
Semiconductor Lasers

ECE 162C

Lecture #10

Prof. John Bowers

Read Kasip, Chapters 3,4

Midterm: May 5. Chapters 1-4

**•ECE 162C: PROBLEM SET #4
DUE FRIDAY, MAY 2, 2004, 4 pm**

(No late submissions because the solution will be emailed out).

•PROBLEM:

Consider a 1.3 μm buried heterostructure InGaAsP /InP bulk laser 400 μm in length with confinement factor $\Gamma=.2$, internal quantum efficiency of 80% and internal loss of 10 cm^{-1} , cleaved facets, 0.2 μm thick active region, 2 μm wide waveguide, 1ns lifetime, linear gain dependence on carrier density with a differential gain $2 \times 10^{16}\text{ cm}^2$ and transparency carrier density of $1 \times 10^{18}\text{ cm}^{-3}$. Assume the index of the active region is 3.5 and the index of the InP cladding is 3.17, and the group index is 3.6. Assume the mirror reflectivity is 0.3.

- Draw the transverse conduction band and valence band diagrams under zero bias and forward bias.
- Sketch the electron and hole carrier densities under zero and forward bias.
- Plot the peak gain versus carrier density.
- What is the mirror loss?
- What is the threshold modal gain?
- What is the threshold current?
- What is the differential quantum efficiency?
- What is the axial mode spacing?
- Calculate the width for a single transverse mode. Is the laser single transverse mode?
- Calculate the width for a single lateral mode. Is the laser single lateral mode? (Use the effective index method).
- Calculate the lateral and transverse widths (FWHM) of the mode.**

Questions

- What is the difference between gain guided and index guided lasers?
- What is the effective index method?
- Are lasers TE or TM? Why?
- What is the expression for differential quantum efficiency?
- How is it related to slope efficiency?
- Compare methods of current confinement
 - Oxide, Homojunction, PN junction, Semi-insulating
- What is a thyristor?
- What is double injection?
- What is the trap filled voltage?

Effective Index Method

1. Do transverse calculation (y)
 - Neglect variation in lateral direction(x)
 - Calculate effective index n in each region
2. Do lateral calculation
 1. Neglect variation in transverse direction(y)
 2. Calculate effective index of mode.

This method is reasonably accurate when

1. The width is much larger than the thickness (lateral > transverse)
2. The lateral confinement is relatively weak.

Symmetric 3 Layer Guide

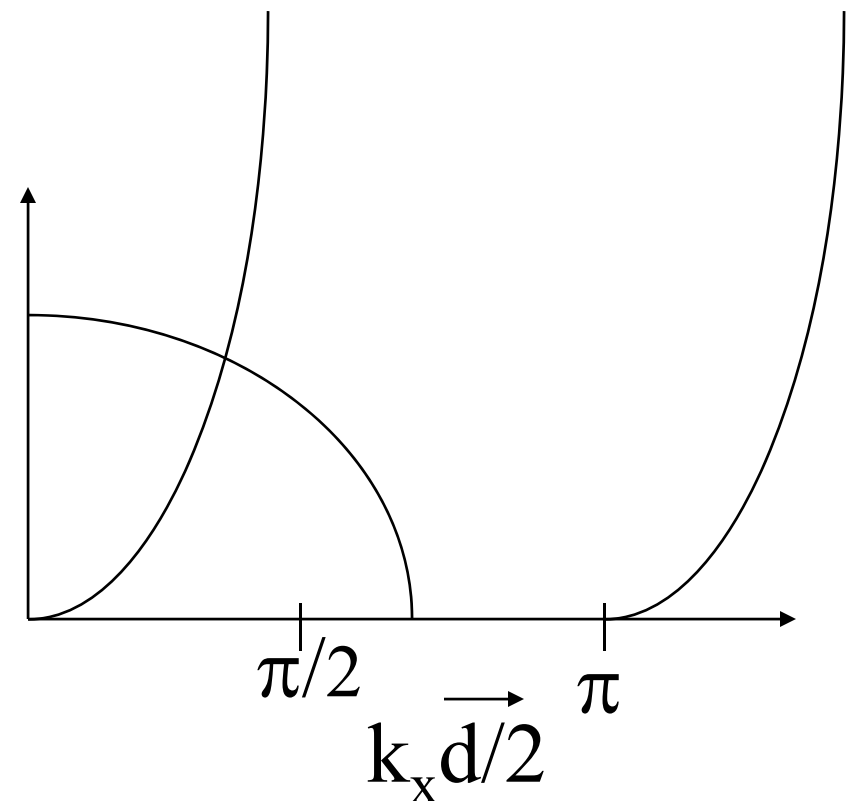
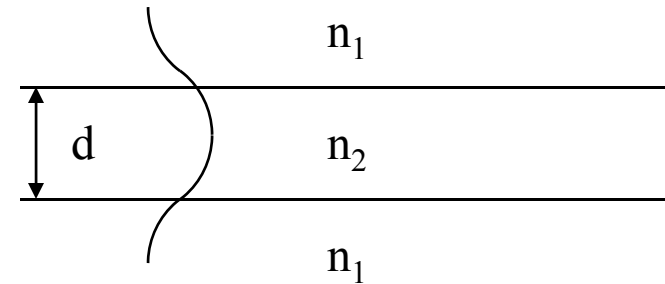
- Plot graphically

$$k_x^2 = k_0^2 n_2^2 - \beta^2$$

$$\gamma^2 = \beta^2 - k_0^2 n_1^2$$

$$k_x \tan \frac{k_x d}{2} = \gamma$$

$$k_x \tan \frac{k_x d}{2} = \sqrt{k_0^2 (n_2^2 - n_1^2) - k_x^2}$$



Symmetric 3 Layer Guide

- Asymmetric solutions

$$k_x^2 = k_0^2 n_2^2 - \beta^2$$

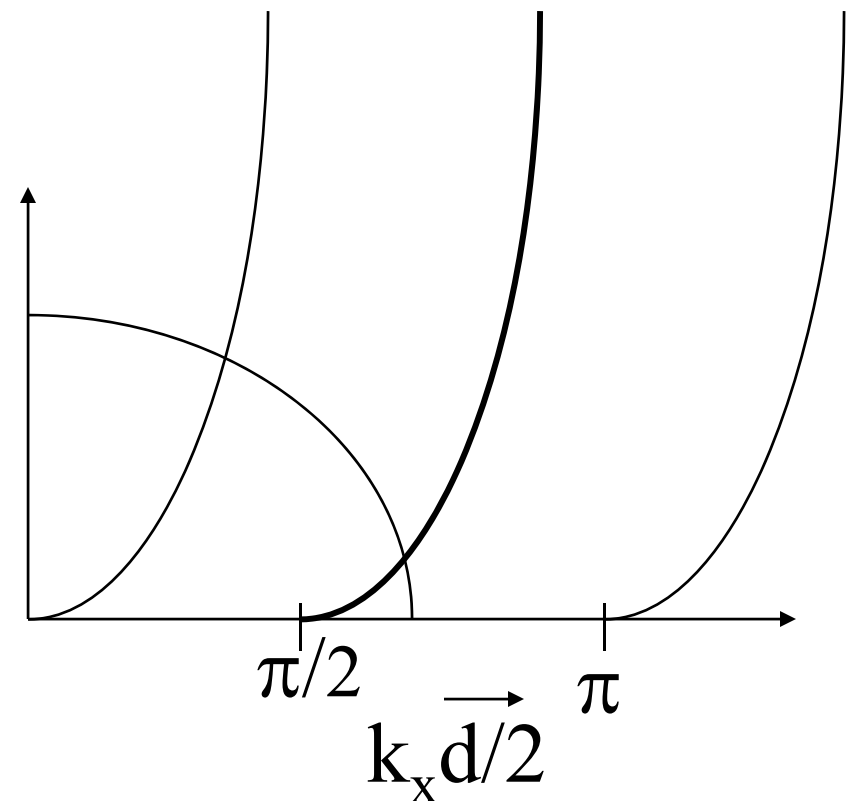
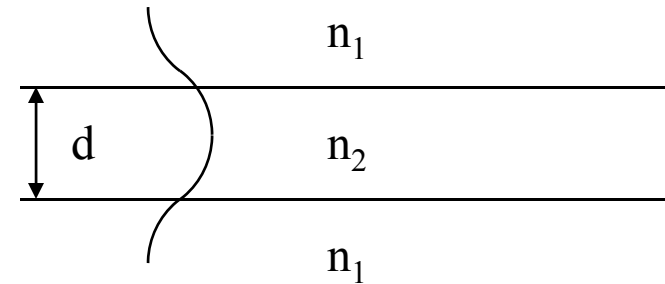
$$\gamma^2 = \beta^2 - k_0^2 n_1^2$$

