The Wave Equation in Birefringent Media, Modes in Optical Fiber

Read: Kasap, Chapter 1,2

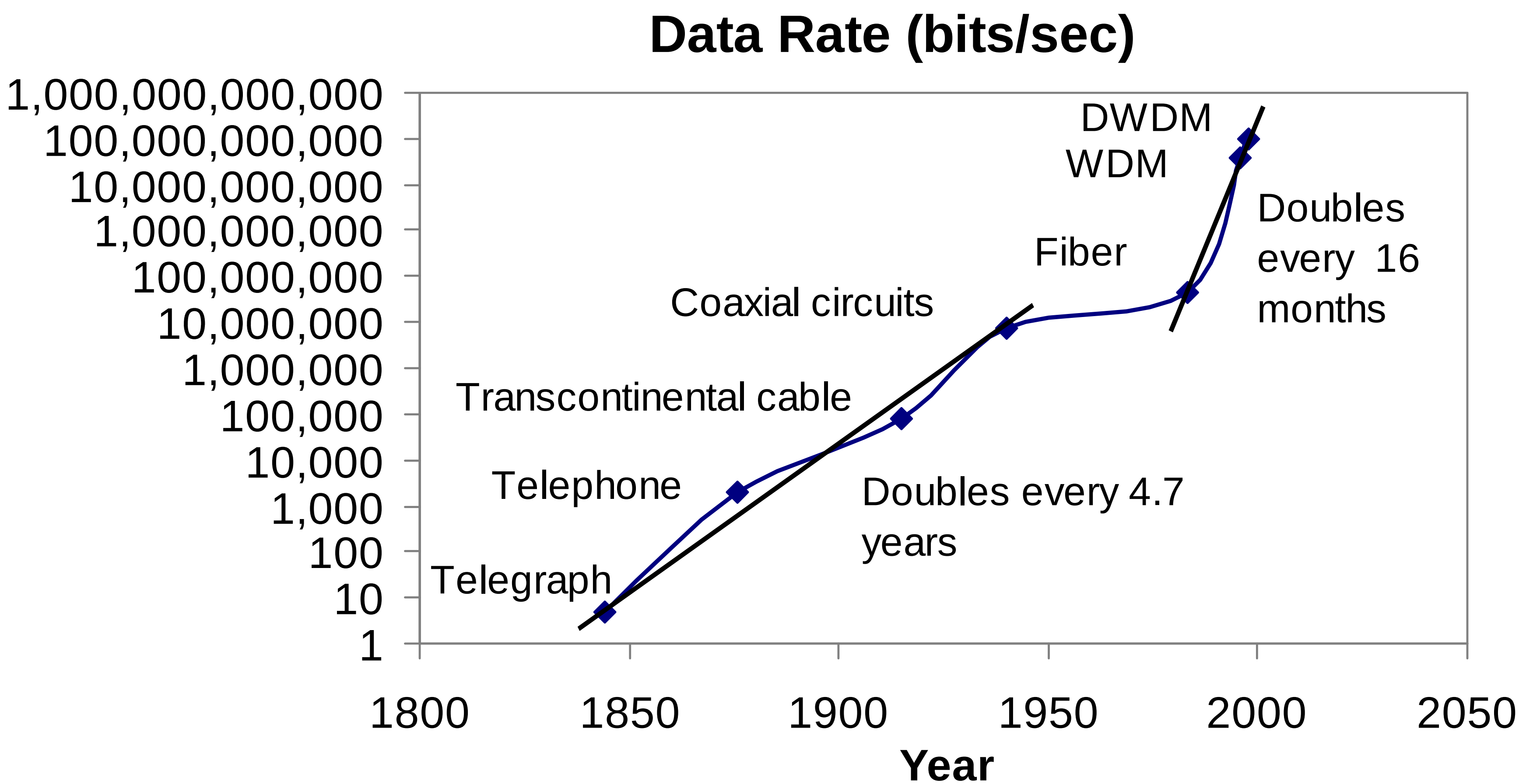
ECE 162C

Lecture #2

Prof. John Bowers

Law of the Photon

Data rate doubles every 16 months



Transmission will be Optical

What do you need to know?

