ECE 162A
Mat 162A

Lecture #3: Early Quantum Theory
Read Chapter 4 of Eisberg,Resnick

Principle of Complementarity

- Neils Bohr: The wave and particle models are complementary; if a measurement proves the wave character of radiation or matter, then it is impossible to prove the particle character in the same experiment.

- Which model is used (wave or particle) is determined by the experiment.

Particle/Wave Duality

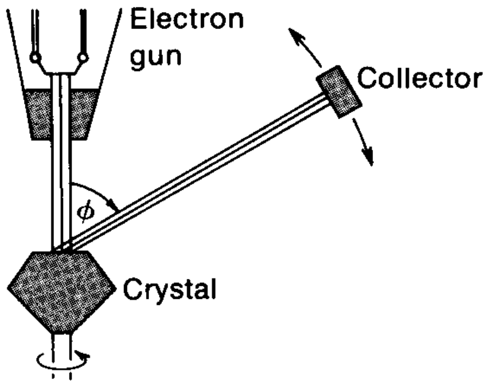
- All material objects show both particle and wave aspects.
  - $E=h\nu$
  - $P=h/\lambda$
- The uncertainty principle means that an experiment to determine particle aspects (for example position) means that momentum is unknown (i.e. wavelength is unknown) and vice versa.

Particles are waves?

- Radical notion proposed by deBroglie.
- To demonstrate, we need demonstrate diffraction or refraction.
- So, calculate wavelength, and find a grating on that scale.
  - What is the wavelength of a 100 eV electron?
  - $E=p^2/2m$
  - 5.4×10^{-24} kg m/s
  - $\lambda=h/p=6.6 \times 10^{-34}/5.4 \times 10^{-24}$ m/s$=10^{-10}$ m$=1$ A
  - Crystal spacing on this order. So, use crystals.

Davidson Germer Experiment

Proof of the wave
Nature of electrons



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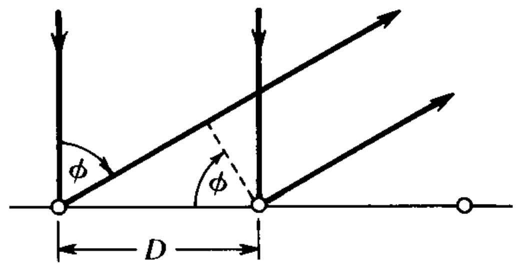

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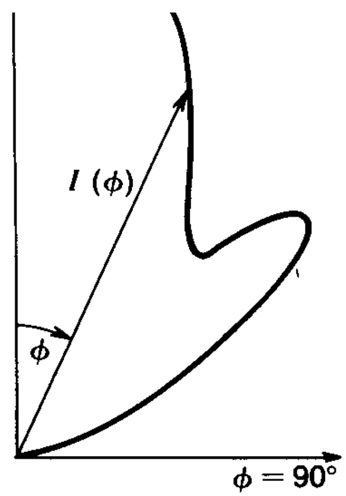
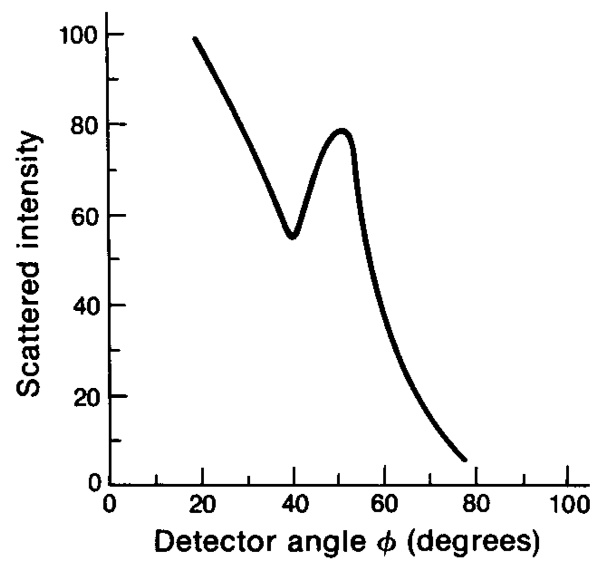


