ECE 162A
Mat 162A

Lecture #15:Spin
E/R: Chapter 8

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Accomplished

- Chapter 1: Thermal radiation, Planck's postulate
- Chapter 2: Light: Wavelike and particlelike (photon)
- Chapter 3: Matter: Wavelike and particlelike
- Chapter 4: Thompson, Rutherford, Bohr Model of Atom
- Chapter 5: Schroedinger Theory: Time dependent, Time independent Schroedinger Equations
- Chapter 6: Solutions of Time Independent SE: Free particle, Step potential, Barrier potential, Infinite square well, finite square well, harmonic oscillator.
- 3D Solutions.
- Chapter 7: Hydrogen atom. Quantum numbers and degeneracy. Angular momentum. Commutator. Simultaneous eigenvalues. 2D harmonic oscillator.

Left to do

- Chapter 8: Spin
- Numerical solutions
- Chapter 9: Exclusion principle, periodic table
- Chapter 11 (11-1 to 11-3): Quantum statistics.
- Chapter 13: Free electrons in metals. Bonds. Periodic potentials. Energy bands.
- Semiconductors. Band offsets. Ternary, Quaternary. Quantum Wells.

- Under what conditions can two or more observable properties of a quantum system have unique eigenvalues for a given quantum state?

- If two operators commute, then the eigenvalues associated with those operators are simultaneous eigenvalues
- If two operators do not commute, then the eigenvalues associated with those two operators typically exhibit an uncertainty relation.
- In general, for every system one may identify at least one complete set of commuting observables.

Specific Case: 2D Harmonic Oscillator

$$V(x,y) = \frac{1}{2}C(x^2+y^2) \equiv \frac{1}{2}M\omega^2(x^2+y^2)$$

$$\frac{-\hbar^2}{2M}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+\frac{1}{2}M\omega^2(x^2+y^2)\psi = E\psi$$

$$\psi(x,y) = f(x)g(y)$$

$$\frac{-\hbar^2}{2M}\left(g\frac{\partial^2 f}{\partial x^2}+f\frac{\partial^2 g}{\partial y^2}\right)+\frac{1}{2}M\omega^2(x^2+y^2)fg = Efg$$

$$\left(\frac{-\hbar^2}{2Mf}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}M\omega^2x^2\right)+\left(\frac{-\hbar^2}{2Mf}\frac{\partial^2 f}{\partial y^2}-\frac{1}{2}M\omega^2y^2\right)=E$$

$$Cons\tan t + Cons\tan t = E$$

$$\frac{-\hbar^2}{2M}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}M\omega^2x^2 f = E_x f$$

$$\frac{-\hbar^2}{2M}\frac{\partial^2 g}{\partial y^2}+\frac{1}{2}M\omega^2y^2 g = E_y g$$

$$E_x + E_y = E$$

2D Harmonic Oscillator Solutions

$$\psi_{n_x n_y} = H_{n_x}(\frac{x}{a}) H_{n_x}(\frac{x}{a}) e^{-(x^2+y^2)/2a^2}$$

$$E = (n_x + n_y + 1)\hbar\omega$$

$$n_x = 0,1,2,...$$

$$n_y = 0,1,2,...$$

Are these solutions of $\hat{L}_z$?

- Yes, if $$\hat{L}_z \psi = L_z \psi$$
$$\hat{L}_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$$

- We need to find linear combinations of degenerate solutions that satisfy the above equation
- Note: Degenerate solutions (solutions with the same energy) do not change in time and are called stationary solutions.

Lowest energy solution

$$n = 0 \qquad n_x = n_y = 0$$

$$\psi = e^{-(x^2+y^2)/2a^2} = e^{-r^2/2a^2}$$

$$\hat{L}_z \psi = \frac{\hbar}{i}\frac{\partial}{\partial \phi} e^{-r^2/2a^2} = 0$$

