1. Consider a **Particle in a Box**:  
\[ V(x) = 0, \quad -L/2 \leq x \leq L/2 \]
\[ = \text{Infinity, elsewhere} \]

a. Sketch the potential  
b. Write down the boundary condition at \( \pm L/2 \) and determine the wave function in different regions.  
c. Use the b.c to show that ‘k’ is discrete.  
d. Write down the expressions and plot the wave functions for the first three eigen states  
e. Normalize the first eigen function  
f. Show that energy is also quantized and give an expression for it.

Where does this quantization come from as opposed the continuous variation in energy of a free electron?

2. **AlAs-GaAs-AlAs Hetero junction quantum well**

It can be shown that the conduction band-edge of a AlAs-GaAs-AlAs quantum well can be modeled as a potential well of depth 1.05eV, with the only difference of a replacement of the electronic mass \( m_e \) with the ‘effective mass’ of the electron in the conduction band
m_{	ext{eff}} = 0.067m_e

a. Looking at the problem as that of a finite-square well, derive the general expression for the wave function an electron of energy ‘E’ inside the quantum well using boundary conditions. ( E < 1.05 eV ).

b. Plot qualitatively the wave function of the first three eigen states.

c. Assuming a well of width 5 nm, (and assuming you have not been given the well depth, which in this case was 1.05 eV ) calculate the 
maximum possible value of the first energy state possible in such a quantum well.

Use the effective mass for all answers.

3. Calculate the phase difference between incident and reflected waves for a step barrier,

   a. $E < V$
   b. $E > V$
   c. $V = \infty$ for a finite $E$