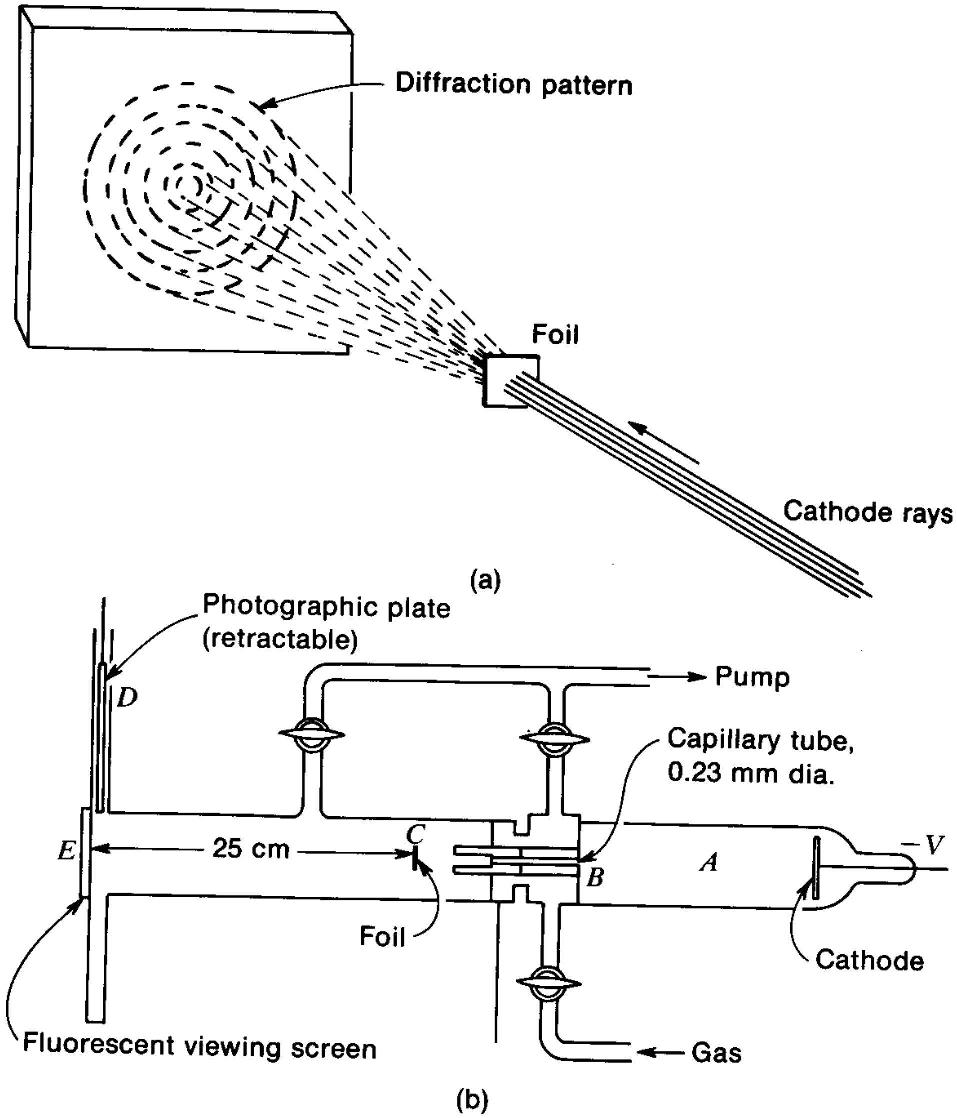
(a)



(b)

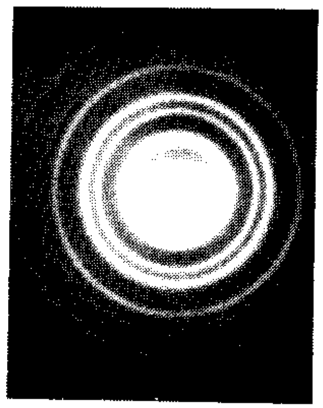
Thompson's Electron Scattering Experiment

Fig. 2-11 (a) Because Thomson's foil consisted of portions of many individual crystals, the diffraction pattern consisted of concentric circles rather than the individual spots obtained when a slice of a single crystal is used. [Recall the single-crystal x-ray diffraction pattern of Figure 1-19(b).] (b) G.P. Thomson's electron diffraction apparatus. Cathode rays generated in tube A passed through collimating tube B before striking thin foil C. The transmitted electrons struck the fluorescent screen E, or a photographic plate D which could be lowered into the path

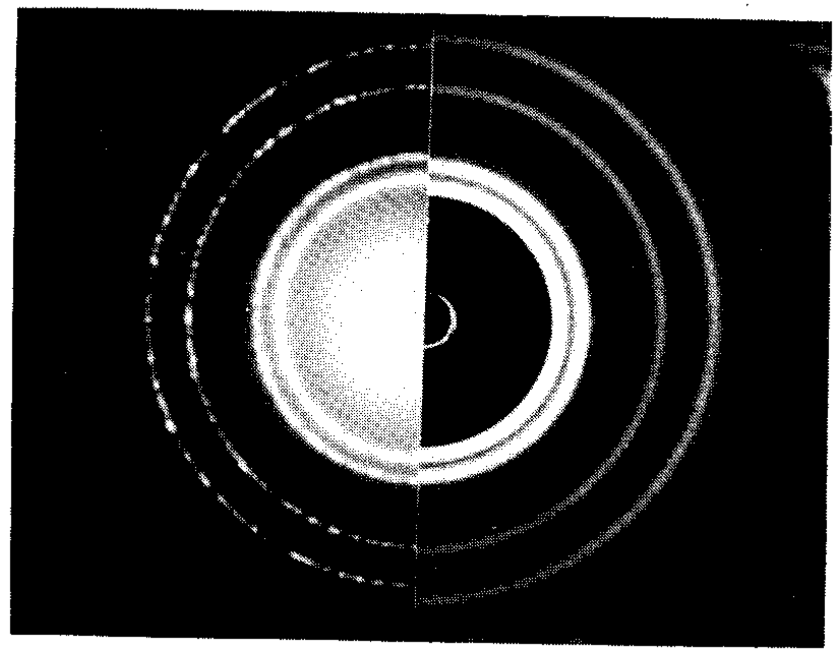


Electron and Xray diffraction

Fig. 2-12 (a) Electron diffraction pattern obtained by G. P. Thomson using a gold foil target. (b) Composite photograph showing diffraction patterns produced with an aluminum foil by x rays and electrons of similar wavelength. (Courtesy of Film Studio, Education Develop-



(a)

