ECE 162A
Mat 162A

Lecture #2: Wavelike Properties of Matter

Read Chapter 3 of Eisberg,Resnick

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- What is the classical theory of light?

Maxwell's Equations

$$\nabla \times H = I + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \bullet D = 0$$

$$\nabla \bullet B = 0$$

where E and H are the electric and magnetic field vecto
D and B are the electric and magnetic displacement vec
No free charge. No current flow.

Constitutive Relations

$$D = \varepsilon_0 E + P$$

$$B = \mu_0 (H + M)$$

P and M are the electric and magnetic polarizations of the medi
ε_0 and μ_0 are the electric and magnetic permeabilities of vacuur
E and H are the electric and magnetic field vectors
D and B are the electric and magnetic displacement vectors

For isotropic media, Electric Susceptibility χ

Isotropic Media: χ is a complex number

$$P = \varepsilon_0 \chi E$$

The real part determines the index (velocity) and the imaginary part determines the gain or absorption.

Isotropic media: Vacuum, gasses, glasses (optical fibers)
Anisotropic media: Semiconductors, crystalline materials.

Wave Propagation in Lossless, Isotropic Media

- Lossless: σ=0, χ is real, ε is real.
- Isotropic: χ, ε are scalars (not tensors).

$$\nabla \times E = i + \frac{\partial B}{\partial t} = 0 + \mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = i + \frac{\partial D}{\partial t}$$

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$$\nabla \times H = i + \frac{\partial D}{\partial t}$$

$$\nabla \times (\nabla \times E) = \mu \frac{\partial (\nabla \times H)}{\partial t} = \mu \frac{\partial^2 D}{\partial^2 t} = \mu\varepsilon \frac{\partial^2 E}{\partial^2 t}$$

$$\nabla \times (\nabla \times E) = \nabla^2 E - \nabla(\nabla \bullet \vec{e})$$

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial^2 t}$$

Wave Equation

Wave Equation

$$E(x,y,z,t) = \mathrm{Re}[E(x,y,z)e^{i\omega t}]$$

$$\nabla^2 \vec{E} + \omega^2 \mu\varepsilon \vec{E} = 0$$

$$\boxed{\nabla^2 \vec{E} + k^2 \vec{E} = 0}$$

where

$$k = \omega\sqrt{\mu\varepsilon} = \omega n / c$$

$$c = 1/\sqrt{\mu_0 \varepsilon_0}$$

$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}}$$

Prequantum Theory
Chapter 1: Thermal Radiation, Plank's Constant

- Light is a wave. Maxwell's equations give rise to a wave equation that explain light propagation quite well. (Undergrad E&M)
- Classical:
  - Wavelength may be quantized (satisfying boundary conditions)
  - Wave can have any energy (continuous)
- Classical theory predicts diffraction, refraction, propagation very well.
- Electromagnetic radiation spreads through space like water waves spread across water.

- What is wrong with classical theory of light?

Chapter 2: Light: particle and wave characteristics

- Photons: Particle like properties of radiation
- Interaction of light with matter:
  - Photoelectric effect
  - Compton effect
  - Pair production
  - Bremstralung
  - Pair annihilation
- All show experimental evidence of particle nature of light when interacting with matter.

What are these effects?

Dual Nature of Radiation

- **Radiation is neither purely a wave phenomenon nor purely a particle phenomenon.**
- A crystal spectrometer used to measure X ray wavelength is using the wave nature.
- A Compton scattering experiment is characterizing the particle nature.

- OK. So, light has both wave and particle nature.
- What is a photon?

So, what is a photon?

- A superposition of cw waves added in phase such that the envelope is a pulse.

