ECE 162A
Mat 162A
Lecture #2: Wavelike Properties of Matter

Read Chapter 3 of Eisberg,Resnick

- What is the classical theory of light?

Maxwell's Equations

$$\nabla \times H = I + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \bullet D = 0$$

$$\nabla \bullet B = 0$$

where E and H are the electric and magnetic field vectors
D and B are the electric and magnetic displacement vectors
No free charge. No current flow.

Constitutive Relations

$$D = \varepsilon_0 E + P$$

$$B = \mu_0 (H + M)$$

P and M are the electric and magnetic polarizations of the medium
ε_0 and μ_0 are the electric and magnetic permeabilities of vacuum
E and H are the electric and magnetic field vectors
D and B are the electric and magnetic displacement vectors

For isotropic media, Electric Susceptibility χ

Isotropic Media: χ is a complex number

$$P = \varepsilon_0 \chi E$$

The real part determines the index (velocity) and the imaginary part determines the gain or absorption.

Isotropic media: Vacuum, gasses, glasses (optical fibers)
Anisotropic media: Semiconductors, crystalline materials.

Wave Propagation in Lossless, Isotropic Media

- Lossless: σ=0, χ is real, ε is real.
- Isotropic: χ, ε are scalars (not tensors).

$$\nabla \times E = i + \frac{\partial B}{\partial t} = 0 + \mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = i + \frac{\partial D}{\partial t}$$

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$$\nabla \times (\nabla \times E) = \mu \frac{\partial (\nabla \times H)}{\partial t} = \mu \frac{\partial^2 D}{\partial^2 t} = \mu\varepsilon \frac{\partial^2 E}{\partial^2 t}$$

$$\nabla \times (\nabla \times E) = \nabla^2 E - \nabla(\nabla \bullet \vec{e})$$

Wave Equation $$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial^2 t}$$

Wave Equation

$$E(x,y,z,t) = \mathrm{Re}[E(x,y,z)e^{i\omega t}]$$

$$\nabla^2 \vec{E} + \omega^2 \mu\varepsilon \vec{E} = 0$$

$$\boxed{\nabla^2 \vec{E} + k^2 \vec{E} = 0}$$

where

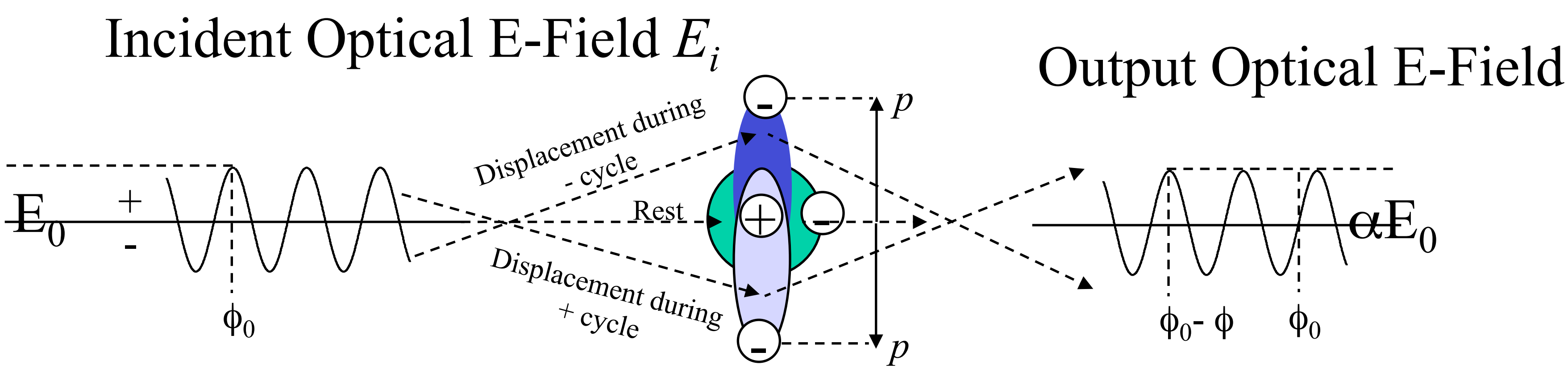
$$k = \omega\sqrt{\mu\varepsilon} = \omega n / c$$ Wave number

$$c = 1/\sqrt{\mu_0 \varepsilon_0}$$ Speed of light

$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}}$$ Index of refraction

Radiation and Atomic Systems

In a dielectric material, an incident optical E-field oscillating at frequency w will induce physical displacement of a bound electron from its rest state at the same oscillation frequency. We call the the ***induced dipole moment or displacement***