- **Modes in optical fibers (wave equation …)**
- **Modes in optical waveguides (lasers, modulators, …wave equation, birefringence)**
- Lasers (gain, absorption, lasing,…)
- Modulators, Photodetectors, Amplifiers
- Multiplexers, Dispersion compensation

Notation

- MKS units
- Lower case for time varying quantities
- Capitals for the amplitudes of time varying quantities
- Complex quantities used to represent amplitude and phase:

$$a(t) = \mathrm{Re}[Ae^{i\omega t}]$$

- Later lectures, and Kasip:
- E(x,y,z,t)=Re [E(x,y,z) $e^{i\omega t}$]

Maxwell's Equations

$$\nabla \times \vec{h} = i + \frac{\partial \vec{d}}{\partial t}$$

$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

$$\nabla \bullet \vec{d} = 0$$

$$\nabla \bullet \vec{b} = 0$$

where e and h are the electric and magnetic field vectors
d and b are the electric and magnetic displacement vectors
No free charge.

Constitutive Relations

$$\vec{d} = \varepsilon_0 \vec{e} + \vec{p}$$

$$\vec{b} = \mu_0 (\vec{h} + \vec{m})$$

p and m are the electric and magnetic polarizations of the medium
ε_0 and μ_0 are the electric and magnetic permeabilities of vacuum
e and h are the electric and magnetic field vectors
d and b are the electric and magnetic displacement vectors

Electric Susceptibility χ (Isotropic)

Isotropic Media: χ is a complex number

$$P = \varepsilon_0 \chi E$$

The **real** part determines the index (velocity) and the **imaginary** part determines the gain or absorption.

Isotropic media: Vacuum, gasses, glasses (optical fibers)
Anisotropic media: Semiconductors, crystalline materials.

Electric Susceptibility χ (Anisotropic media)

Anisotropic Media: χ is a complex second rank tensor

$$\vec{P} = \varepsilon_0 \overleftrightarrow{\chi} \vec{E}$$

$$P_i = \varepsilon_0 \sum \chi_{ij} E_j$$

$$P_x = \varepsilon_0 (\chi_{xx} E_x + \chi_{xy} E_y + \chi_{xz} E_z)$$

One can always choose a coordinate system such that off axis elements are zero. These are the principal dielectric axes of the crystal. We will only use the principal coordinate system.

$$P_x = \varepsilon_0 \chi_{11} E_x$$

$$P_y = \varepsilon_0 \chi_{22} E_y$$

$$P_z = \varepsilon_0 \chi_{33} E_z$$

Principal Axes

D, E and P are not parallel in general. D and E are related by the electric permeability tensor ε

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = \varepsilon \vec{E}$$

Principal axes can always be chosen such that D and E are parallel and the off diagonal elements of ε are zero.

$$\varepsilon_{11} = \varepsilon_0 (1 + \chi_{11})$$

$$\varepsilon_{22} = \varepsilon_0 (1 + \chi_{22})$$

$$\varepsilon_{33} = \varepsilon_0 (1 + \chi_{33})$$

Wave Propagation in Lossless, Isotropic Media

- Lossless: σ=0, χ is real, ε is real.
- Isotropic: χ, ε are scalors (not tensors).