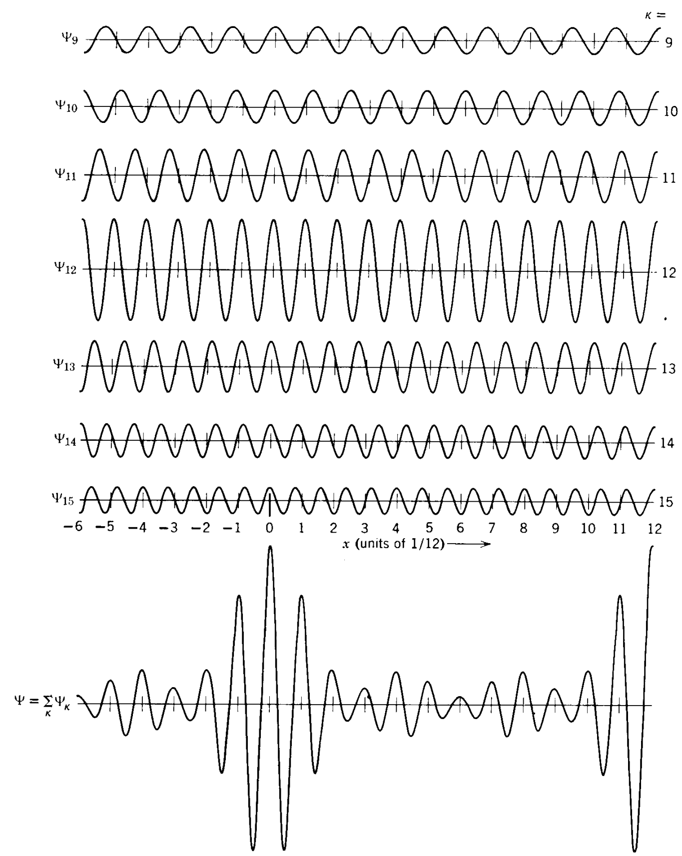


Figure 3-9 Showing, at $t = 0$, the superposition of seven cosine waves $\Psi_\kappa = A_\kappa \cos 2\pi(\kappa x - \nu t)$ with uniformly spaced reciprocal wavelengths drawn from the range $\kappa = 9$ to $\kappa = 15$. Their amplitudes A_κ maximize at the value $A_{12} = 1$ for the wave whose κ lies in the center of the range, and they decrease symmetrically through the values 1/2, 1/3, and 1/4 for the other waves as their κ approach the ends of the range. The sum $\Psi = \sum_\kappa \Psi_\kappa$ of these waves consists of a group centered on $x = 0$, plus repeating groups of the same shape periodically spaced along the x axis in both directions from $x = 0$. With Δx defined as the maximum amplitude to half-maximum amplitude width of Ψ, and $\Delta\kappa$ defined as the range of reciprocal wavelengths of the components of Ψ from maximum amplitude to half-maximum amplitude, we have $\Delta x \simeq 1/12$, $\Delta\kappa \simeq 1$, and $\Delta x \Delta\kappa \simeq 1/12$.

What does a 2 slit diffraction pattern look like?

What happens when you send a single photon through a slit?

What happens if you then send 1 million photons one at a time through the slit?

Chapter 3: Wave Nature of Matter

- de Broglie: Ph.D. thesis: Wave nature of matter.
- The dual wave-particle behavior of light applies equally well to matter.
- Received the Nobel prize 5 years later.
- Energy: $E=h\nu$
- Momentum: $p=h/\lambda$
- Wavelength: $\lambda=h/p$

Examples

- What is the wavelength of a fastball?

  m= 1 kg. v=10 m/s

- What is the wavelength of a 100 eV electron?

(used in MBE for low energy electron scattering)

$h=6.6 \times 10^{-34} Js$

Matter Wavelengths

- Baseball wavelength:
  - $\lambda = h/p = h/(mv) = 6.6 \times 10^{-34}/(1\ \text{kg}\ 10\ \text{m/s}) = 6.6 \times 10^{-35}$ m = 10^{-25} angstrom
- Electron wavelength: ($E = p^2/2m$)
  - 5.4×10^{-24} kg m/s
  - $\lambda = h/p = 6.6 \times 10^{-34}/5.4 \times 10^{-24}$ m/s $= 10^{-10}$ m $= 1$ A

Validation of matter waves: Davisson/Germer experiment

- Electrons incident on a single crystal sample (nickel).
- Angular dependence only explainable through diffraction, not particle scattering.

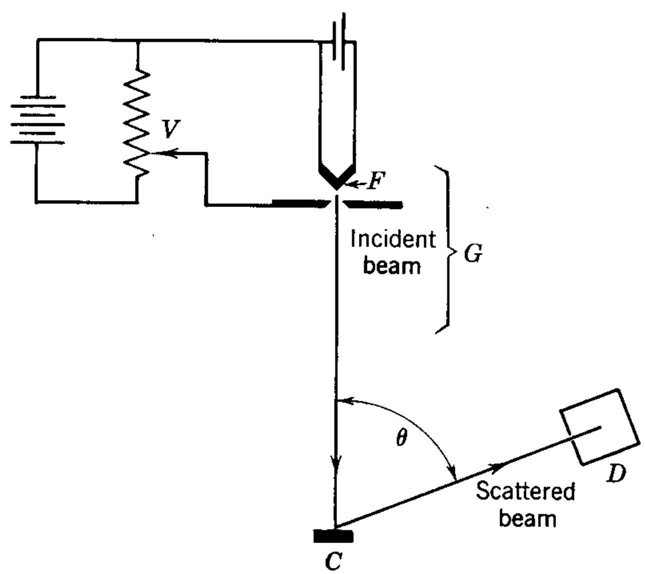
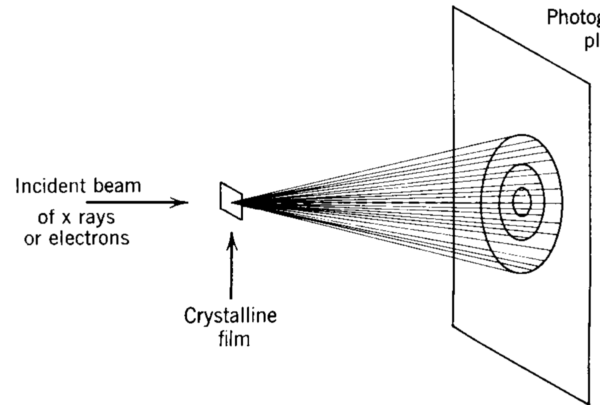