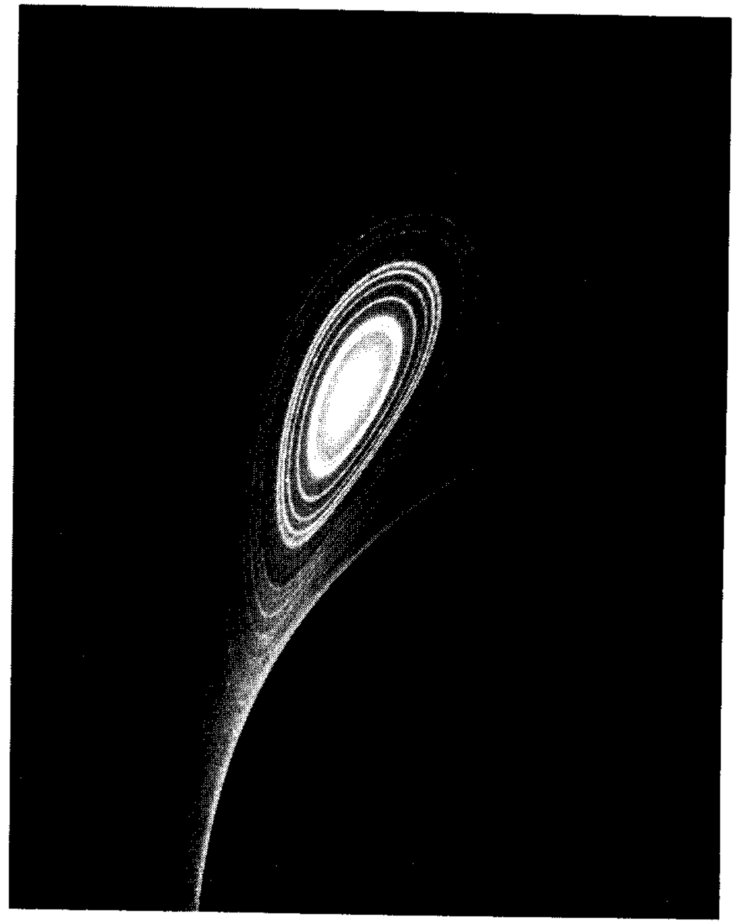


(b)

Effect of magnet on diffraction pattern:

ment Center, Newton, Mass.) (c) Diffraction pattern produced by 40 keV electrons passing through zinc oxide powder. The distortion of the pattern was produced by a small magnet which was placed between the sample and the photographic plate. An x-ray diffraction pattern would not be affected by a magnetic field! (Photograph made by Dr. Darwin W. Smith; reproduced by permission of R. B. Leighton.)

(c)



Xray propagation not affected by dc magnetic field
Electron path is distortion by magnetic field

Heisenberg Uncertainty Principle

- One cannot simultaneously measure energy and time better than

  $\Delta t \, \Delta E > \frac{1}{2} \hbar$

- One cannot simultaneously measure moment and position better than

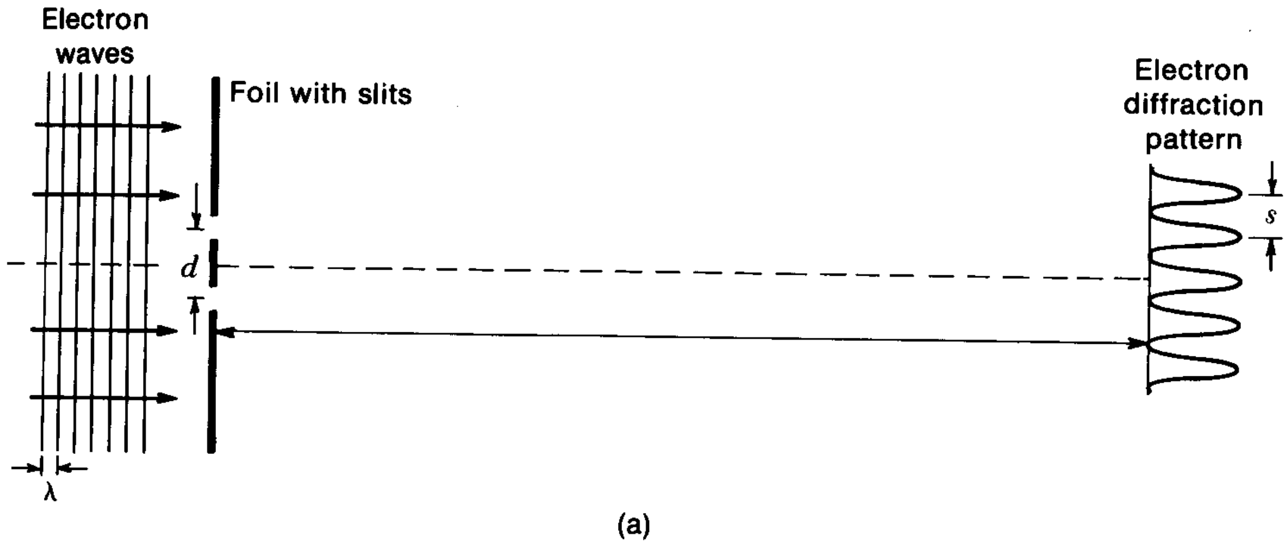
  $\Delta p_x \, \Delta x > \frac{1}{2} \hbar$

  $\Delta p_y \, \Delta y > \frac{1}{2} \hbar$

  $\Delta p_z \, \Delta z > \frac{1}{2} \hbar$

We will define Δx more carefully later. For now, it is the uncertainty in position.