$$k_x \cot \frac{k_x d}{2} = -\gamma$$

$$k_x \cot \frac{k_x d}{2} = \sqrt{k_0^2 (n_2^2 - n_1^2) - k_x^2}$$

$$d < \frac{\pi}{k_0^2 \sqrt{(n_2^2 - n_1^2)}}$$

Single mode condition



Single Mode Lasers

- At least 3 cm^{-1} of gain difference between the dominant mode and other modes is required.
 - Less gain difference: The laser may lase cw in a single mode, but lases in multiple modes when modulated.
 - More gain difference necessary to achieve 40 dB sidemode suppression under 100% modulation.

L-I Curve

- Use the electron rate equation (neglect ϵS)

$$\frac{dN}{dt} = \eta_i \frac{I}{qV} - \frac{v_g a(N - N_{tr})}{1 + \epsilon S} S - \frac{N}{\tau_n}$$

$$0 = \eta_i \frac{I}{qV} - v_g g_{th} S - \frac{N}{\tau_n}$$

$$v_g g_{th} S = \eta_i \frac{I}{qV} - \frac{N}{\tau_n}$$

$$S = \frac{\eta_i (I - I_{th})}{qV v_g g_{th}}$$

L-I Curve

- Convert from photon density to output power

The number of photons in the cavity is SV_p

The energy of photons in the cavity is $h\nu SV_p$

The output power is

$$P = v_g \alpha_m h\nu SV_p$$

$$S = \frac{\eta_i (I - I_{th})}{qVv_g g_{th}}$$

$$P = \frac{v_g \alpha_m h\nu V_p \eta_i (I - I_{th})}{qVv_g g_{th}}$$

$$P = \frac{\eta_i \alpha_m}{\alpha_i + \alpha_m} \frac{h\nu}{q} (I - I_{th})$$

External differential quantum efficiency

$$P = \frac{\eta_i \alpha_m}{\alpha_i + \alpha_m} \frac{h\nu}{q} (I - I_{th})$$

The differential quantum efficiency is (%)

$$\eta_d = \frac{\eta_i \alpha_m}{\alpha_i + \alpha_m}$$

The slope efficiency is (W/A)

$$R_l = \frac{\eta_i \alpha_m}{\alpha_i + \alpha_m} \frac{h\nu}{q}$$

$$\frac{dP}{dI} = R_l$$

Laser Requirements

- Confinement of the optical mode
 - Transverse
 - Lateral (rib, strip, gain guided)
 - Longitudinal (cleaved facets, rings, DFB, DBR,...)
- Confinement of carriers
 - Heterojunction, etched, no confinement (implanted, diffused,...)
- Confinement of current
 - Oxide
 - Homojunction
 - PN junction
 - Semi-insulating

Laser Structure

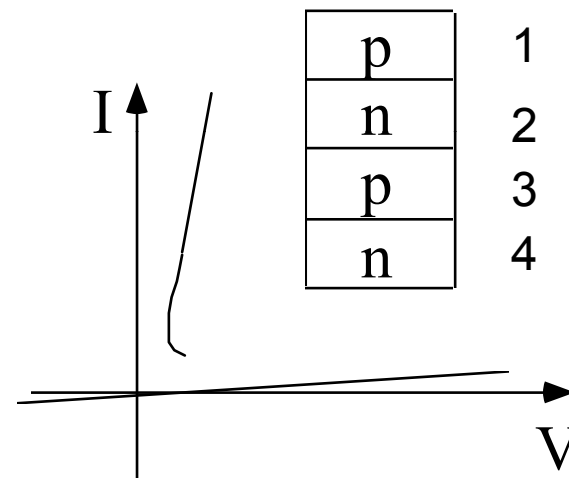
Current Confinement

- Dielectric layers, e.g. SiO_2 , SiN_x , polyimide or oxidized AlGaAs
 - Very low capacitance
 - Possible reliability problem
 - Poor thermal characteristics
- Reverse biased p-n junctions
 - Good high power and high T capability
 - Large depletion capacitance
- Larger bandgap homojunctions
 - Simple fabrication
 - Leakage current and diffusion capacitance are high
- Semi-insulating semiconductor regions
 - Low capacitance and good high T and high power performance
 - Growth of high quality semi-insulating layers not trivial

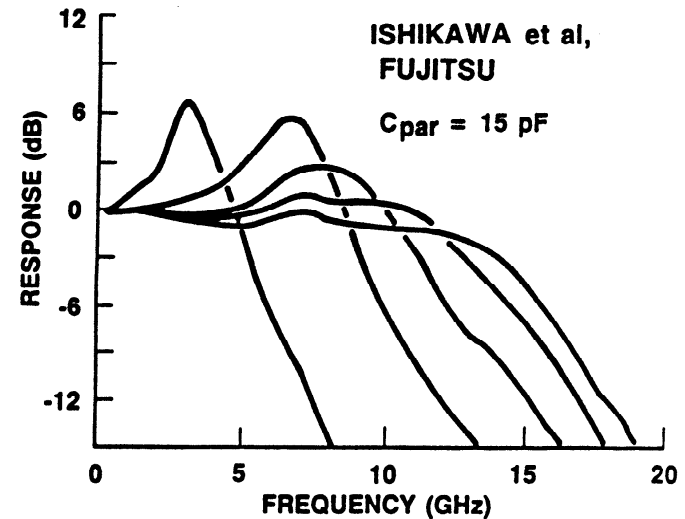
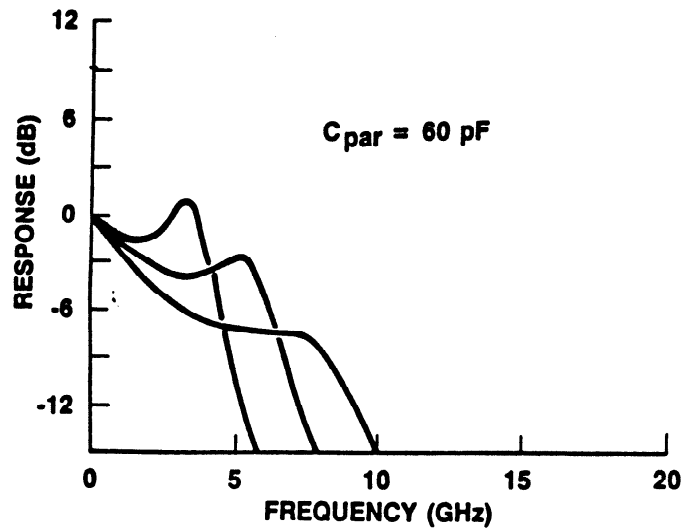
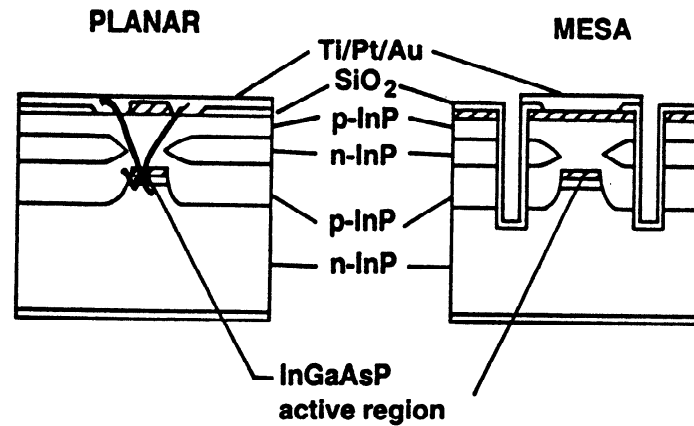
Parasitic Thyristor

Problem for devices where current confinement is obtained with reverse biased p-n junctions.