$$\nabla \times \vec{e} = i + \frac{\partial \vec{b}}{\partial t} = 0 + \mu \frac{\partial \vec{h}}{\partial t}$$

$$\nabla \times \vec{h} = i + \frac{\partial \vec{d}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{e}) = \mu \frac{\partial (\nabla \times \vec{h})}{\partial t} = \mu \frac{\partial^2 \vec{d}}{\partial^2 t} = \mu\varepsilon \frac{\partial^2 \vec{e}}{\partial^2 t}$$

$$\nabla \times (\nabla \times \vec{e}) = \nabla^2 \vec{e} - \nabla (\nabla \bullet \vec{e})$$

$$\nabla^2 \vec{e} = \mu\varepsilon \frac{\partial^2 \vec{e}}{\partial^2 t}$$

Wave Equation

Wave Equation

$$e(x, y, z, t) = \mathrm{Re}[E(x, y, z)e^{i\omega t}]$$

$$\nabla^2 \vec{E} + \omega^2 \mu\varepsilon \vec{E} = 0$$

$$\boxed{\nabla^2 \vec{E} + k^2 \vec{E} = 0}$$

where

$$k = \omega\sqrt{\mu\varepsilon} = \omega n / c$$

$$c = 1/\sqrt{\mu_0 \varepsilon_0}$$

$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}}$$

Step Index Circular Waveguide (lossless, isotropic)

- Simplest type of fiber
- (Most fiber these days is far more complex)
- Cylindrical symmetry

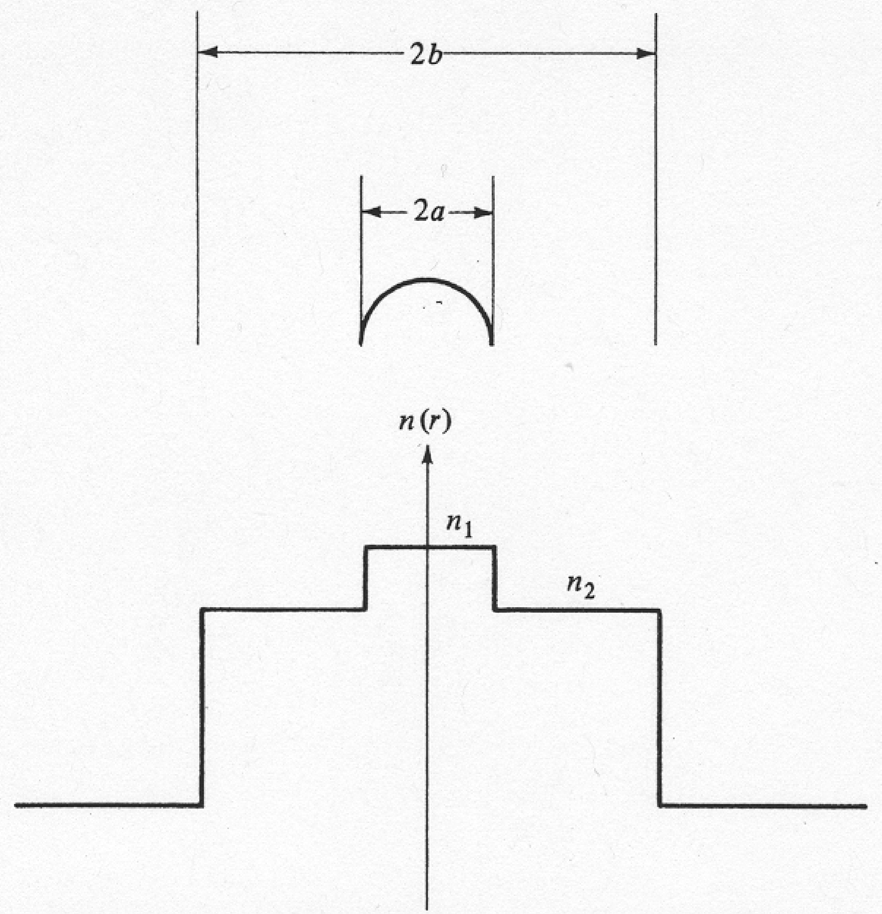


Figure 3-1 Structure and index profile of a step-index circular wa

Step Index Circular Waveguide (lossless, isotropic)

