ECE 162A
Homework 1 Solutions

1. From Merriam-Webster

   Determinism - a theory or doctrine that acts of the will, occurrences in nature, or social or psychological phenomena are causally determined by preceding events or natural laws
   Causality - the relation between a cause and its effect or between regularly correlated events or phenomena

   The probabilities associated with games of chance may support or reject a strictly deterministic world, depending on the position of the observer. If there are natural laws governing the outcome of these games, an unaware observer might assign various probabilities to make reasonable predictions. On the other hand, it is the relative frequency of the random outcomes in a series of events that allow us to make predictions in games of chance. If the events are truly random, then games of chance do not support determinism.

   While games of chance have associated probabilities, the definitive outcome of a future event is typically unknown. On a large scale, this randomness rejects any notion of causality. If a six-sided die is not perfectly symmetric or other irregularities exist, the outcome might change which introduces a certain degree of causality.

2. An electron and photon have a wavelength (λ) of 2 Å (2×10^{-10} m)

   A) Electron and photon momentum is defined as:

$$k = \frac{h}{\lambda}$$

   where h is Planck's constant. The electron and photon both have a momentum of:

$$k = \frac{6.626 \times 10^{-34}\, m^2 \cdot kg \cdot s^{-1}}{2 \times 10^{-10}\, m} = 3.313 \times 10^{-24}\, kg \cdot m \cdot s^{-1}$$

   B) The photon energy, which is entirely kinetic, is defined as:

$$KE = E = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34}\, J \cdot s\right) \cdot \left(3 \times 10^{8}\, m \cdot s^{-1}\right)}{2 \times 10^{-10}\, m} = 9.939 \times 10^{-16}\, J$$

   Since an electron has rest mass, the following equations needs to be used to calculate total energy:

$$E^2 = (cp)^2 + (Mc)^2 = \left(3\times10^8 m\cdot s^{-1}\cdot 3.313\times10^{-24} J\cdot s\right)^2 + \left(9.11\times10^{-31} kg\cdot 3\times10^8 m\cdot s^{-1}\right)^2$$

$$E = \left((cp)^2 + (Mc)^2\right)^{1/2} = \left(\left(9.939\times10^{-16} J\right)^2 + \left(8.19\times10^{-14} J\right)\right)^{1/2} = 8.19\times10^{-14} J$$

The kinetic energy of an electron can be written in many forms, such as (cp) above and the one shown below:

$$KE = \frac{h^2}{2m_e\lambda} = \frac{\left(6.626\times10^{-34} J\cdot s\right)^2}{2\cdot 9.11\times10^{-31} kg\left(3\times10^8 m\cdot s^{-1}\right)} = 6.031\times10^{-18} J$$

C) The kinetic energy for an electron is three orders of magnitude smaller than that of a photon with equal de Broglie wavelengths of 2Å.

3. XRay emission from a TV is well below hazardous levels, possibly due to a number of reasons. These include:
   a. A fairly thick fluorescent screen has a large number of atoms, any of which can reduce the momentum of a high energy electron. This process is known produce higher energy photons such as XRays (see Bremsstrahlung). These photons have a reasonably high probability of colliding with other atoms before reaching the viewing side of the TV, greatly reducing in energy/momentum on the way.
   b. Besides the fluorescent screen, TV manufacturers added a thin lead coating to their screens. Lead has a high atomic number (82), which means that it has a large number of electrons that serve as scattering mechanisms for high energy photons.

4. For 1.94 cal/(cm^{2}min) of incident sunlight with an average wavelength of 5500 Å, photocurrent can be found as follows:

   Convert calories, a unit of heat, to energy:

   $$1cal = 4.184J$$

   $$1.94 cal\cdot cm^{-2}\cdot \min^{-1}\cdot \left(\frac{4.184J}{cal}\right)\left(\frac{1\min}{60\sec}\right) = 13.52\times10^{-2} J\cdot cm^{-2}\cdot s$$

   We can now calculate the energy of a photon with λ = 5500 Å

   $$E_{ph} = \frac{6.626\times10^{-34} m^2\cdot kg\cdot s^{-1}\cdot\left(3\times10^8 m\cdot s^{-1}\right)}{5500\times10^{-10} m} = 3.61\times10^{-19} J\cdot photon^{-1}$$

   The number of photons is the total energy divided by the energy per photon:

$$N_{ph} = \frac{13.52 \times 10^{-2} J \cdot cm^{-2} \cdot s}{3.61 \times 10^{-19} J \cdot photon^{-1}} = 3.75 \times 10^{17} photons \cdot cm^{-2} \cdot s^{-1}$$

Assuming that every photon is absorbed and generates an electron that can be immediately collected, the photocurrent is given as:

$$J_{ph} = 1.602 \times 10^{-19} C \cdot 3.75 \times 10^{17} photons \cdot cm^{-2} \cdot s^{-1} \approx 60 mA \cdot cm^{-2}$$

5. The Compton wavelength is given as:

$$\lambda_C \equiv \frac{h}{m_o c}$$

which, for a proton scattering a photon, equals:

$$\lambda_C = \frac{6.626 \times 10^{-34} m^2 \cdot kg \cdot s^{-1}}{1.673 \times 10^{-27} kg \cdot \left(3 \times 10^8 m \cdot s^{-1}\right)} = 1.32 \times 10^{-15} m$$

The wavelength shift associated with Compton scattering is:

$$\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_C \left(1 - \cos(\phi)\right)$$

which has a maxima when $\cos(\phi) = -1$, or when $\phi = \pi$. This angle corresponds to a "head-on" collision of the photon with the proton.

$$\Delta\lambda_{\max} = \lambda_C \left(1 - \cos(\pi)\right) = 2\lambda_C = 2.64 \times 10^{-15} m$$