Electron 2 slit diffraction (Jonsson)



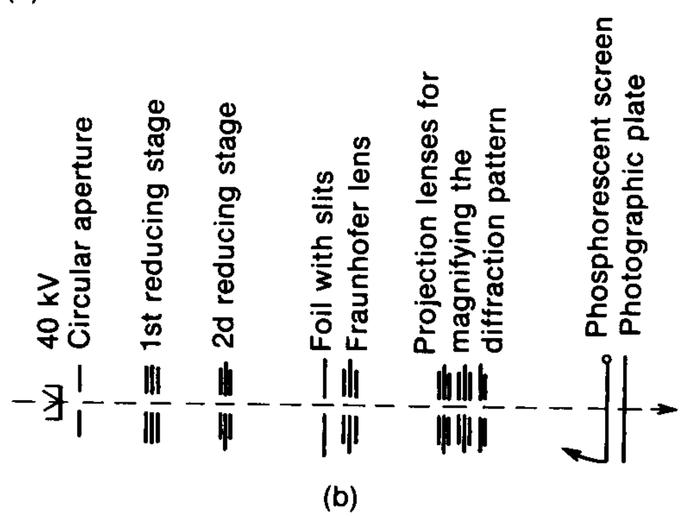


Fig. 2-14 (a) Electron diffraction analog to Young's double-slit experiment. (b) Jönsson's actual arrangement, including the electrostatic lenses which were used to magnify the tiny fringe pattern. [Note: *These drawings are not to scale.*]

Two slit interference

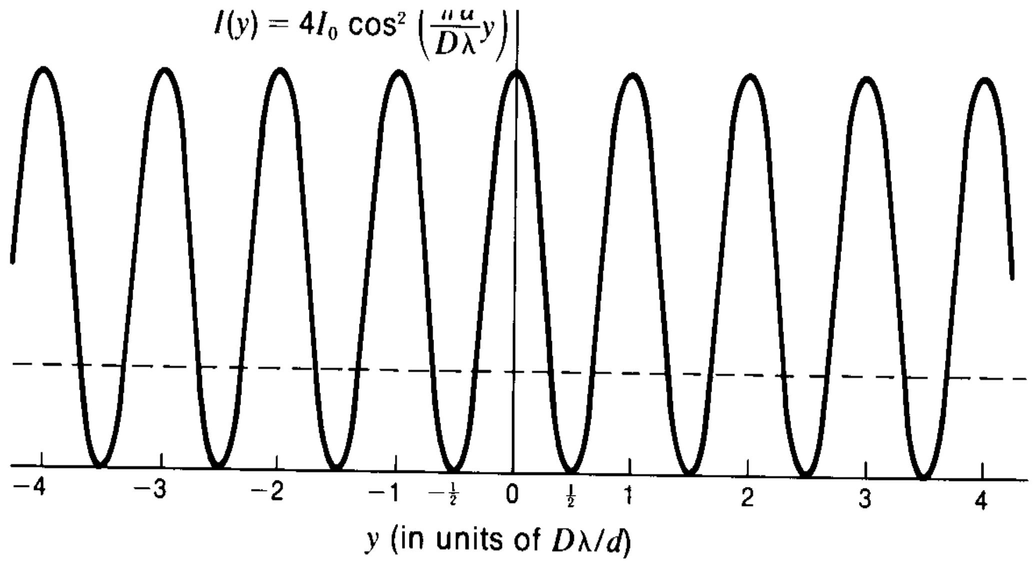
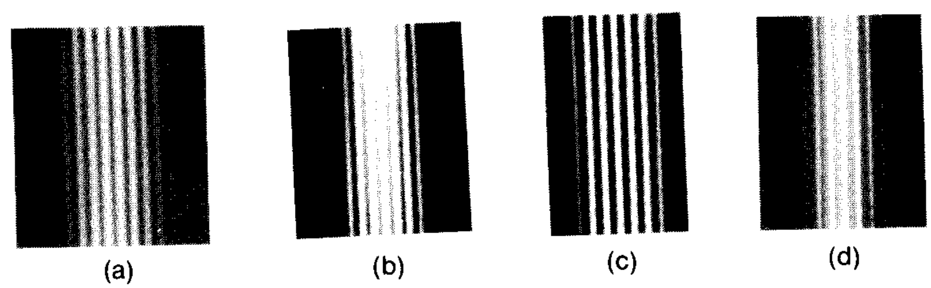