- Simplest type of fiber
- (Most fiber these days is far more complex)
- Cylindrical symmetry
- Express Laplacian operator in cylindrical coordinates

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$

- Separate variables

$$E_r = \psi(r)\Phi(\phi)e^{i(\omega t - \beta z)}$$

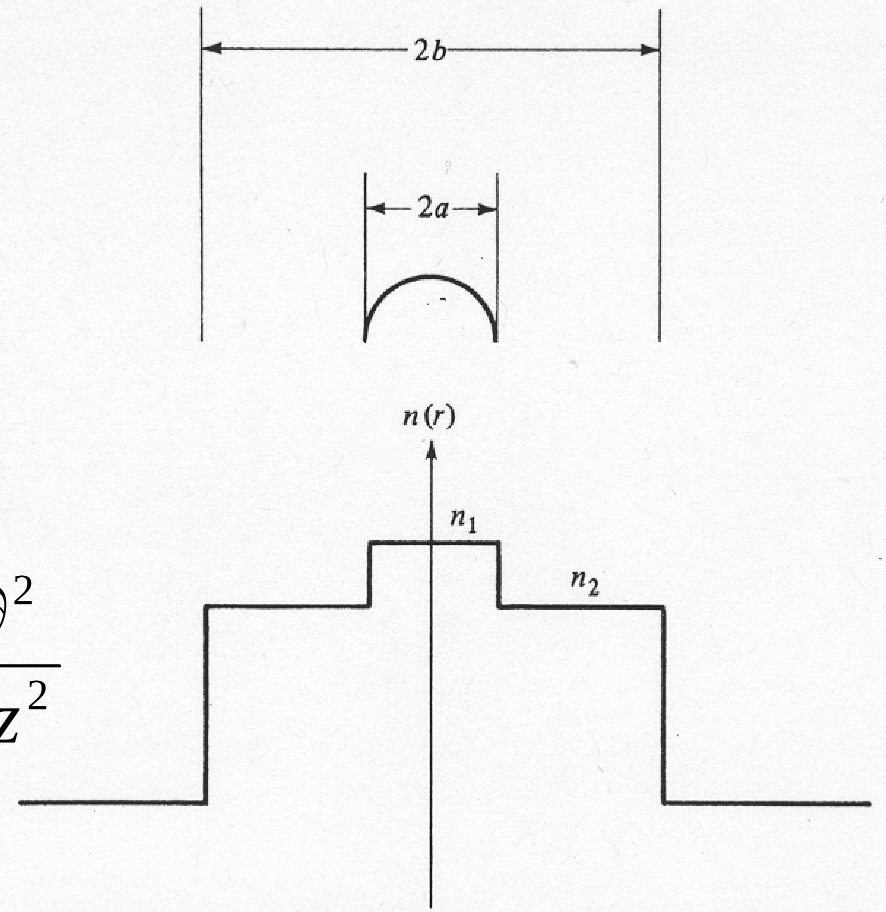


Figure 3-1 Structure and index profile of a step-index circular wa

Separable Solutions

$$[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + (k^2 - \beta^2)]E_z = 0$$

$$E_r = \psi(r)\Phi(\phi)e^{i(\omega t - \beta z)}$$

$$\Phi(\phi) = e^{\pm il\phi} \quad where \quad l = 0,1,2...$$

Separable Solutions

$$[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}+(k^2-\beta^2)]E_z=0$$

$$E_r=\psi(r)\Phi(\phi)e^{i(\omega t-\beta z)}$$

$$\Phi(\phi)=e^{\pm il\phi} \quad where \quad l=0,1,2...$$

$$[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+(k^2-\beta^2-\frac{l^2}{r^2})]\psi=0$$

