162A
Homework 4

Due Oct 28, 2 pm.
No late papers accepted. Solutions given in class.

1. Consider an electron in an infinite potential well of width L. At time y = 0, the state is described by

$$\Psi(x,0) = A\,(\,\psi_1(x) + \psi_2(x)\,)$$

Where ψ_1 and ψ_2 are the first two eigenstates of the well and A is the normalization coefficient.

a. Normalize $\Psi(x)$.
(Make use of the shape of ψ_1 and ψ_2 to simplify the calculations)

b. Draw a sketch of $\Psi(x)$ at $t=0^+$

c. Write an expression of the subsequent time evolution of the system $\Psi(x,t)$ in terms of energies E_1 and E_2 of ψ_1 and ψ_2

d. What is the time period (T) of the oscillations ?

e. Plot Ψ at t= 0 and at half the time period, t=T/2.

f. What is the expectation value of momentum ($\langle p \rangle$) ? Think…

2. Consider an infinite potential extending from –L/2 to +L/2. with a barrier of height V_0 inside the well extending from –a/2 to +a/2 (of course $a<L$)

a. Sketch the wave function for the case (E – energy of particle in the well)

i. $E > V_0$
ii. $E = V_o$

b. Determine the minimum barrier height V_0 such that the first eigen state has energy less than that of the barrier height V_0

3. A rectangular potential well in one dimension is bounded by a wall of height $5V_0$ (at x=0) on one side and a wall of height $2V_0$ (at x=L)on the other.

Suppose the second energy state for a particle of a mass m has energy $E_2 = V_0$

a. Plot ψ_2

b. Does the node of this wave function occur to the left or the right of the centre of the well (i.e to the right of L/2 or to the left).. and why ?

4. Consider a infinite potential extending from 0 to L

Write down the general solution for the state m, i.e $\Psi_{m.}$
For $\mathbf{m \neq n}$ prove that

$$\int \Psi_n (x)^* \Psi_m(x)\, dx = 0.$$

Note that this not true for m=n. This is a very important property of the solutions of the S.W.E –that they are **mutually orthogonal**