α is field attenuation

p is the induced dipole moment

α_p is the atomic polarizability

ϕ_0 is the input field phase reference

ϕ is the output field phase

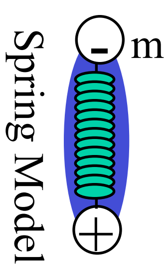
$$\mathbf{p} = \alpha_p \mathbf{E}_i$$

Classical Electron Model

- Now we need a basic model that describes the motion, or displacement of the electron (dipole) in the presence of an external force (incident optical field)
- Consider the classical equation of motion that describes displacement (position X) of an electron with charge *q* and mass *m* attached to an atom (as if connected by a spring) with a dampening coefficient (loss) γ and resonant frequency ω_0 in response to an applied field

$$m\frac{d}{dt^2}X + m\gamma\frac{d}{dt}X + m\omega_0^2 X = -qE_a$$

⇨ Lets assume the atom can be described as a two level system with energy levels E_1 and E_2 such that the atomic resonance is $\omega_0 = (E_2\text{-}E_1)/\hbar$ and the applied electric field is of the form $E=E_0e^{i\omega t}$. Then the eletron position (relative to rest) and induced dipole moment of the atom can be described as



$$X = \frac{-qE_0}{m\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}e^{i\omega t}$$

$$p = -qX = \frac{-q^2E_0}{m\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}e^{i\omega t}$$

Classical Electron Model

- The dielectric constant can now be written in terms of the spring model

$$p = \alpha_p E_a$$

$$\alpha_p = \frac{q^2}{m\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

$$n^2 = 1 + \chi = 1 + {N\alpha_p}/{\varepsilon_0} = 1 + \frac{Nq^2}{m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

⇨ And we can now re-write the electric susceptibility in terms of the classical electron model and define the real and imaginary parts of χ as χ' and χ''

$$\chi = \chi' - i\chi'' = \frac{Nq^2}{m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

Near Resonance Condition

- Near resonance, $\omega \approx \omega_0$, the second term in χ is small compared to the first term and the index of refraction can be approximated by $n=1+\chi/2$

$$n \approx 1 + \frac{\chi}{2} = 1 + \frac{Nq^2}{2m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

$$= 1 + \frac{Nq^2}{4\omega_0 m\varepsilon_0\left(\omega_0 - \omega + i\gamma / 2\right)}$$

Complex Refractive Index and Dispersion

- The real and imaginary parts of the electric susceptibility and index of refraction give rise to ***optical phase delay*** and ***optical absorption***.

⇨ We can separate the real and imaginary parts by multiplying numerator and denominator by the complex conjugate

$$\chi' - i\chi'' = \frac{Nq^2}{m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)} \frac{\omega_0^2 - \omega^2 - i\gamma\omega}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$= \frac{Nq^2\left(\omega_0 - \omega\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]} - i\frac{Nq^2\left(\gamma/2\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]}$$

$$\chi' = \frac{Nq^2\left(\omega_0 - \omega\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]}$$

$$\chi'' = \frac{Nq^2\left(\gamma/2\right)}{2m\omega_0\varepsilon_0\left[\left(\omega_0 - \omega\right)^2 + \left(\gamma/2\right)^2\right]}$$

Complex Refractive Index and Dispersion

- The complex index is (not assuming near resonance)

$$n' = n - i\kappa = 1 + \frac{Nq^2\left(\omega_0^2 - \omega^2\right)}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]} - i\frac{Nq^2\gamma\omega}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]}$$

⇨ For a propagating field, we can write

$$E = Ae^{i(\omega t - k'z)} = Ae^{i(\omega t - \frac{2\pi}{\lambda}(n - i\kappa)z)}$$

$$= Ae^{i(\omega t - \frac{2\pi}{\lambda}nz)}e^{-\frac{2\pi}{\lambda}\kappa z}$$

⇨ Where we can now define the attenuation coefficient α

$$\alpha = \frac{1}{I}\frac{dI}{dz}$$

$$I(z) = I(0)e^{-\alpha z}$$

$$\alpha = \frac{4\pi}{\lambda}\kappa$$

Dispersion and Complex Refractive Index

- Looking more closely at *n* from previous lecture (harmonic oscillator model),

$$n \approx 1 + \frac{\chi}{2} = 1 + \frac{Nq^2}{2m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

$$= 1 + \frac{Nq^2}{4\omega_0 m\varepsilon_0\left(\omega_0 - \omega + i\gamma / 2\right)}$$