Bessel differential equation

$$\psi=c_1 J_l(hr)+c_2 Y_l(hr) \quad k^2-\beta^2=h^2>0$$

$$\psi=c_1 I_l(qr)+c_2 K_l(qr) \quad k^2-\beta^2=-q^2>0$$

J Bessel function of the first kind

Y Bessel function of the second kind

I Modified Bessel function of the first kind

K Modified Bessel function of the second kind

Boundary Conditions

Decaying fields for r>a
q>0

$$q^2 = \beta^2 - k^2 = \beta^2 - n_2{}^2 k_0{}^2$$
$$k_0 = \omega / c$$

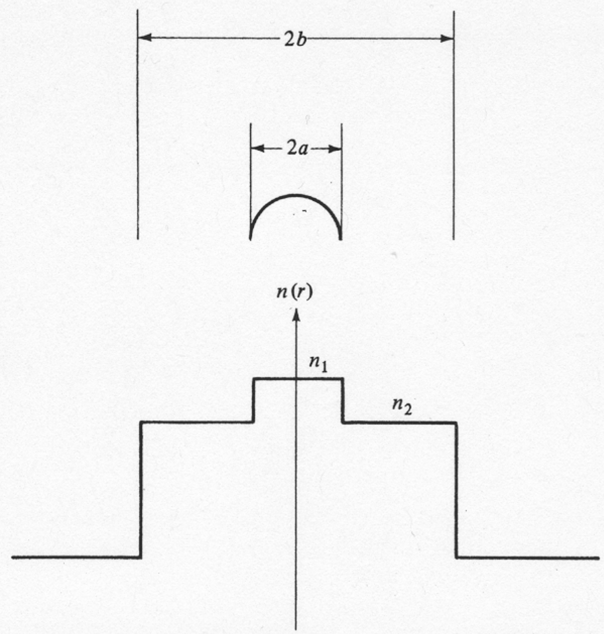


Figure 3-1 Structure and index profile of a step-index circular waveguide.

For fields in the core r<a, we need finite fields
(which eliminates Y and K which go to infinity as r approaches 0.