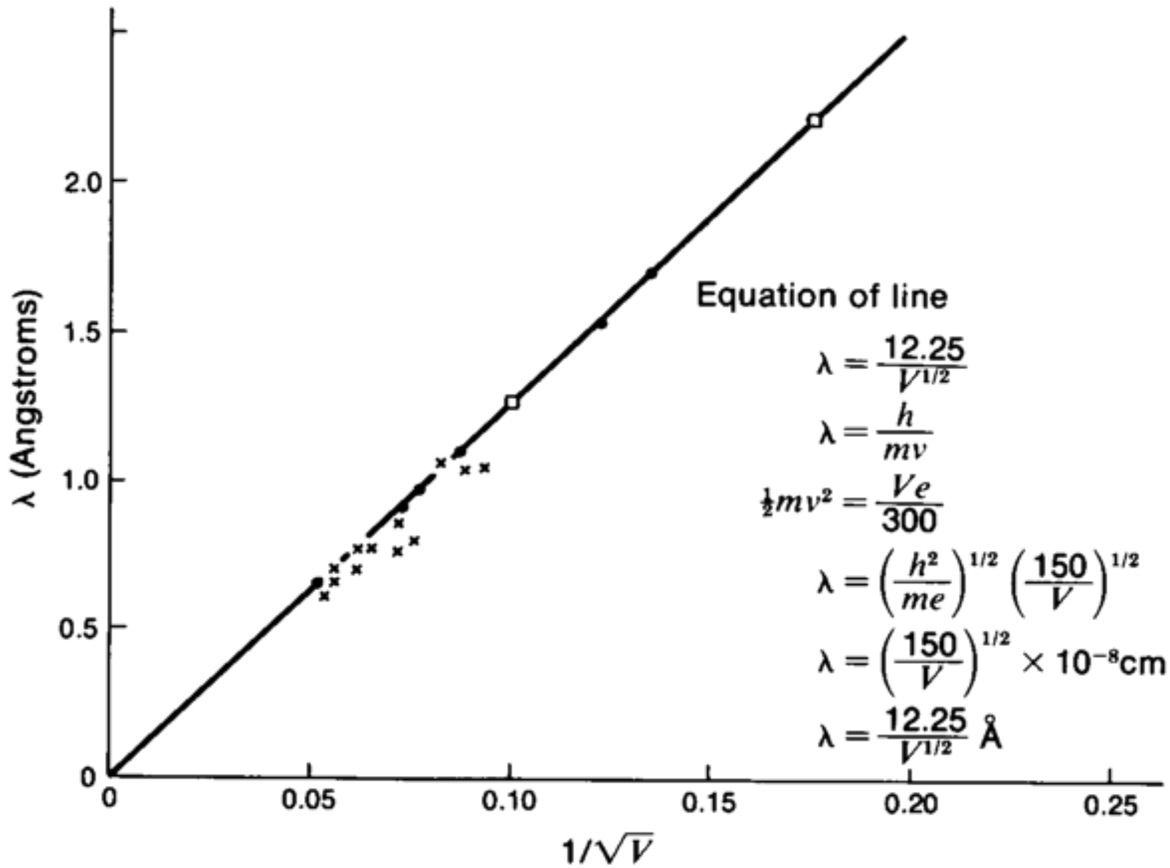
Fig. 2-15 Multislit diffraction patterns for electrons obtained by Jönsson. (a) two-slit pattern, (b) three-slit pattern, (c) four-slit pattern, (d) five-slit pattern. (From Jönsson, op. cit. Reproduced with permission of the author)



2 3 4 5 Slits

Verification of de Broglie Formula

Fig. 2-5 Experimental test of the de Broglie formula $\lambda = h/p$ as applied to electrons. The abscissa is $1/\sqrt{V}$, where V is the accelerating voltage. The ordinate is observed wavelength, as obtained from diffraction experiments. The solid line is the prediction of the de Broglie relation, $\lambda = \sqrt{150/V}$. (Figure adapted from Nobel Lectures: Physics, *Elsevier, Amsterdam and New York, © Nobel Foundation, 1965.)*



Early Quantum Theory

Chapter 4: Bohr's Model of the Atom

- 1910 Thompson's model:
  - Atoms contain electrons (as shown by Compton effect, photoelectric effect,…)
  - Atoms are neutral so their charge must be balanced by something.
  - Experimental evidence that the number of electrons in an atom is equal to atomic weight/2
  - Thompson proposed that the electrons existed within a continuous sea of positive charge.

Rutherford Scattering

- Rutherford, a former student of Thompson, tested this theory by scattering experiments of alpha particles (He ++) on thin foils of metal.
- Most alpha particles showed small angle scattering. A few showed large angle scattering.
- An alpha particle is 10,000 times heavier than an electron.
- This is analogous to firing a “15 inch shell at a piece of tissue paper and it came back and hit you.” – Rutherford.

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How is this possible?
Think about pool.
There must be particles of similar mass to He++

Rutherford Model

– Rutherford proposed that all of the positive charge and essentially all of the mass were concentrated on a small region in the center called the nucleus.
– Most scattering is electron-alpha scattering (small angle).
– Occasionally, an alpha particle scatters off a nucleus giving a large angle (pool ball like) scattering.

Rutherford model

- This explains alpha particle scattering, but raises questions:
- The electron is attracted to the nucleus, so why doesn’t it spiral into the nucleus?
- Classically, the electron must be circling the nucleus analogous to planets revolving round the sun. **But, bending the path of an electron is known (experimentally and theoretically) to cause the electron to emit light. Why doesn’t the electron emit light, lose energy, and spiral into the nucleus? (All accelerated charges radiate electromagnetic energy).**
- (Rutherford scattering shows that the electron radii must be of the order of the atom to atom spacing).
- Further data is needed.

Atomic Spectra

- When atoms are excited by an electric discharge, they emit light at specific frequencies (emission spectrum).
- The visible part of the atomic spectrum of hydrogen is shown below
- Balmer found the wavelengths in the visible follow the formula
- $\lambda = 3646\, n^2/(n^2-4)$ Å.