Figure 3-1 The apparatus of Davisson and Germer. Electrons from filament F are accelerated by a variable potential difference V. After scattering from crystal C they are collected by detector D.

X Ray Diffraction Electron Diffraction

- Bragg relation:
- $n\lambda = 2d \sin \varphi$



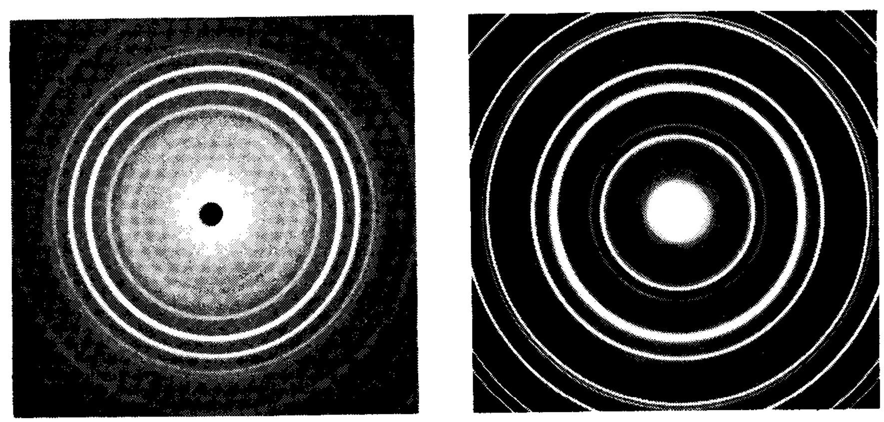


Figure 3-4 *Top:* The experimental arrangement for Debye-Scherrer diffraction of x rays or electrons by a polycrystalline material. *Bottom left:* Debye-Scherrer pattern of x-ray diffraction by zirconium oxide crystals. *Bottom right:* Debye-Scherrer pattern of electron diffraction by gold crystals.

Classical particle motion

- For large objects, the mass is large so the wavelength is very small, smaller than can be observed experimentally.

□ $\lambda = h/p = h/(mv)$

Principle of Complementarity

- Neils Bohr: The wave and particle models are complementary; if a measurement proves the wave character of radiation or matter, then it is impossible to prove the particle character in the same experiment.

- Which model is used (wave or particle) is determined by the experiment.

Light Particle/Wave Link

- Link provided by a probability interpretation.
- Wave: Intensity proportional to average electric field squared.
- Particle: Intensity equal to Nhν where N is the average number of photons per unit time per unit area (perpendicular to propagation vector).

 $I=\overline{E^2}/(\mu_0 c) = N\,h\nu$

Matter Particle/Wave Link

- Link provided by a probability interpretation.
- Particle: Probability density is proportional to average wave function squared.

- Classical physics: Laws of physics are deterministic. The laws of motion can be solved exactly and the position and momentum can be known for all time.

- Quantum physics: Laws of physics determine probability of finding a particle. The position and momentum cannot be known exactly at any time, much less all time.

- (Einstein famous objection: God does not play dice with the universe).

Fourier Analysis

- A gaussian transforms to a gaussian.
- Spectral analysis: A gaussian pulse in time is composed of a variety of frequencies, with an envelope that is a gaussian.
- Narrow frequency distribution means large time distribution
- Narrow pulse in time requires a large range of frequencies.