- Using the notation for *n* as *n´* to denote a complex number, we see that as ω approaches ω_0 and assuming χ<<1, the index of refraction increases. This change in refractive index as a function of optical frequency is called ***Chromatic Dispersion***.
- Looking at the imaginary part of *n´* we see that the optical E-field will be attenuated as ω approaches ω_0 giving rise to ***Optical Absorption***.

$$n' = n - i\kappa = 1 + \frac{Nq^2\left(\omega_0^2 - \omega^2\right)}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]} - i\frac{Nq^2\gamma\omega}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]}$$

Chromatic Dispersion Absorption

Dielectric Constant and Index of Refraction

- For a homogeneous material of N atoms per unit volume, the polarization is defined by the contribution of all atoms in that volume that interact with the optical E-field

$$\mathbf{P} \approx N\mathbf{p} = N\alpha_p \mathbf{E_i} \equiv \varepsilon_0 \chi_e \mathbf{E}_i$$

⇨ Where we have defined the ***vacuum permittivity*** ε_0 and ***material electric susceptibility*** χ_ε by

$$N\alpha_p \equiv \varepsilon_0 \chi_e$$

⇨ Using the constitutive relation $\mathbf{D}=\varepsilon\mathbf{E}=\varepsilon_0\mathbf{E}+\mathbf{P}$, the ***dielectric constant*** is defined as

$$\varepsilon = \varepsilon_0 + \frac{\mathbf{P}}{\mathbf{E}} = \varepsilon_0 + \varepsilon_0\chi_e = \varepsilon_0\left(1+\chi_e\right)$$

$$= \varepsilon_0\left(1 + {N\alpha_p}/{\varepsilon_0}\right)$$

⇨ And we can now define for a non-magnetic medium (which dielectrics are at optical frequencies), the ***optical index of refraction n***

$$n^2 = 1 + \chi_e = 1 + {N\alpha_p}/{\varepsilon_0}$$

$$n = \sqrt{1+\chi_e} = \sqrt{1 + {N\alpha_p}/{\varepsilon_0}}$$

Prequantum Theory
Chapter 1: Thermal Radiation, Plank's Constant

- Light is a wave. Maxwell's equations give rise to a wave equation that explain light propagation quite well. (Undergrad E&M)
- Classical:
  - Wavelength may be quantized (satisfying boundary conditions)
  - Wave can have any energy (continuous)
- Classical theory predicts diffraction, refraction, propagation very well.
- Electromagnetic radiation spreads through space like water waves spread across water.

- What is wrong with classical theory of light?

Chapter 2: Light: particle and wave characteristics

- Photons: Particle like properties of radiation
- Interaction of light with matter:
  - Photoelectric effect
  - Compton effect
  - Pair production
  - Bremstralung
  - Pair annihilation
- All show experimental evidence of particle nature of light when interacting with matter.

What are these effects?