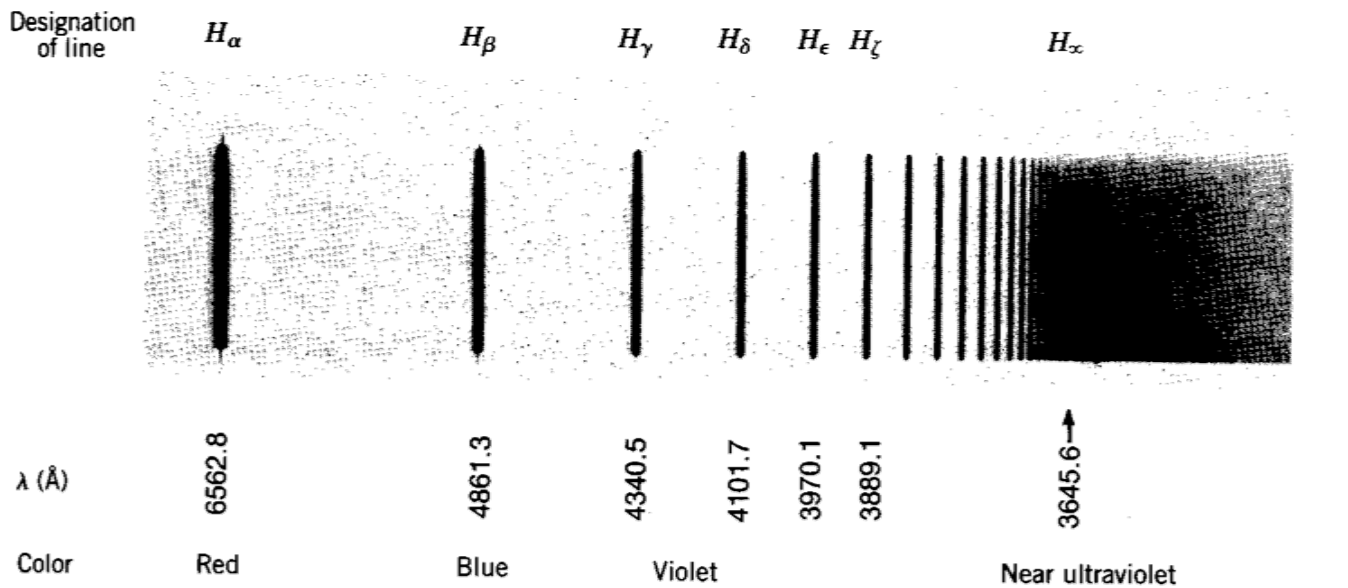


Figure 4-10 A photograph of the visible part of the hydrogen spectrum. (*Spectrum from W. Finkelnburg, Structure of Matter, Springer-Verlag, Heidelberg, 1964.*)

Balmer Formula

- Additional lines were discovered, and found by Rydberg to follow the formula (in reciprocal wavelengths κ)

Rydberg constant (exp)

- $\kappa = 1/\lambda = R_H\,(1/p^2 - 1/n^2)$
- P=1 Lyman series (ultraviolet)
- P=2 Balmer series (visible)
- P=3 Paschen series (infrared)
- P=4 Brackett series (infrared)
- P=5 Pfund series (infrared)
- Emission: all series observed
- Absorption spectrum: Only Lyman series is observed. **Why?**

Frank hertz exp

Tube with mercury vapor

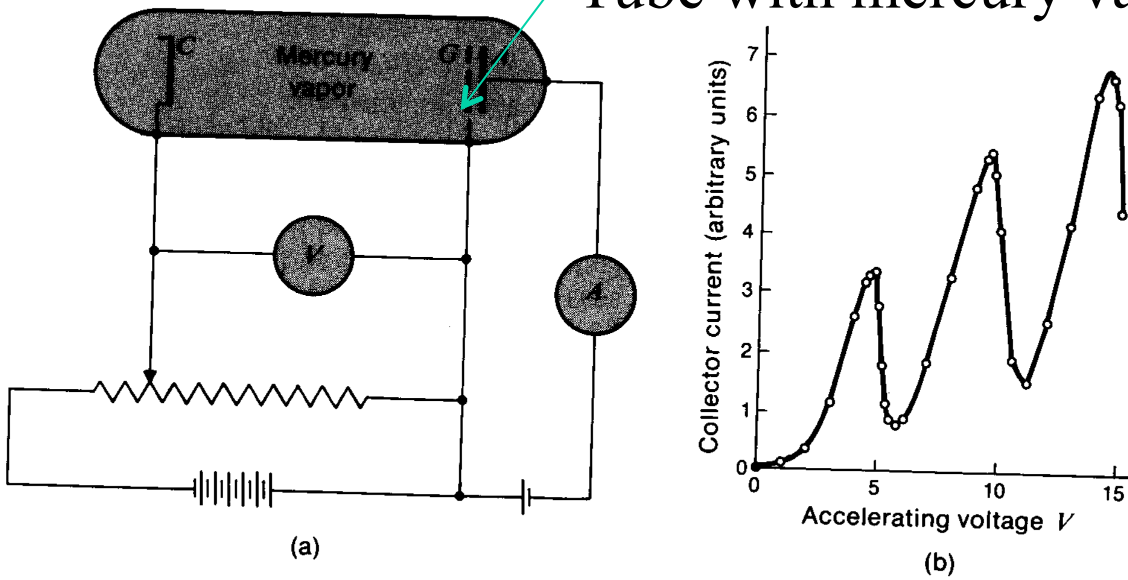


Fig. 1-11 The Franck-Hertz experiment.
(a) Schematic diagram of experimental arrangement.
(b) Collector current versus accelerating voltage.
The sharp decreases in current near $V = 10$ V and again near $V = 15$ V result when electrons undergo multiple (two and three, respectively) inelastic collisions.

At a voltage of 4.9V, light is emitted from Mercury vapor with the appropriate wavelength. Evidence of quantization of energy.

Problems

- Why doesn't the electron emit radiation and spiral into the nucleus?
- Why does hydrogen emit in wavelengths given by the Balmer formula?
- Why are the emission and absorption spectra different?
- Why do other atoms have different spectra?
  Alkali: $\kappa = 1/\lambda = R_H\ (1/(p\text{-}a)^2 - 1/(n\text{-}b)^2)$
- Others: More complicated.