$\Delta t \, \Delta\omega > 1/2$

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$\Delta t \; \Delta\omega > ½$

If $E = h\nu = h\omega$, then

$\Delta t \; \Delta E > ½ \; h$

Heisenberg Uncertainty Principle

- One cannot simultaneously measure energy and time better than

  $\Delta t \, \Delta E > \frac{1}{2} \hbar$

- One cannot simultaneously measure moment and position better than

  $\Delta p_x \, \Delta x > \frac{1}{2} \hbar$

  $\Delta p_y \, \Delta y > \frac{1}{2} \hbar$

  $\Delta p_z \, \Delta z > \frac{1}{2} \hbar$

We will define Δx more carefully later. For now, it is the uncertainty in position.

- An attempt to measure position accurately requires high energy light (for example) which makes the momentum uncertain.

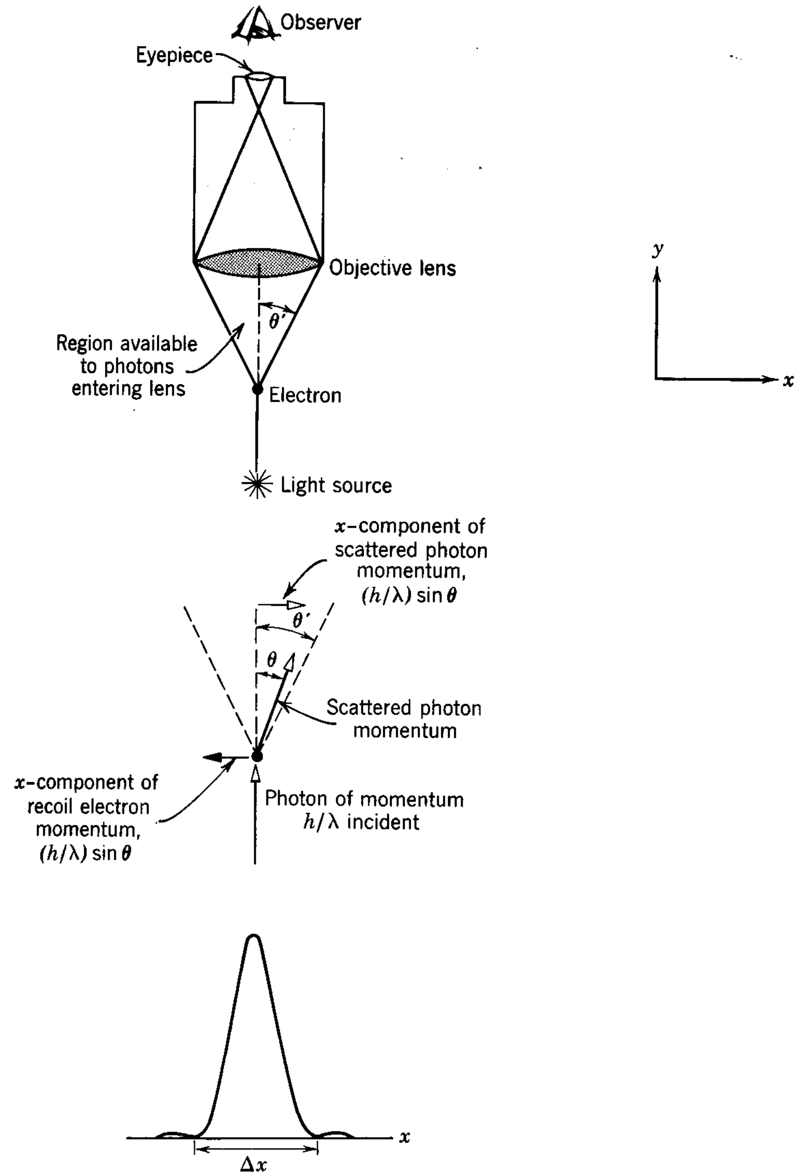

Figure 3-6 Bohr's microscope thought experiment. *Top:* The apparatus. *Middle:* The scattering of an illuminating photon by the electron. *Bottom:* The diffraction pattern image of the electron seen by the observer.

Particle/Wave Duality

- All material objects show both particle and wave aspects.
- The uncertainty principle means that an experiment to determine particle aspects (for example position) means that momentum is unknown (i.e. wavelength is unknown) and vice versa.

- Lectures posted at http://www.ece.ucsb.edu/courses/ECE162/162A_F07Bowers
- Assignment #1: Due next Tuesday in class
- There will be a homework set assigned every Tuesday which will be due at the beginning of class the following Tuesday. You are encouraged to work together on solving the homework problems but the final write up must be your own.

- Homework which is one day late can earn a maximum of 75 % of the total score, two days late 50 %, three days late 0. Homework turned in after the Tuesday class is considered 1 day late. The purpose of this policy is to allow Sid to discuss solutions during the Thursday discussion time. Your lowest score won't count towards your grade, so you can skip one homework completely if you are sick/travelling/busy, etc.