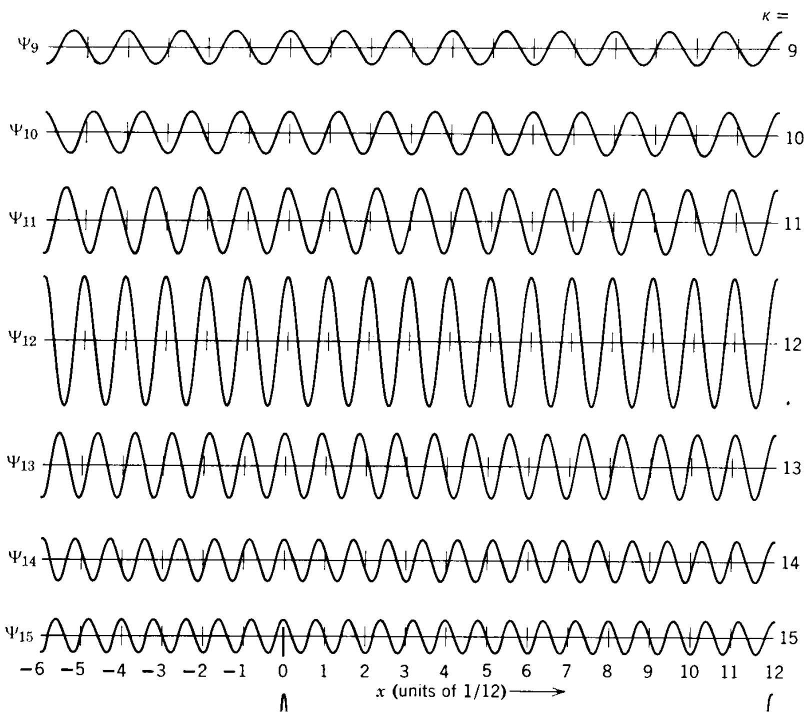
Dual Nature of Radiation

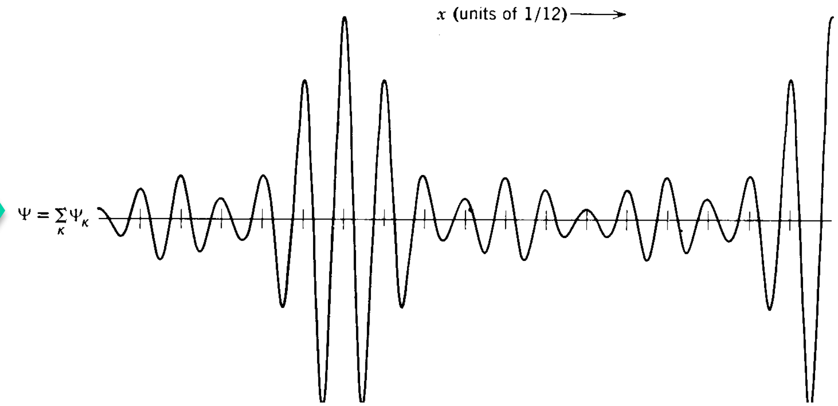
- **Radiation is neither purely a wave phenomenon nor purely a particle phenomenon.**
- A crystal spectrometer used to measure X ray wavelength is using the wave nature.
- A Compton scattering experiment is characterizing the particle nature.

- OK. So, light has both wave and particle nature.
- What is a photon?

So, what is a photon?

- A superposition of cw waves added in phase such that the envelope is a pulse.





What does a 2 slit diffraction pattern look like?

What happens when you send a single photon through a slit?

What happens if you then send 1 million photons one at a time through the slit?

Chapter 3: Wave Nature of Matter

- de Broglie: Ph.D. thesis: Wave nature of matter.
- The dual wave-particle behavior of light applies equally well to matter.
- Received the Nobel prize 5 years later.
- Energy: $E=h\nu$
- Momentum: $p=h/\lambda$
- Wavelength: $\lambda=h/p$

Examples

- What is the wavelength of a fastball?
  m= 1 kg. v=10 m/s

- What is the wavelength of a 100 eV electron?

(used in MBE for low energy electron scattering)

$h=6.6 \times 10^{-34} Js$

Matter Wavelengths

- Baseball wavelength:
  - λ=h/p=h/(mv)=6.6 x 10^{-34}/(1 kg 10 m/s)=6.6 x 10^{-35} m = 10^{-25} angstrom
- Electron wavelength: (E=p^2/2m)
  - 5.4 x 10^{-24} kg m/s
  - λ=h/p=6.6 x 10^{-34}/5.4 x 10^{-24} m/s=10^{-10} m=1 A

Validation of matter waves: Davisson/Germer experiment

- Electrons incident on a single crystal sample (nickel).
- Angular dependence only explainable through diffraction, not particle scattering

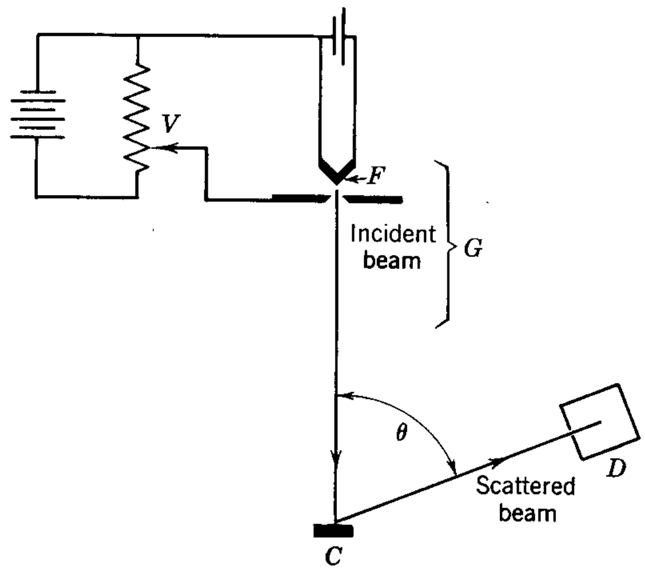


Figure 3-1 The apparatus of Davisson and Germer. Electrons from filament *F* are accelerated by a variable potential difference *V*. After scattering from crystal *C* they are collected by detector *D*.