TE l=0 Modes

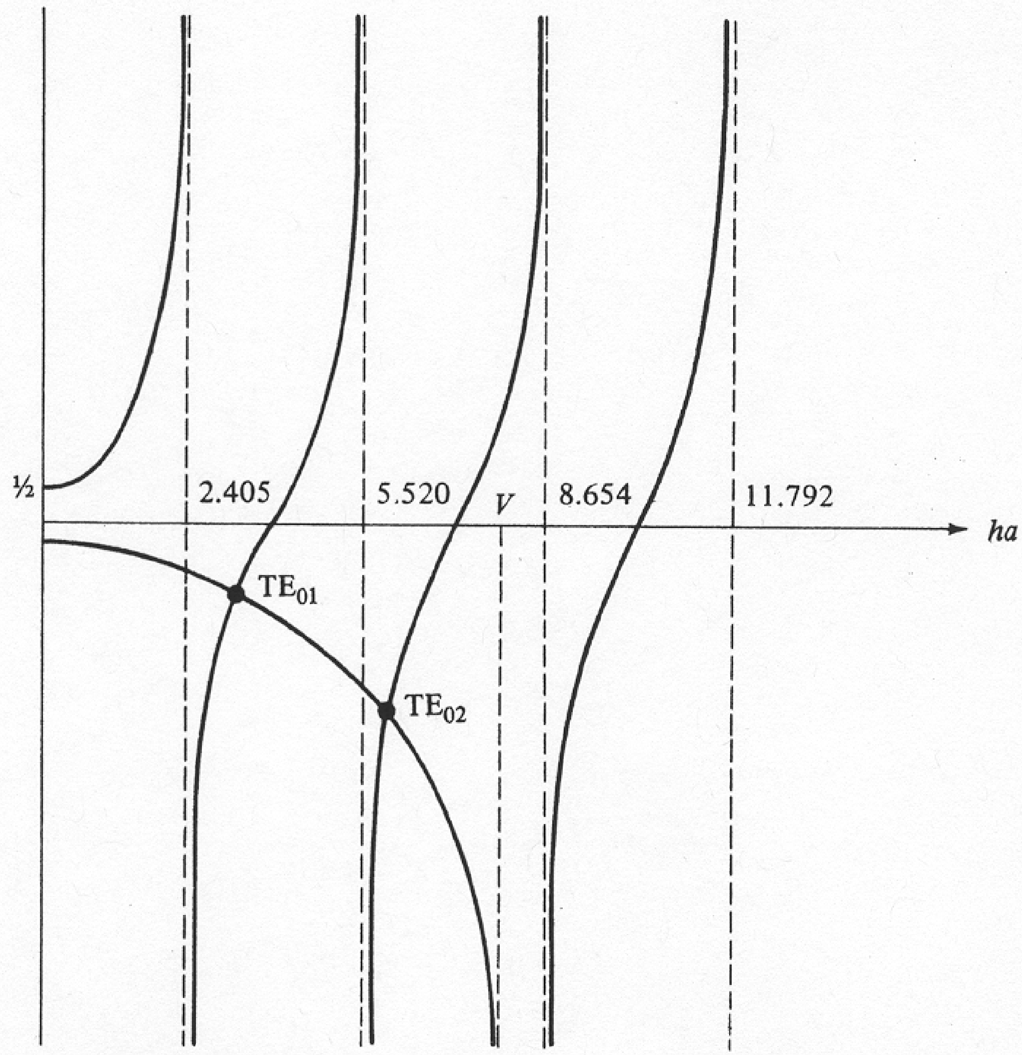


Figure 3-2 Graphical determination of the propagation constants of TE modes ($l = 0$) for a step-index waveguide.

l=1 (not TE or TM, but EH)