- At low bias junction 2-3 is reverse biased, preventing current flow
- When bias is increased we get hole injection from 3->1
- Same thing happens with transistor 2-3-4



Laser Structure : High Speed Planar Laser



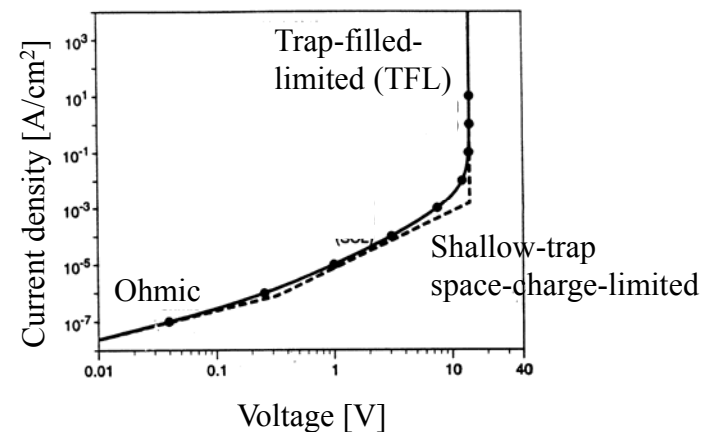
Semi-Insulating Fe:InP

Fe is a deep level acceptor in InP, making the material highly resistive to electron current. However, if the Fe-doped layer is placed between p- and n- material, holes are injected and can recombine with electrons, i.e. current is flowing.

Use n-SI-n structure

- avoids double injection
- At high bias levels all traps are filled.

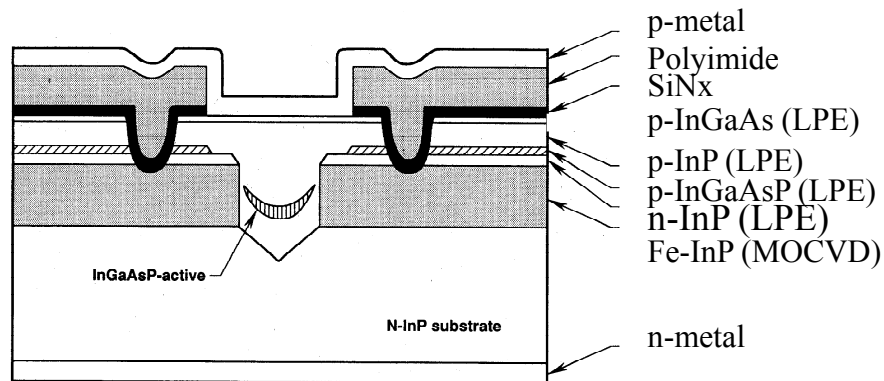
$$V_{TFL} = \frac{eN_t L^2}{2\epsilon}$$



Laser Structures

Semi-insulating buried crescent laser

Current confinement achieved with Fe-doped semi-insulating layer

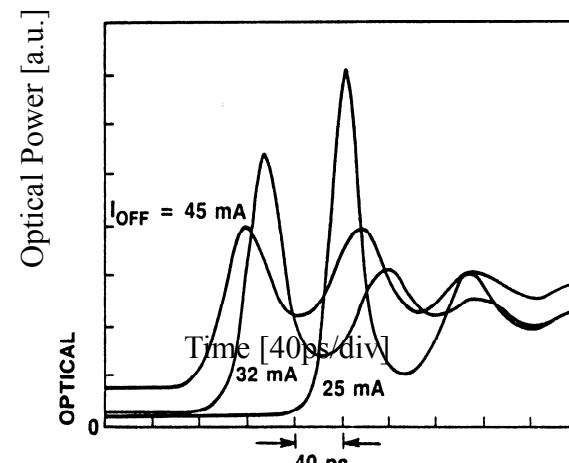
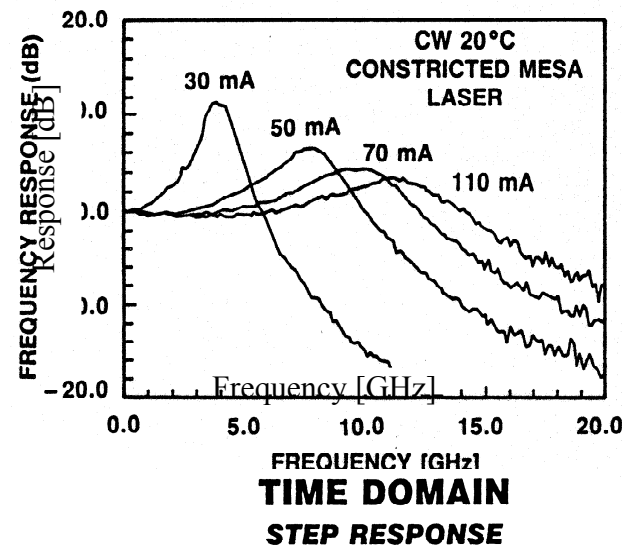


Laser Requirements

- Large Bandwidth
- High Modulation efficiency
- Low Intensity Noise
- Large Temperature Range
- Low Distortion
- Low Reflection Sensitivity
- Low Chirp

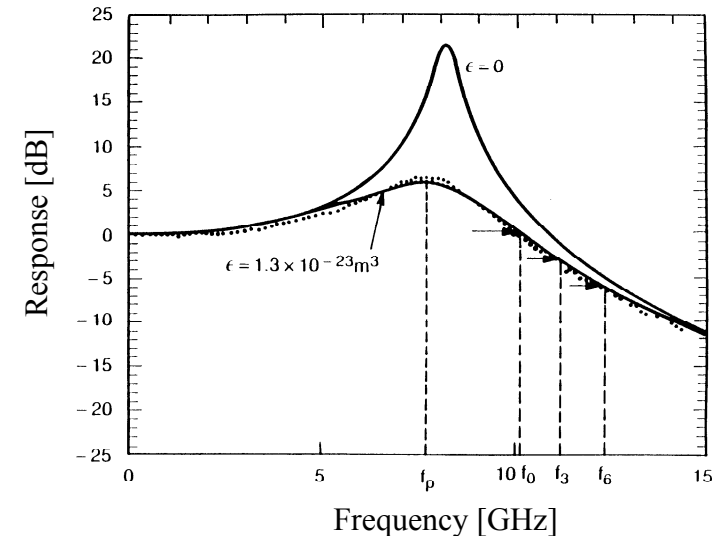
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**FREQUENCY DOMAIN
SMALL SIGNAL RESPONSE**



Bandwidth Limiting Factors

- Resonance Frequency
 - Current or power limited
- Damping
 - Spectral hole burning or
 - carrier heating limited
- Transport
 - Diffusion or tunneling limited
- Parasitics
 - Capacitance and resistance limited
- Microwave Effects
 - Microwave loss limited



Rate Equations

Neglecting the phase of the optical field, the length dependence of the carrier and photon densities, and the modal dependence; the rate equations for the averaged photon and carrier densities become:

$$\frac{dS}{dt} = \frac{\Gamma v_g a(N - N_{tr})}{1 + \epsilon S} S - \frac{S}{\tau_p} + \frac{\beta \Gamma N}{\tau_n}$$

$$\frac{dN}{dt} = \eta_i \frac{I}{qV} - \frac{v_g a(N - N_{tr})}{1 + \epsilon S} S - \frac{N}{\tau_n}$$

