6.1 (i) $F(E) = [1 + \exp(E - E_F)/kT]^{-1}$ (1)

For $E - E_F = kT$, $F(E) = (1 + e)^{-1} = \underline{0.270}$

(ii) When $E - E_F = 0.5$ eV and $F(E) = 1\%$,

from eqn (1), $T = \dfrac{E - E_F}{k \log(1/F - 1)} = \dfrac{0.5}{8.62\times10^{-5} \log(1/0.01-1)} = \underline{1262.3\ K}$

6.3 The energy of the incident light = hc/λ

$= 6.62\times10^{-34}\times3\times10^{8}/0.2\times10^{-6}$ J = 6.2 eV

Since this energy is larger than the work functions of all the metals listed in [Table 6.2], all of them will emit electrons in response to the input light.

6.4 The density of occupied states = density of states x Fermi function
At an energy kT above Fermi level E_F,

the density of occupied states: $C\sqrt{E_F + KT}/(1+e)$

where C is a constant given by [eqn 6.10].
For the same density of occupied states at an energy E below the Fermi level, $E' = E_F - E$

$$\sqrt{E_F - E} / [1+\exp(-E/kT)] = \sqrt{E_F + kT} / (1+e) \qquad (1)$$

Since the change of the density of states is much slower than the change in Fermi function for small E, the solution of eqn (1) must be at a large value of E, so that the Fermi function becomes fairly constant and the density of states can compensate the difference due to the change in the Fermi function. Hence eqn (1) can be written approximately as

$E \gg KT$, $\exp(-E/kT) \to 0$

$$\sqrt{E_F - E} \times 1 = \sqrt{E_F + KT}/(1+e)$$

$E_F \gg KT$
Giving $E = 0.928\ E_F$; thus $E' = 0.072E_F$

6.8 The number of photons in the laser beam $= \frac{\text{laser power}}{\text{energy of a photon}} s^{-1}$

$$= 2\times10^{-3}/\left(\frac{hc}{\lambda}\right) = \frac{2\times10^{-3}\times632.8\times10^{-9}}{6.62\times10^{-34}\times3\times10^{8}} = 6.37\times10^{15}\ s^{-1}$$

Each photon induces 10^{-4} electrons. Thus the total current induced by the laser beam = $6.37\times10^{15}\times10^{-4}\times1.6\times10^{-19}$ coulomb per second = $\underline{1.02\times10^{-7}\ A}$. Assume that all the electrons come from the Fermi level. Then they come out with a kinetic energy $E_{KE} = hf - \phi$. The electron flow may thus be stopped with an anode voltage so negative that $-eV_a = E_{KE}$. Hence, the work function may be obtained from $f = hf - |eV_a|$.

6.9 no. of Cu atoms in 1 m^3, N_a, $= \frac{9.4\times10^{3}}{63.5}\times6.02\times10^{26} = 8.91\times10^{28}$

Each Cu atom contributes one electron, therefore $N_e = 8.91\times10^{28} m^{-3}$.

Hence $E_F = \frac{h^2}{2m}\left(\frac{3N_e}{8\pi}\right)^{2/3} = \underline{7.27\ eV}$

Electronic specific heat $= \frac{\pi^2 k^2 T}{2E_F} N_e$/density of Cu (from [eqn 6.25])

= $2.24\ Jkg^{-1}K^{-1}$ (taking room temp. as 293 K)

Lattice specific heat $= 3N_a k$/density of Cu

= $3.92\times10^{2}\ Jkg^{-1}K^{-1}$

So the total specific heat $= 2.24 + 3.92\times10^{2} = \underline{394\ Jkg^{-1}K^{-1}}$, and $2.24/394$; $\underline{0.5\%}$ is contributed by the electrons.

In reference books, they give a value of about 398 $Jkg^{-1}K^{-1}$ which is close to our estimation.