7.2 From the free electron theory [eqn 6.16], $E_F = \frac{h^2}{2m^*}\left(\frac{3N}{8\pi}\right)^{2/3}$ (note that we replace the actual mass m by the effective mass m*).

Now N = no. of electrons available for conduction per unit volume

= density of atoms × no. of free electrons contributed by each atom

$$= \left(\frac{530}{6.94}\times 6.02\times 10^{26}\right)\times 1 = 4.60\times 10^{28}\,\text{m}^{-3}$$

and $E_F = 4.2$ eV

Hence $m^* = \frac{h^2}{2E_F}\left(\frac{3N}{8\pi}\right)^{2/3} = 1.01\times 10^{-30}$ kg or $\underline{1.11\ m_0}$

7.4 The group velocity $V_g = \frac{1}{h}\frac{\partial E}{\partial k} = \frac{1}{h}\frac{\partial}{\partial k}\left(\frac{h^2\alpha^2}{2m}\right)$ (from [eqn 7.5])

$$= \frac{hy}{a^2 m}\frac{\partial y}{\partial k} \quad \text{where } y = \alpha a$$

Differentiate [eqn 7.3] with respect to k

$$\therefore -a \sin ka = P\,\frac{y\cos y - \sin y}{y^2}\frac{dy}{dk} - \sin y\,\frac{dy}{dk}$$

Hence at $k = n\pi/a$, $\frac{dy}{dk} = \left.\frac{-a\sin ka}{P\frac{y\cos y - \sin y}{y^2} - \sin y}\right|_{k=n\pi/a} = 0$

$\therefore$ the group velocity of the electron is zero.

7.5 In three dimension, $\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{h^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$

Hence the equation of motion $\frac{d\mathbf{v}}{dt} = \frac{1}{m}\mathbf{F}$ should be modified to

$$\begin{bmatrix} \frac{dv_x}{dt} \\ \frac{dv_y}{dt} \\ \frac{dv_x}{dt} \end{bmatrix} = \frac{1}{h^2}\begin{bmatrix} \frac{\partial^2 E}{\partial k_x^2} & \frac{\partial^2 E}{\partial k_x \partial k_y} & \frac{\partial^2 E}{\partial k_x \partial k_z} \\ \frac{\partial^2 E}{\partial k_y \partial k_x} & \frac{\partial^2 E}{\partial k_y^2} & \frac{\partial^2 E}{\partial k_y \partial k_z} \\ \frac{\partial^2 E}{\partial k_z \partial k_x} & \frac{\partial^2 E}{\partial k_z \partial k_y} & \frac{\partial^2 E}{\partial k_z^2} \end{bmatrix}\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

7.7 The potential energies of the electrons in the forward- and backward-travelling wave functions are given by [eqn 7.19]:

$V_\pm = \pm\frac{1}{a}\int_0^a \cos 2kx\ V(x)\,dx$ where $V(x)$ is the actual potential of the lattice.

For V(x) as shown in [Fig. 7.2] and with 2w = a,

$$V_\pm = \pm\frac{2}{a}\left\{\int_0^{a/4} -\frac{V_0}{2}\cos 2kx\,dx + \int_{a/4}^{a/2}\frac{V_0}{2}\cos 2kx\,dx\right\}$$
$$= \pm\frac{V_0}{2ka}\left(\sin\frac{3ka}{2} - \sin\frac{ka}{2} - \frac{1}{2}\sin 2ka\right)$$

In the first energy band, k=π/a (from [eqn 7.15]). Therefore $V_\pm = V_0/\pi$.

The total energies of the electrons = kinetic energy + potential energy

$$= h^2k^2/2m \pm V_0/\pi$$

∴ the width of the first forbidden band is $2V_0/\pi$.