Rate equations: Small Signal Approximation

Making the usual small signal approximation $I = I_0 + ie^{j\omega t}$

$$S = S_0 + se^{j\omega t}$$

$$N = N_0 + ne^{j\omega t}$$

We find the small-signal modulation response of the laser:

$$H(f) = \frac{s(f)}{i(f)} = \frac{s(0)}{i(0)} \frac{f_0^2}{f_0^2 - f^2 + jff_d}$$

Rate Equation: Intrinsic Frequency Response

The damping frequency is:

$$f_d \approx \frac{\epsilon S}{2\pi\tau_p}$$

The resonance frequency is at the geometric mean of the photon and carrier lifetimes:

$$f_0 = \frac{1}{2\pi\sqrt{\tau_p\tau_n^{stim}}}$$

Rate Equations: Resonance Frequency

The photon lifetime, typically on the order of 1ps, is given by:

$$\tau_p = \frac{1}{v_g \left(\alpha_i + \frac{1}{2L} \ln \left(\frac{1}{R_1 R_2} \right) \right)}$$

Far above threshold, spontaneous emission can be neglected, and the stimulated electron lifetime becomes:

$$\tau_n^{stim} = \frac{1}{v_g a S}$$

Resonance Frequency

From the previous we get the following expression for the resonance frequency:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{v_g a S}{\tau_p}} = \frac{1}{2\pi} \sqrt{\frac{v_g a \eta}{\tau_p} (I - I_{th})}$$

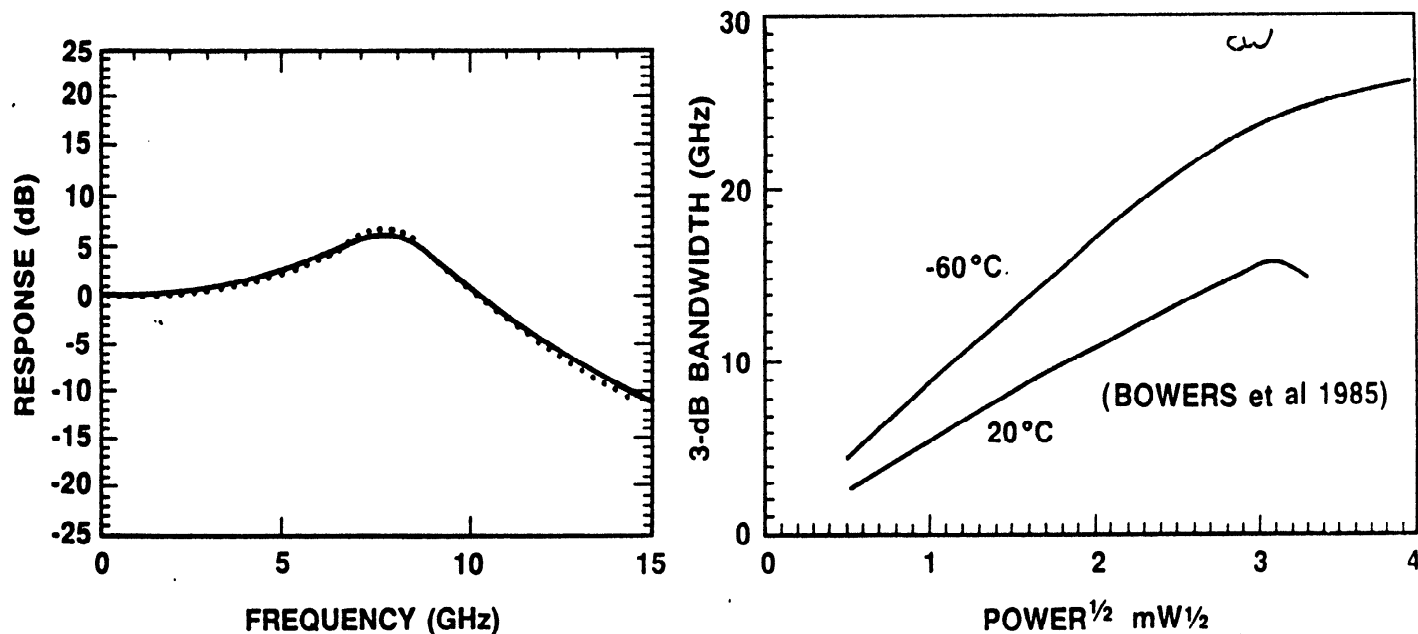
Note that the peak in modulation response is not at f_0 , but at f_p , given by:

$$f_p^2 = f_0^2 - \frac{f_d^2}{4}$$

Modulation Response

The resonance frequency and modulation bandwidth depends on the output power:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{v_g a S}{\tau_p}} = D\sqrt{P}$$



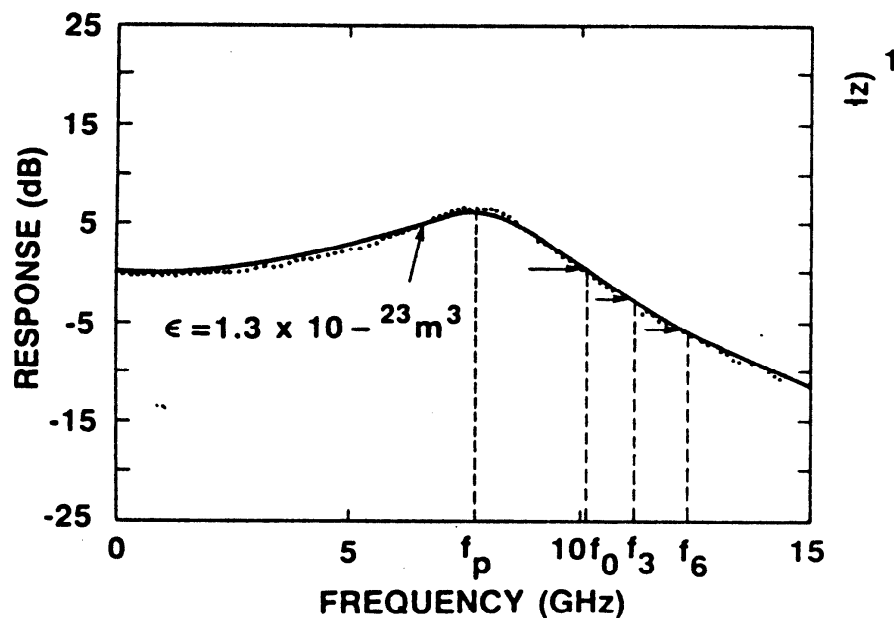
Bandwidth Relations

Depending on the application, different bandwidths are specified. From the general expression for the modulation response we find:

$$f_{0dB} = \sqrt{2}f_0$$

$$f_{3dB} \approx \sqrt{1 + \sqrt{2}}f_0$$

$$f_{6dB} \approx \sqrt{3}f_0$$



Resonance Frequency Limitations

For most lasers the resonance frequency is limited by the current density due to:

- Leakage currents due to breakdown of p-n junctions or semi-insulating layers
- Conduction across the active layer at high current densities
- Heating (proportional to I^2R)

For a given current density higher resonance frequency is obtained with:

- Higher differential gain
- Shorter cavity length

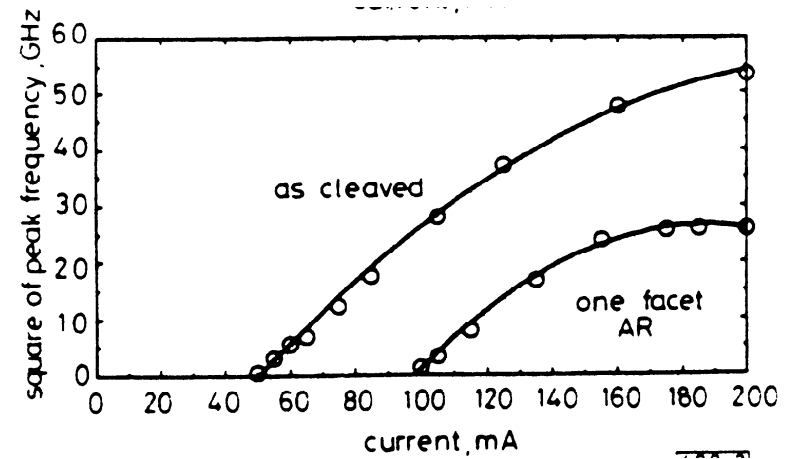
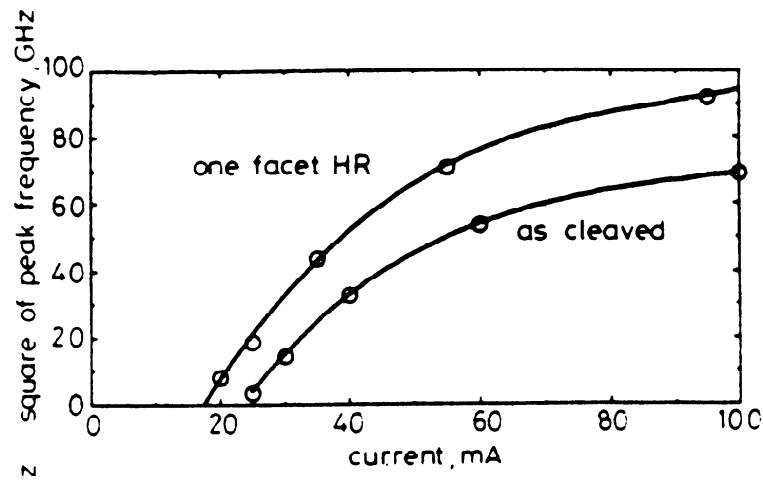
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- Small mode volume

Facet Coatings

Can we coat the facets to achieve higher bandwidth?

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{\Gamma v_g a \eta_i I}{eV} - \frac{\Gamma v_g a \eta_i N_{tr}}{\tau_n} - \frac{\eta_i v_g}{\tau_n} \left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right)}$$



HR coating increases modulation bandwidth

Bandwidth Limiting Factors

- Resonance Frequency
 - Current or power limited
- **Damping**
 - **Spectral hole burning or carrier heating limited**
- Transport
 - Diffusion or tunneling limited
- Parasitics
 - Capacitance and resistance limited
- **Microwave Effects**
 - Microwave loss limited

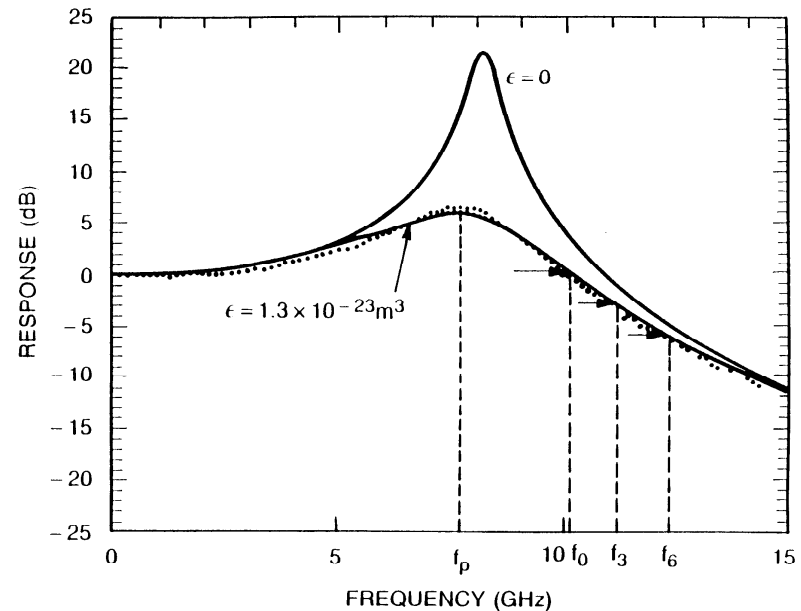
Limits to Bandwidth: Damping

Modulation bandwidth is limited by damping due to non-linear gain.

$$g = g_0 / (1 + \epsilon S)$$

Important causes of non-linear gain are:

- Spectral hole burning
- Carrier heating



Damping

Damping is described by the damping factor K , defined as:

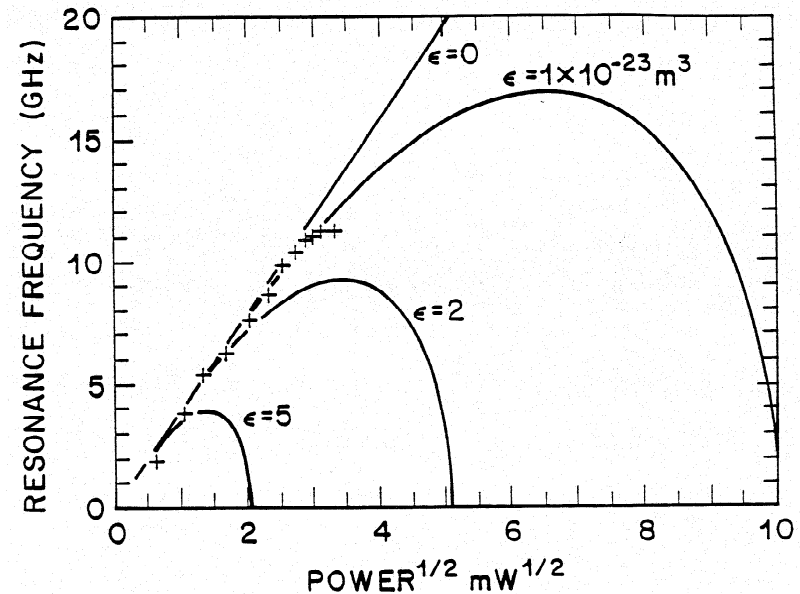
$$f_d = Kf_0^2$$

It can be shown that

$$K = 4\pi^2 \left(\tau_p + \frac{\epsilon}{a} \right)$$

Damping limits the maximum bandwidth to:

$$f_{3dB}^{\max} = \frac{2\pi\sqrt{2}}{K} \cong \frac{8.8}{K}$$

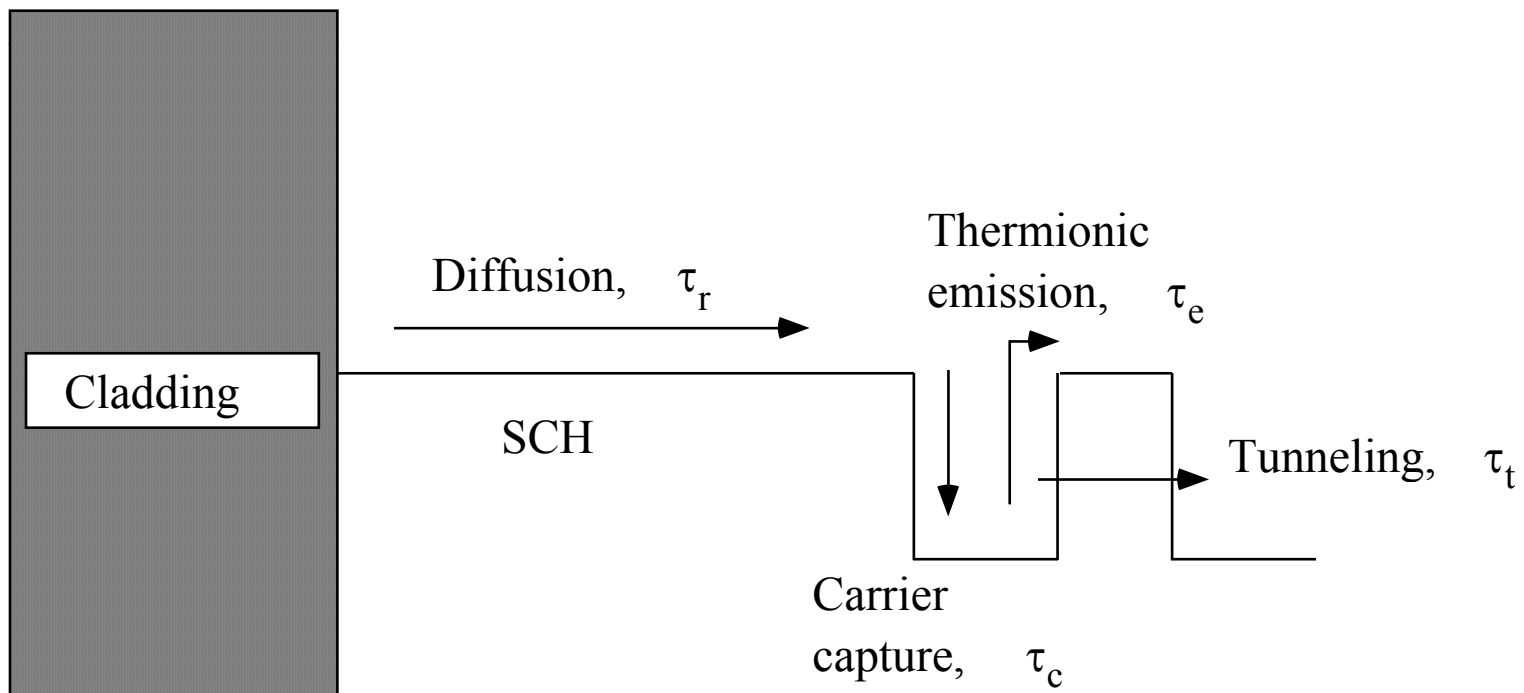


Bandwidth Limiting Factors

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Transport Effects

Important transport processes in separate confinement heterostructure (SCH) laser



Transport Effects : Rate Equations

When transport effects are included, the rate equations are modified

$$\frac{dN_B}{dt} = \frac{I}{qV_{SCH}} - \frac{N_B}{\tau_r} + \frac{N_W(V_W/V_{SCH})}{\tau_e}$$

$$\frac{dN_W}{dt} = \frac{N_B(V_{SCH}/V_W)}{\tau_r} - \frac{N_W}{\tau_n} - \frac{v_g a(N_W - N_{tr})S}{(1 + \epsilon S)}$$

$$\frac{dS}{dt} = \frac{\Gamma v_g (N_W - N_{tr})S}{(1 + \epsilon S)} - \frac{S}{\tau_p} + \beta \Gamma \frac{N_W}{\tau_n}$$

TRANSPORT EFFECTS: Modulation Response

The modulation response and damping factor then become:

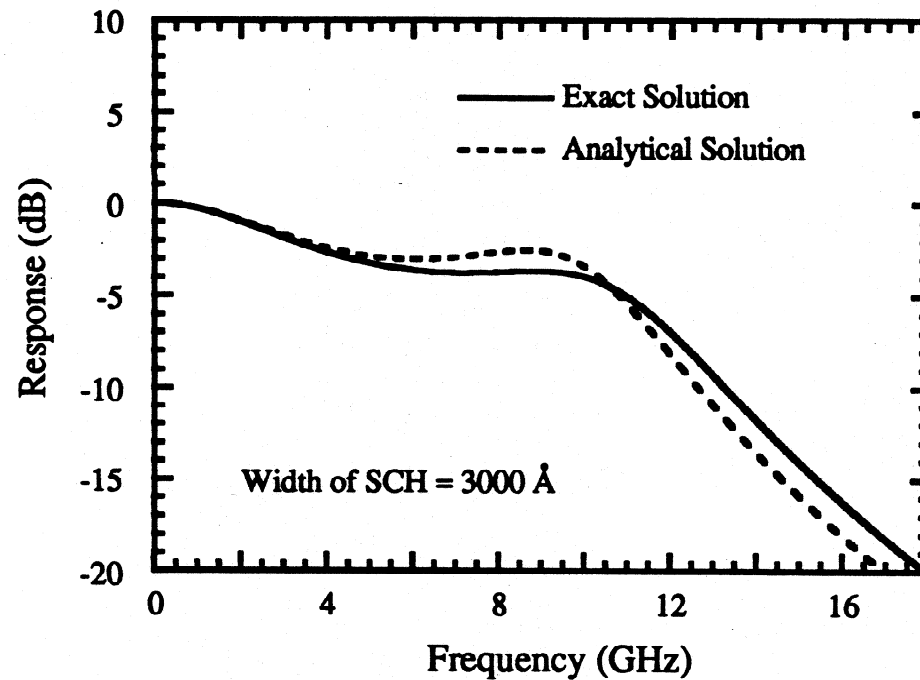
$$R(\omega) = \frac{\omega_0^2}{(1 + j\omega\tau_r)(\omega_0^2 - \omega^2 + j\omega\gamma)}$$

$$\omega_0^2 = \frac{\left(\frac{a}{\chi}\right)S}{\tau_p(1 + \epsilon S)} \quad \chi = 1 + \frac{\tau_r}{\tau_e}$$