This is a solution of energy and L_z

N=1 Solutions

$$n=1 \quad n_x=1 \quad n_y=0 \quad \psi_{10}=\frac{2x}{a}e^{-r^2/a^2}$$

$$n=1 \quad n_x=0 \quad n_y=1 \quad \psi_{01}=\frac{2y}{a}e^{-r^2/a^2}$$

These are not solutions that satisfy:

$$\hat{L}_z\psi = L_z\psi$$

$$\hat{L}_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$

N=1 Solutions

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$$n = 1 \quad n_x = 0 \quad n_y = 1 \quad \psi_{01} = \frac{2y}{a} e^{-r^2/a^2}$$

$$Note:$$

$$re^{i\phi} = r\cos\phi + ir\sin\phi = x + iy$$

$$re^{-i\phi} = r\cos\phi - ir\sin\phi = x - iy$$

$$So$$

$$\psi = \psi_{01} + i\psi_{01} = \frac{2(x+iy)}{a} e^{-r^2/a^2} = \frac{2r}{a} e^{i\phi} e^{-r^2/a^2}$$

$$\psi = \psi_{01} - i\psi_{01} = \frac{2(x-iy)}{a} e^{-r^2/a^2} = \frac{2r}{a} e^{-i\phi} e^{-r^2/a^2}$$

These are both solutions with L_z= +1 and -1 respectively.

Dirac Notation

$\psi_{n_x n_y}$ Is represented by the Dirac ket vector

$| n_x, n_y >$

This notation is a useful shorthand:

$$| n = 1, m = 1 >=| 1,0 > +i | 0,1 >$$

The projection of onto all possible positions is the wave function

$$< x, y | n_x, n_y >= \psi_{n_x n_y}$$

Magnetic moments

- CLASSICALLY, an electron moving in a loop produces a current

$$i = \frac{e}{period}$$

$$period = \frac{distance}{velocity} = \frac{2\pi r}{v}$$

$$i = \frac{ev}{2\pi r}$$

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- A current in a loop produces a magnetic dipole moment

$$\mu = current \ \times \ area = iA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

Bohr Magneton

- Classically,

$$L = mrv$$

$$\mu = \frac{evr}{2} = \frac{eL}{2m}$$

$$If$$

- (Bohr Magneton)

$$\mu_b \equiv \frac{e\hbar}{2m} = .927 \times 10^{-23}\, Am^2$$

$$\mu = \frac{\mu_b L}{\hbar}$$

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- The correct quantum mechanical result is
- (g_l is the orbital g factor and g_l=1)

$$\vec{\mu} = -\frac{g_l \mu_b}{\hbar}\vec{L}$$

Dipole in a Magnetic Field

- The effect of a magnetic field on a magnetic dipole is to exert a torque

$$\vec{T} = \vec{\mu} \times \vec{B}$$

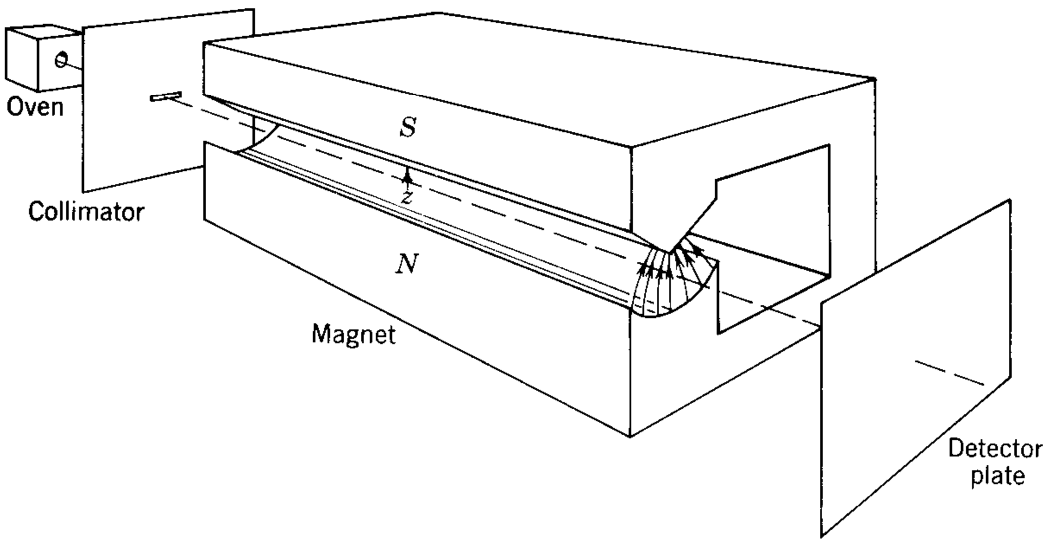
- The potential energy is lowest when the dipole is aligned with the magnetic moment

$$\Delta E = -\vec{\mu} \bullet \vec{B}$$

- Uniform magnetic field: precession, but no translation.
- Converging magnetic field: translational force.