Solution: Niels Bohr

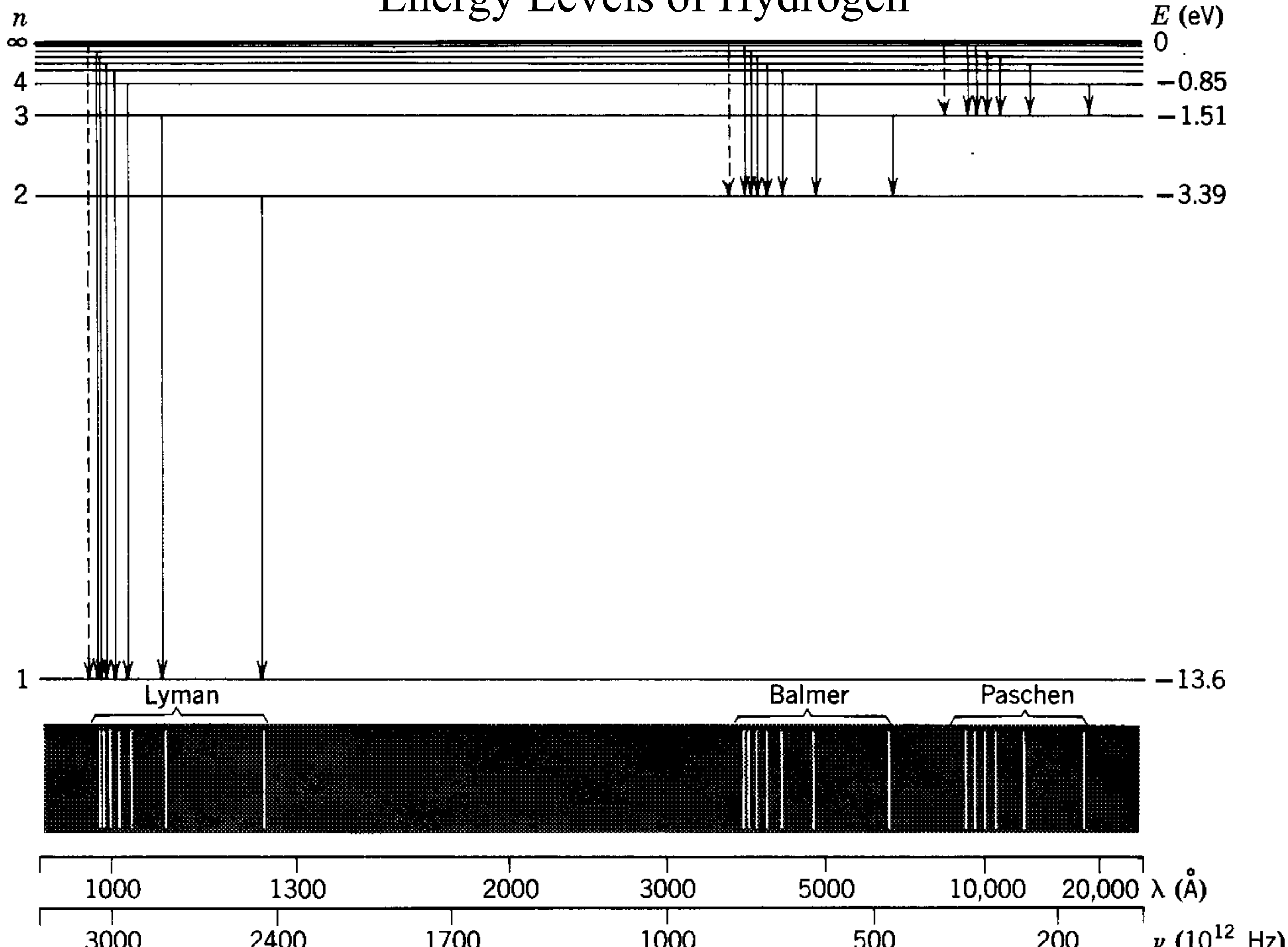
- Bohr Postulates:
- Electrons move in a circular orbit obeying the laws of classical mechanics: F=ma or

$1/(4\pi\varepsilon_0)\ Ze^2/r^2 = m\ v^2/r$

- The only orbits allowed are the ones where its orbital angular momentum L is an integral multiple of ħ:

  $mrv = n\hbar$ (Plank proposed that energy is quantized).
- An electron in an allowed orbit does not radiate.
- An electron can absorb or emit photons with energy $E=h\nu$ and energy is conserved $E_i-E_f=h\nu$
- Result: $E=E_0/n^2$
- So $k=1/\lambda = \nu/c = (E_i-E_f)/(hc) = R_H/(hc)\ (1/p^2 - 1/n^2)$ where p is the number of the initial state and n is the number of the final state.

Energy Levels of Hydrogen



Bohr Model

- Correctly explained the known Balmer Paschen series (Hydrogen)
- Predicted the Lyman, Brackett and Pfund series, which were subsequently discovered. (Hydrogen)
- It also worked for He+

Fine structure splitting

When the spectral lines of the hydrogen spectrum are examined at very high resolution, they are found to be closely-spaced doublets. This splitting is called fine structure (and was one of the first experimental evidences for electron spin).

How to explain with Bohr theory?