$$K = 4\pi^2 \left(\tau_p + \chi \frac{\epsilon}{a} \right)$$

Transport Effects

Exact numerical solution to rate equations compared to analytical approximation



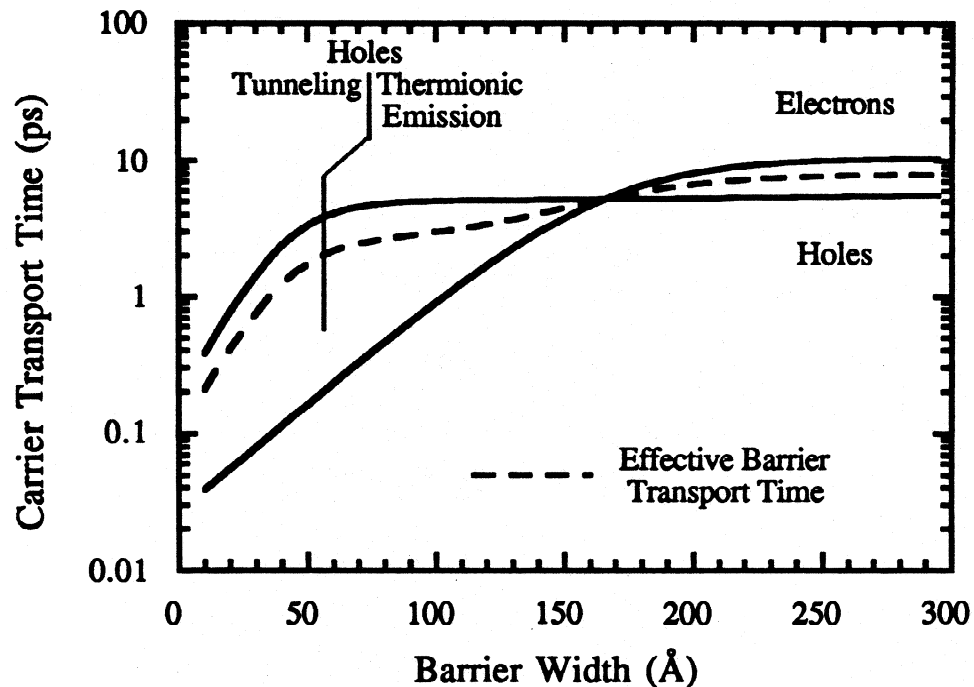
Transport Effects

Results from the Model

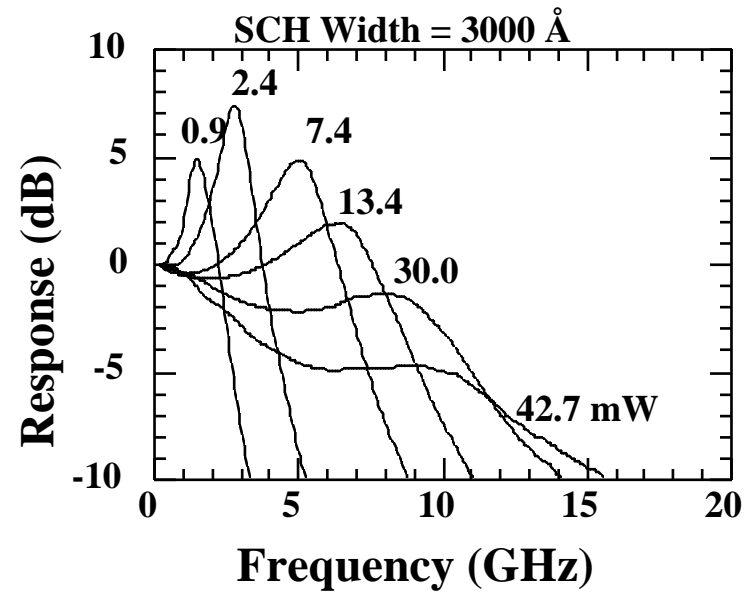
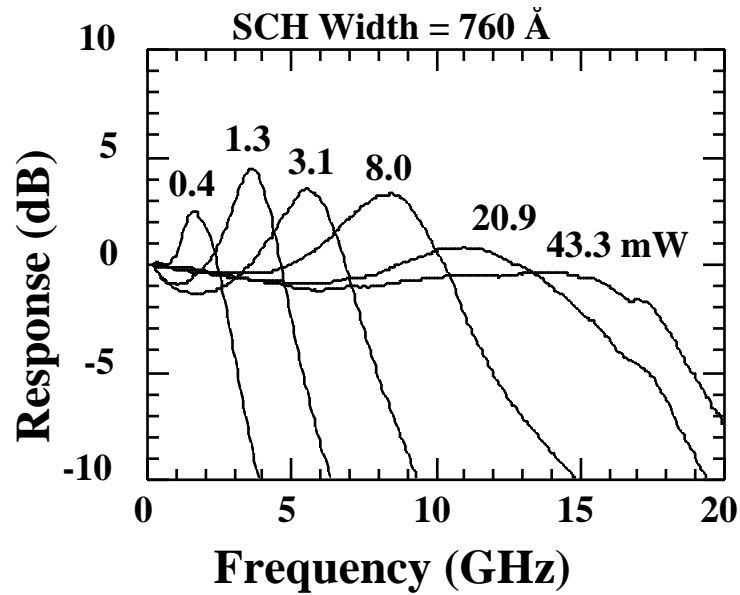
- Severe roll off in the modulation response due to transport across the SCH region (τ_r). This roll off is independent of:
 - Reduction of differential gain
 - Gain compression
 - Device parasitics
- Transport effects causes reduction of effective differential gain $a \rightarrow \frac{a}{\chi}$
- Gain compression factor is independent of SCH width

Transport effects

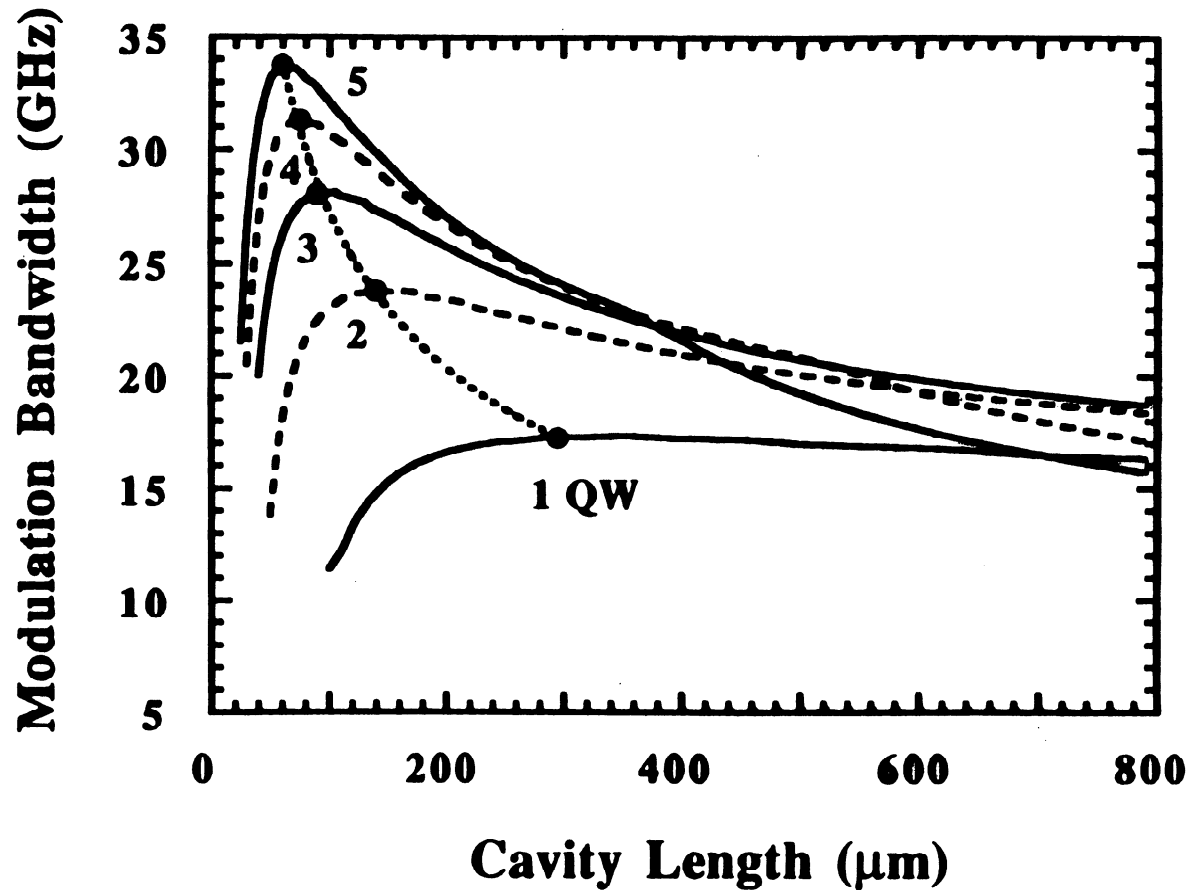
Carrier transport time between two quantum wells in a multiple quantum well (MQW) laser



Transport Effects: SCH width



Transport Effects : Cavity Length



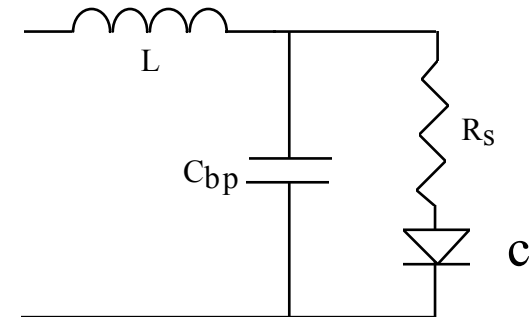
Bandwidth Limiting Factors

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Laser Parasitics

Device and package parasitics can cause significant reduction in modulation response

- Junction capacitance C
- Bond pad capacitance C_{bp}
- Series resistance R_s
- Bond wire inductance L