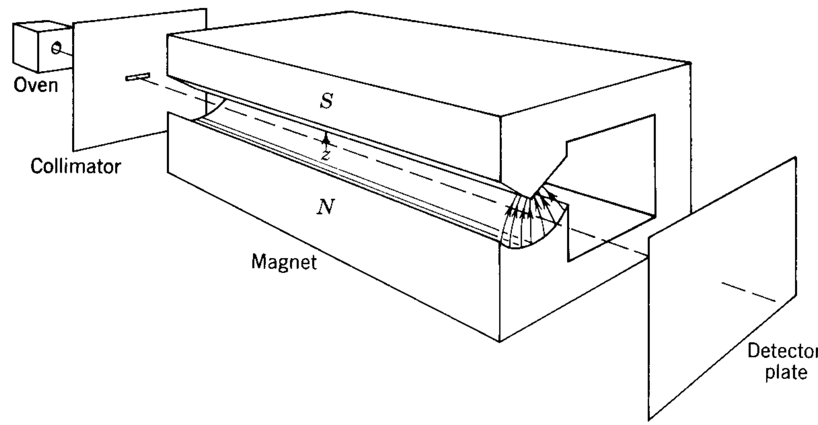
$$F_z = \frac{\partial B}{\partial z}\mu_z$$

Stern Gerlach Experiment



- Stern Gerlach Exp: Pass a beam of silver atoms through a nonuniform magnetic field and record deflections
- Classical prediction: ?
- Quantum mechanical prediction: ?

Stern Gerlach Experiment



- Stern Gerlach Exp: Pass a beam of silver atoms through a nonuniform magnetic field and record deflections
- Classical prediction: a range of deflections corresponding to μ_z ranging from + μ to - μ.
- Quantum mechanical prediction: discrete deflections corresponding to m_l=–l,…0…l
- Result: 2 discrete components: one positive, one negative.

Phipps/Taylor (1927) Experiment

- Repeat Stern Gerlach Exp with hydrogen atoms in the ground state. l=0. m_l=0.
- Quantum mechanical prediction: No deflection corresponding to m_l=0

Phipps/Taylor (1927) Experiment

- Repeat Stern Gerlach Exp with hydrogen atoms in the ground state. l=0. m_l=0.
- Quantum mechanical prediction: No deflection corresponding to m_l=0
- Result: 2 beams, one deflected positive, one negative.
- Something is missing in the theory.

- Size of deflection: 2000x bigger than Bohr magneton for a proton.
- The atom is not responsible for the deflection. The electron is!

Electron Spin

- Goudsmit and Uhlenbeck (1925), grad students.
- Explain fine splitting of Hydrogen lines by assuming the electron is a small spinning sphere with surface charge i.e. a magnetic moment.

$$Quantum\ numbers\ s = 1/2$$

$$Intrinsic\ angular\ momentum\ S = \sqrt{s(s+1)}\hbar$$

- The z component of spin is quantized:

$$S_z = m_s \hbar$$

$$m_s = -1/2, 1/2$$

$$g_s = 2$$

g_s is the spin g factor

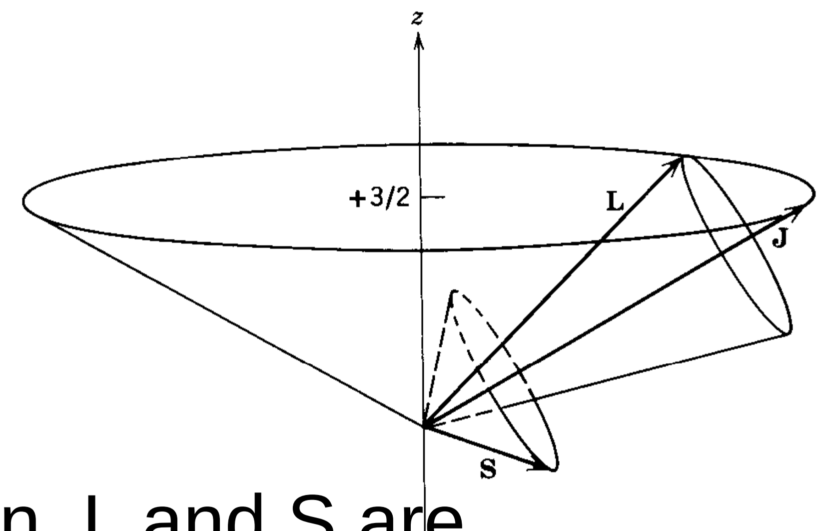
Spin Orbit Interaction

- The spin orbit interaction is the result of the electron spin magnetic moment and the internal magnetic field of the atom due to the electrons angular momentum.

$$\Delta E = \frac{1}{2m^2c^2r}\frac{dV}{dr}\vec{S} \bullet \vec{L}$$

- This is typically about 10^{-4} eV. This splitting is called fine structure.

Total Angular Momentum



$$\vec{J} = \vec{L} + \vec{S}$$

- Due to the spin orbit interaction, L and S are not independent. The spin orbit interaction causes a coupling between L and S and a precession about the axes.
- The total angular momentum is fixed and quantized.
- The z component of total angular momentum is quantized.

Total Angular Momentum

$$\vec{J} = \vec{L} + \vec{S}$$

The total angular momentum in terms of quantum number j

$$J = \sqrt{j(j+1)}\hbar$$

The z component of angular momentum is

$$J_z = m_j \hbar$$

Where the quantum number m_j is

$$m_j = -j, \ldots 0 \ldots j$$

How does j relate to l and s?

$$\left|\vec{L}+\vec{S}\right| \geq \left|\vec{J}\right| \geq \left|\vec{L}-\vec{S}\right|$$

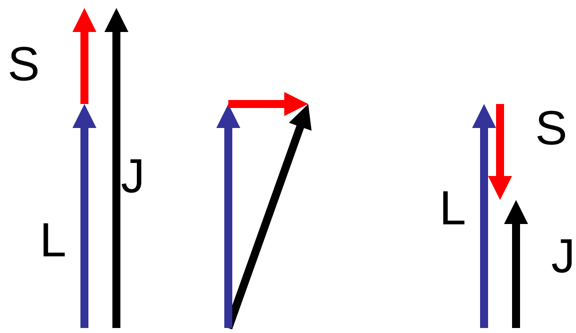
$$\sqrt{l(l+1)}+\sqrt{s(s+1)} \geq \sqrt{j(j+1)} \geq \sqrt{l(l+1)}-\sqrt{s(s+1)}$$

The result of this inequality is that j can have two values:

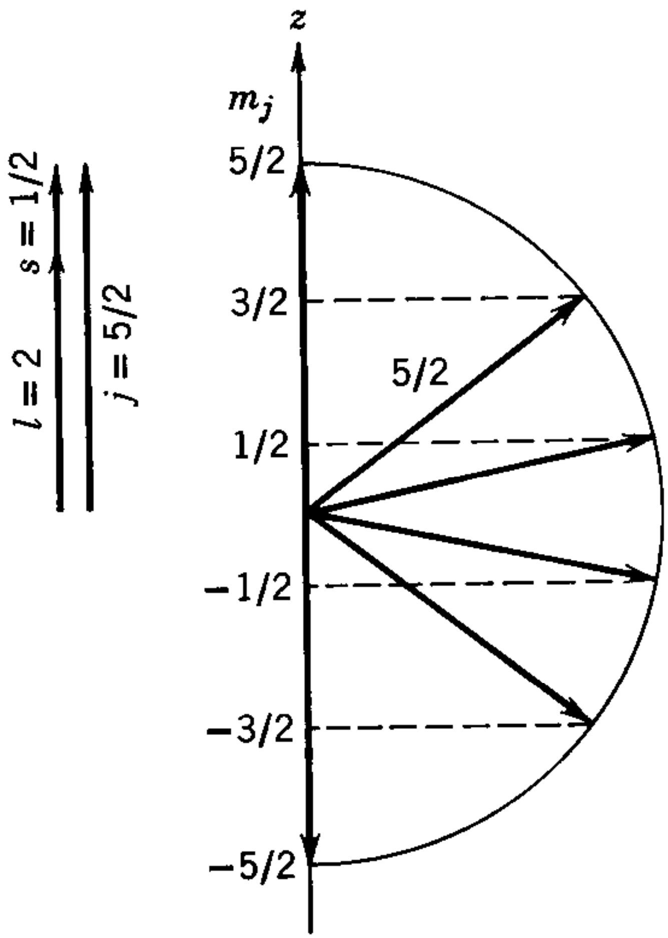
$$j = l+1/2, l-1/2$$

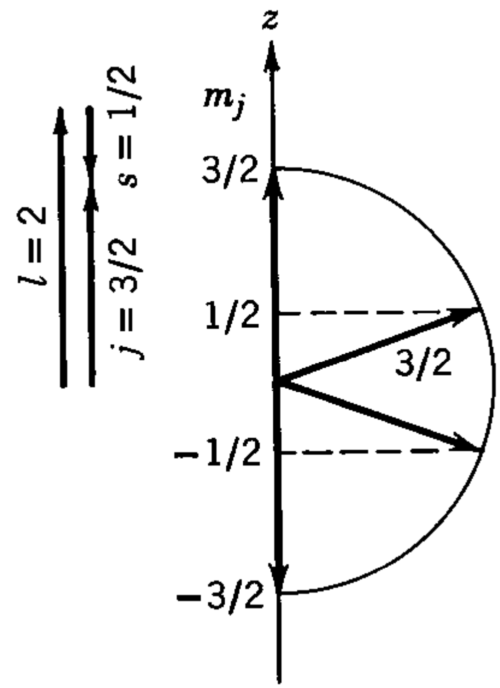
When l=0, then there is only one value of j:

$$j = 1/2$$



Example for l=2





Hydrogen spectra

- The spectra of hydrogen show a fine structure which is well explained by Schroedinger's equation with spin.
- There is also a hyperfine structure, which is due to the spin of the nucleus.

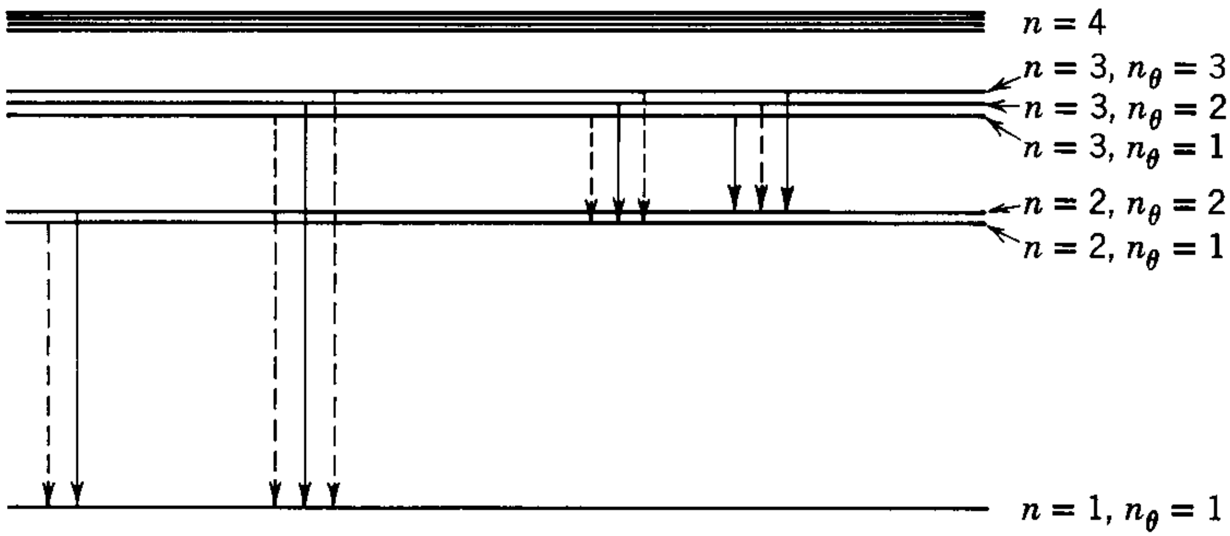


Figure 4-19 The fine-structure splitting of some energy levels of the hydrogen atom. The splitting is greatly exaggerated. Transitions which produce observed lines of the hydrogen spectrum are indicated by solid arrows.