X Ray Diffraction Electron Diffraction

- Bragg relation:
- $n\lambda = 2d \sin \varphi$

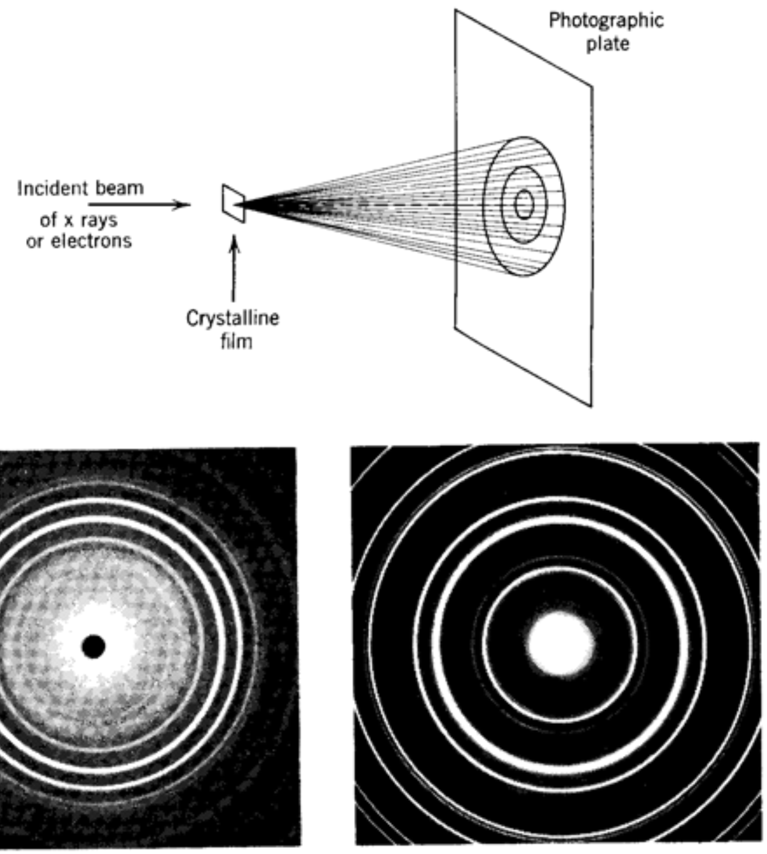


Figure 3-4 *Top:* The experimental arrangement for Debye-Scherrer diffraction of x rays or electrons by a polycrystalline material. *Bottom left:* Debye-Scherrer pattern of x-ray diffraction by zirconium oxide crystals. *Bottom right:* Debye-Scherrer pattern of electron diffraction by gold crystals.

Classical particle motion

- For large objects, the mass is large so the wavelength is very small, smaller than can be observed experimentally.
- $\lambda = h/p = h/(mv)$

Principle of Complementarity

- Neils Bohr: The wave and particle models are complementary; if a measurement proves the wave character of radiation or matter, then it is impossible to prove the particle character in the same experiment.

- Which model is used (wave or particle) is determined by the experiment.

Light Particle/Wave Link

- Link provided by a probability interpretation.
- Wave: Intensity proportional to average electric field squared.
- Particle: Intensity equal to $Nh\nu$ where N is the average number of photons per unit time per unit area (perpendicular to propagation vector).
- $I=\overline{E^2}/(\mu_0 c) = N\, h\nu$

Matter Particle/Wave Link

- Link provided by a probability interpretation.
- Particle: Probability density is proportional to average wave function squared.

Classical and Quantum Physics

- Classical physics: Laws of physics are deterministic. The laws of motion can be solved exactly and the position and momentum can be known for all time.
- Quantum physics: Laws of physics determine probability of finding a particle. The position and momentum cannot be known exactly at any time, much less all time.
- (Einstein famous objection: God does not play dice with the universe).

Fourier Analysis

- A gaussian transforms to a gaussian.
- Spectral analysis: A gaussian pulse in time is composed of a variety of frequencies, with an envelope that is a gaussian.
- Narrow frequency distribution means large time distribution
- Narrow pulse in time requires a large range of frequencies.

