuming that the incident and the reflected waves have the same wavelength λ. This assumption is equivalent to elastic collision of the electrons with the atoms.

(c) Let the incident beam have a wave-vector $\mathbf{k_1}$ and the reflected wave have a wave-vector $\mathbf{k_2}$. Both wave-vectors have the same magnitude (what is it?) but different directions. Write $\mathbf{k_1}$ and $\mathbf{k_2}$ in terms of the unit vectors in the $\mathbf{k_x}$ and the $\mathbf{k_y}$ directions, namely $\mathbf{e_{kx}}$ and $\mathbf{e_{ky}}$.

(d) Show that for constructive interference, it is necessary that

$$(\mathbf{k_1} - \mathbf{k_2}) = m\,(2\pi/d)\,\mathbf{e_{ky}}$$

where 'm' is an integer.