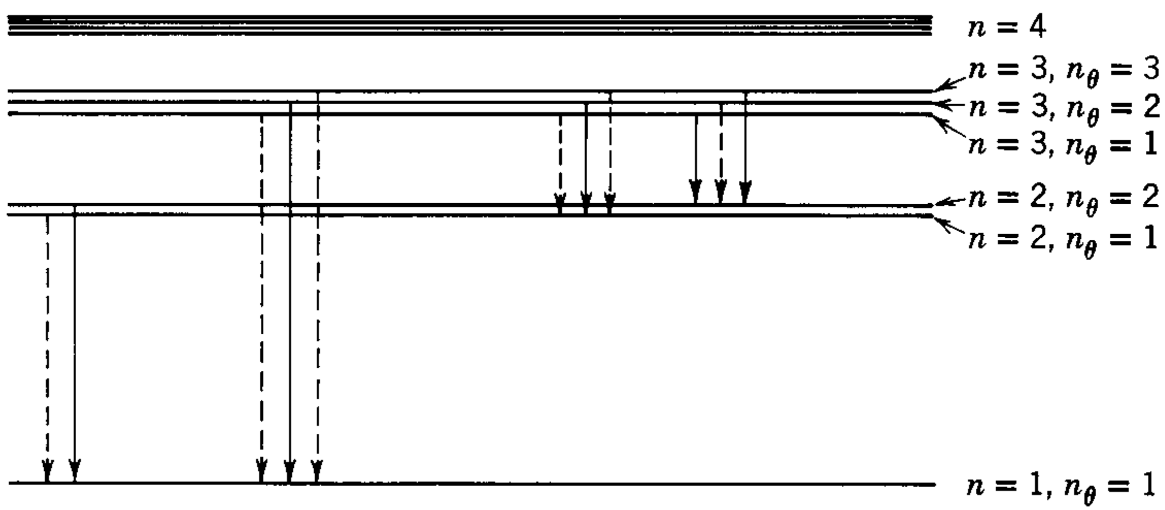


Figure 4-19 The fine-structure splitting of some energy levels of the hydrogen atom. The splitting is greatly exaggerated. Transitions which produce observed lines of the hydrogen spectrum are indicated by solid arrows.

Fine structure splitting

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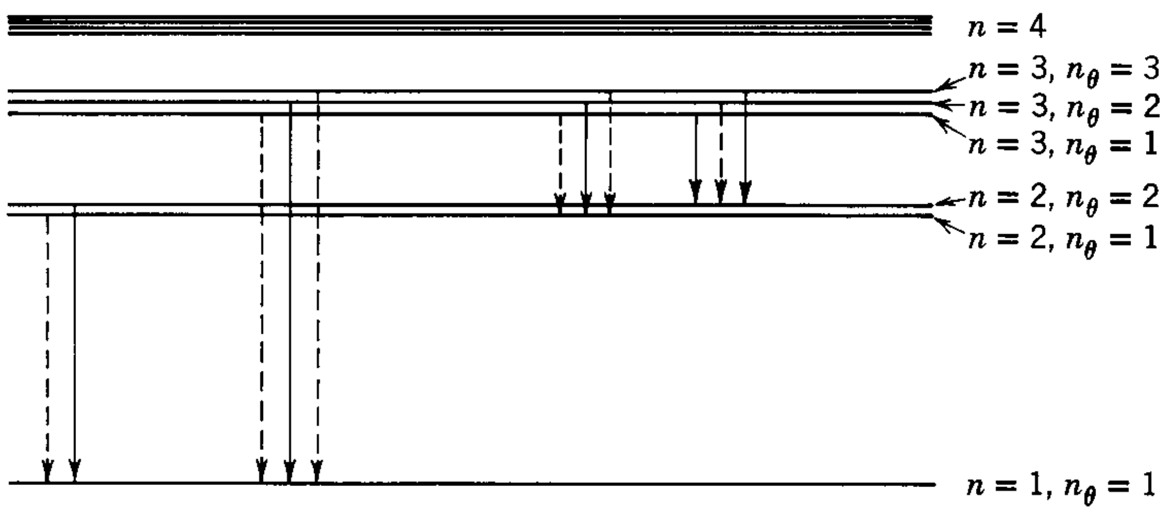
How to explain with Bohr theory?
Sommerfeld's model: Attempt to explain using elliptical orbits. . Treat relativistically.



However, dashed lines don't appear experimentally. Why? Selection rules….

Figure 4-19 The fine-structure splitting of some energy levels of the hydrogen atom. The splitting is greatly exaggerated. Transitions which produce observed lines of the hydrogen spectrum are indicated by solid arrows.

Correspondence Principle

- Bohr, 1923.
- The predictions of the quantum theory for the behavior of any physical system must correspond to the prediction of classical physics in the limit in which the quantum numbers specifying the state of the system become very large.
- A selection rule holds true over the entire range of the quantum number concerned. Thus, any selection rules which are necessary to obtain the required correspondence in the classical limit (large n) also apply in the quantum limit (small n).

Old (Bohr Model) Quantum Theory Problems

- It only works for one electron systems:
  - Hydrogen, alkali elements
  - It fails for He, and most other elements.
  - It allows the calculation of energies, but not of rates of transition.
- It is intellectually unsatisfying; why is momentum quantized? Why don’t electrons radiate?

Summary

- Experimental evidence for quantization of energy and angular momentum
- Bohr theory works for one electron atoms (H, He+, Li, Na, K,…)
- Bohr theory fails for He and most other atoms.
- Rates of transitions not predicted.
- Next: Schroedinger's Theory of Quanum Mechanics (Chapter 5)