$\Delta t \, \Delta\omega > \frac{1}{2}$

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If $E = h\nu = h\omega$, then

$\Delta t \, \Delta E > \frac{1}{2} h$

Heisenberg Uncertainty Principle

- One cannot simultaneously measure energy and time better than

$$\Delta t \, \Delta E > \tfrac{1}{2} \hbar$$

- One cannot simultaneously measure moment and position better than

$$\Delta p_x \, \Delta x > \tfrac{1}{2} \hbar$$

$$\Delta p_y \, \Delta y > \tfrac{1}{2} \hbar$$

$$\Delta p_z \, \Delta z > \tfrac{1}{2} \hbar$$

We will define Δx more carefully later. For now, it is the uncertainty in position.

- An attempt to measure position accurately requires high energy light (for example) which makes the momentum uncertain.

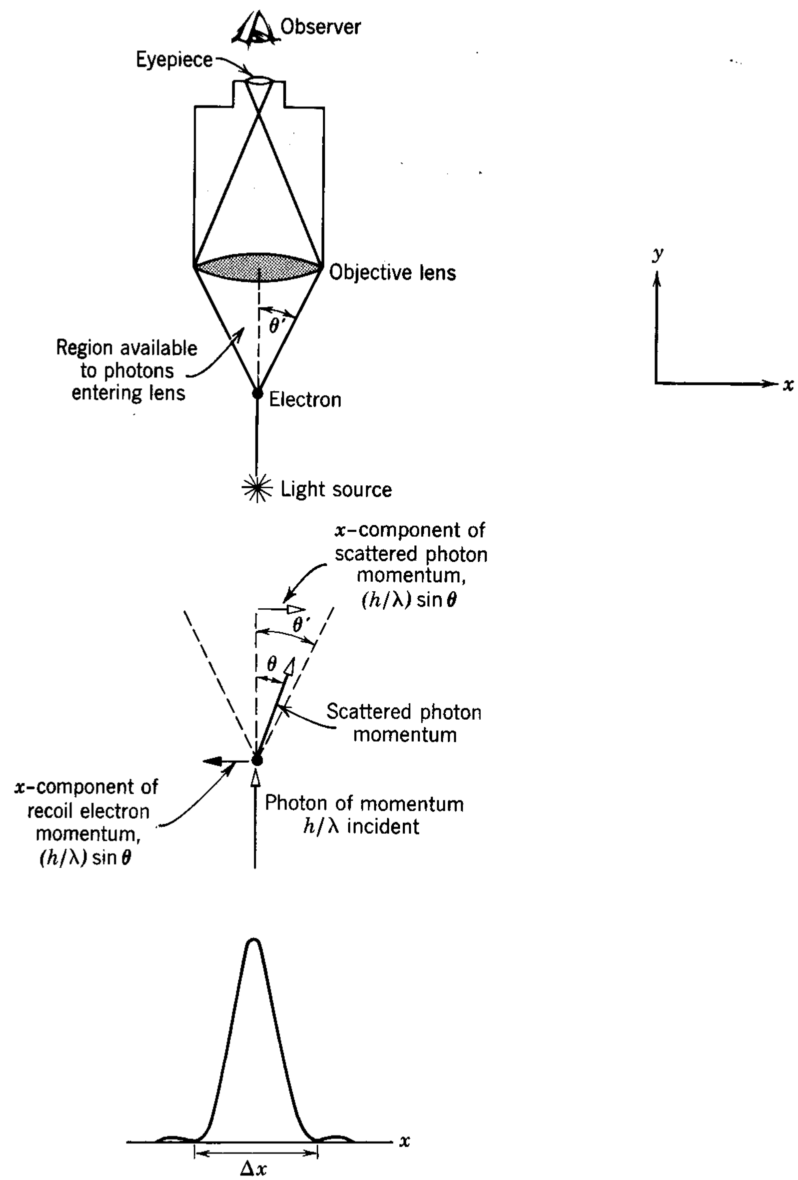

Figure 3-6 Bohr's microscope thought experiment. *Top:* The apparatus. *Middle:* The scattering of an illuminating photon by the electron. *Bottom:* The diffraction pattern image of the electron seen by the observer.

Particle/Wave Duality

- All material objects show both particle and wave aspects.
- The uncertainty principle means that an experiment to determine particle aspects (for example position) means that momentum is unknown (i.e. wavelength is unknown) and vice versa.

- Assignment #1: Assigned Tuesday Oct. 6th on the web
- No class next Tuesday
- Recitation only on Thursday Oct. 8th
- Makeup class Friday Oct. 9th
- Assignment #1 due Tuesday 13th