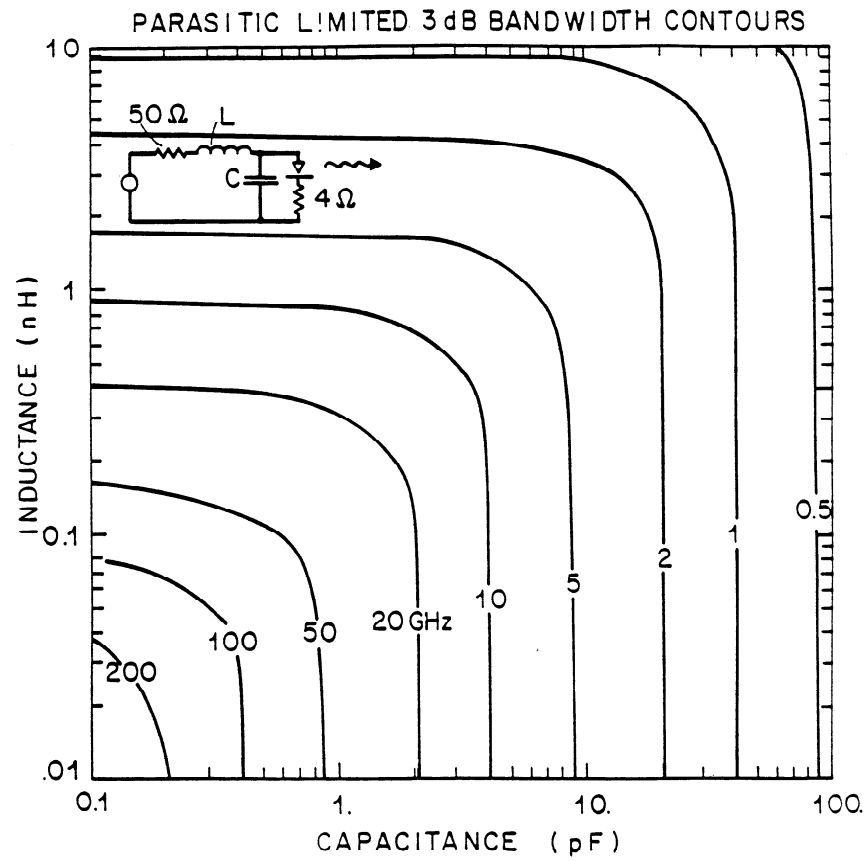


Example: To achieve 15 GHz bandwidth:

$$\begin{aligned}C &< 2\text{pF} \\C_{bp} &< 2\text{pF} \\R_s &< 4\Omega \\L &< 0.3\text{nH}\end{aligned}$$

Parasitics

Parasitic limited 3 dB bandwidth contours



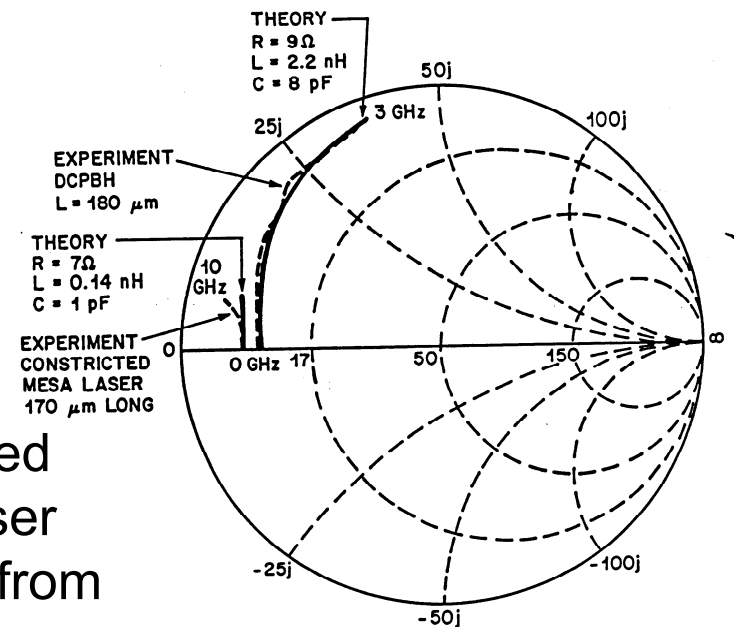
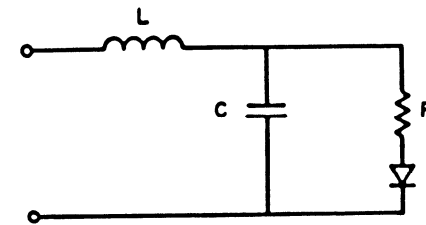
Laser Impedance

Model for laser impedance, consisting of bond-wire inductance, parasitic capacitance and series resistance.

Measurements show microwave characteristics of two mounted laser structures:

- a constricted mesa laser
- a high power dual-channel planar buried heterostructure laser (DCPBH)

Inductance and resistance are determined from measurement of forward biased laser (as shown). Capacitance can be found from reverse biased measurement



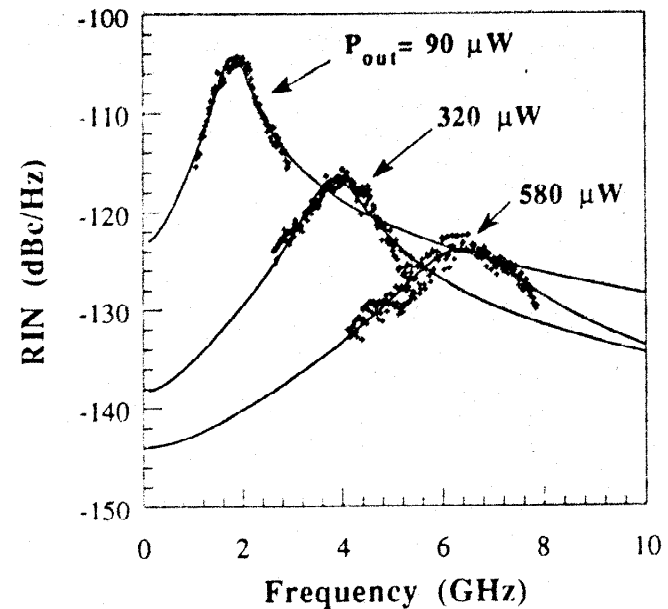
Relative Intensity Noise

- Recombination and generation are stochastic processes -> variations in output power
- Relative Intensity Noise:

$$RIN \equiv \frac{\langle \delta P(t)^2 \rangle}{P^2}$$

- RIN spectrum related to modulation response:

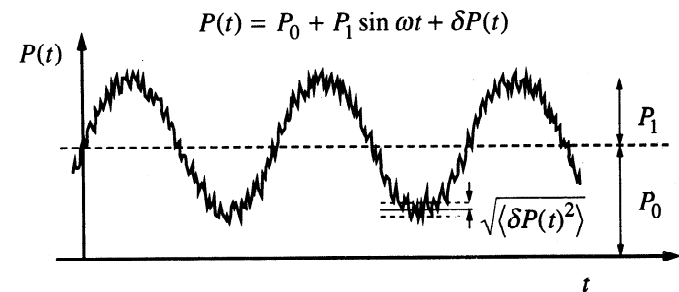
$$RIN(\omega) = \frac{2h\nu}{P_0} \left[\frac{a_1 + a_2\omega^2}{\omega_R^4} |H(\omega)|^2 + 1 \right]$$



RIN

Optical Fiber Communication

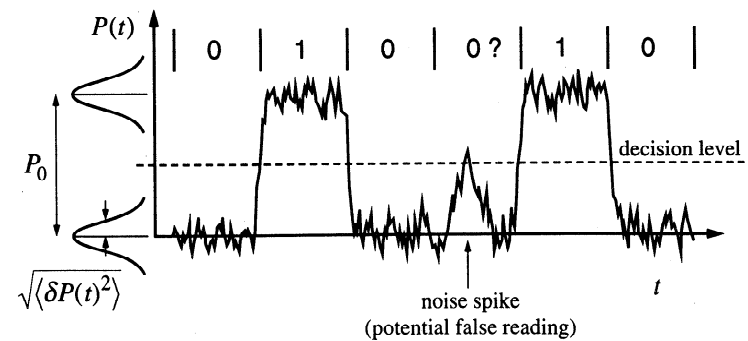
- Relative intensity noise cause degradation of Signal-to-Noise Ratio in analog systems, and errors in digital systems
- Analog signal:



$$SNR = \frac{2}{m^2} \frac{1}{RIN}$$

- Digital signal, for $BER < 10^{-9}$

$$RIN < (11.89)^{-2}$$



Analog Transmission

- High-speed analog transmission for CATV and wireless communication
- Issues:
 - Noise (RIN)
 - Chirp
 - Linearity
- Non-linearity causes Inter-Modulation Distortion (IMD)
 - Typical system requirement
 - $\text{IMD3} < -80\text{dBc}$