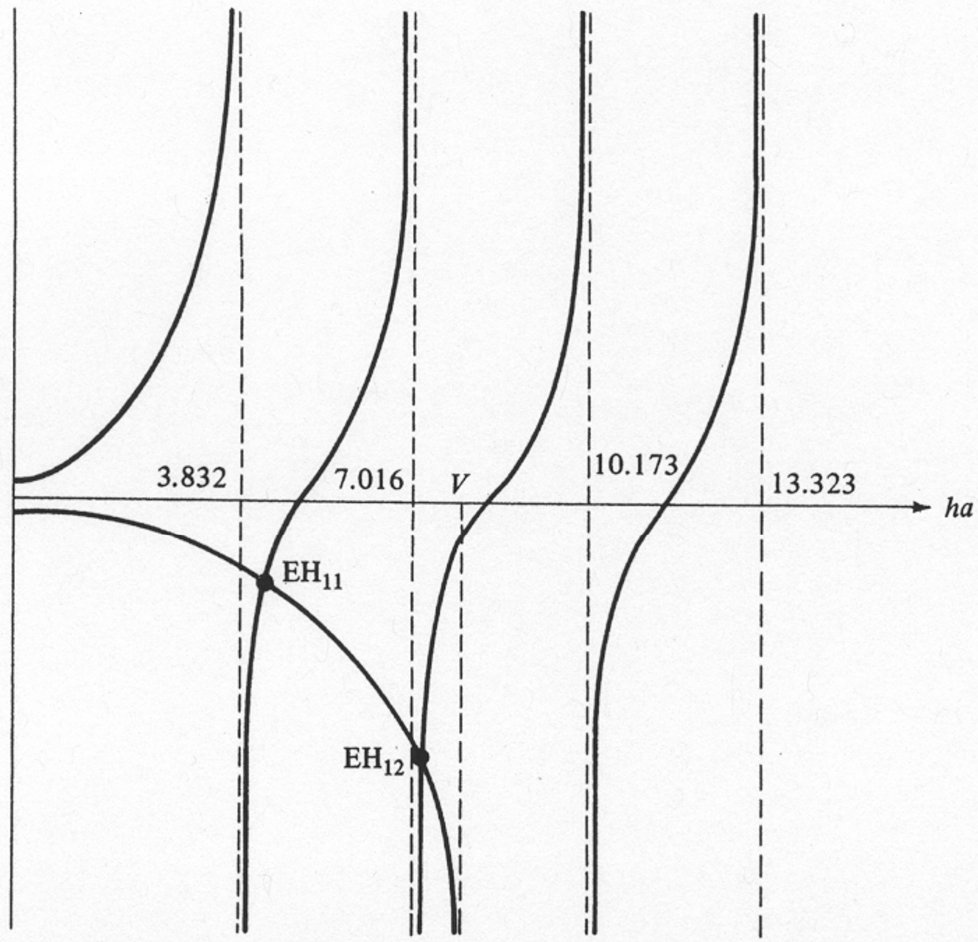


Figure 3-3 Graphical determination of the propagation constants of l = 1 EH modes for a step-index fiber.

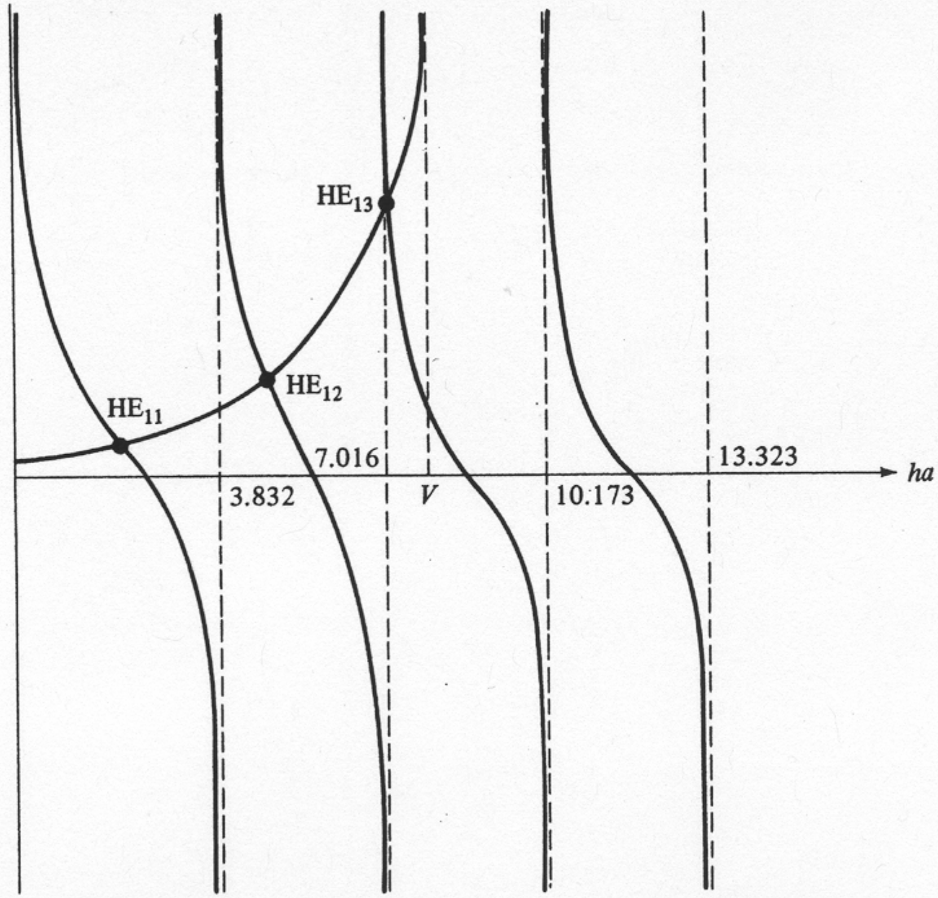


Figure 3-4 Graphical determination of the propagation constants of the $l = 1$ HE modes for a step-index dielectric waveguide.

V parameter

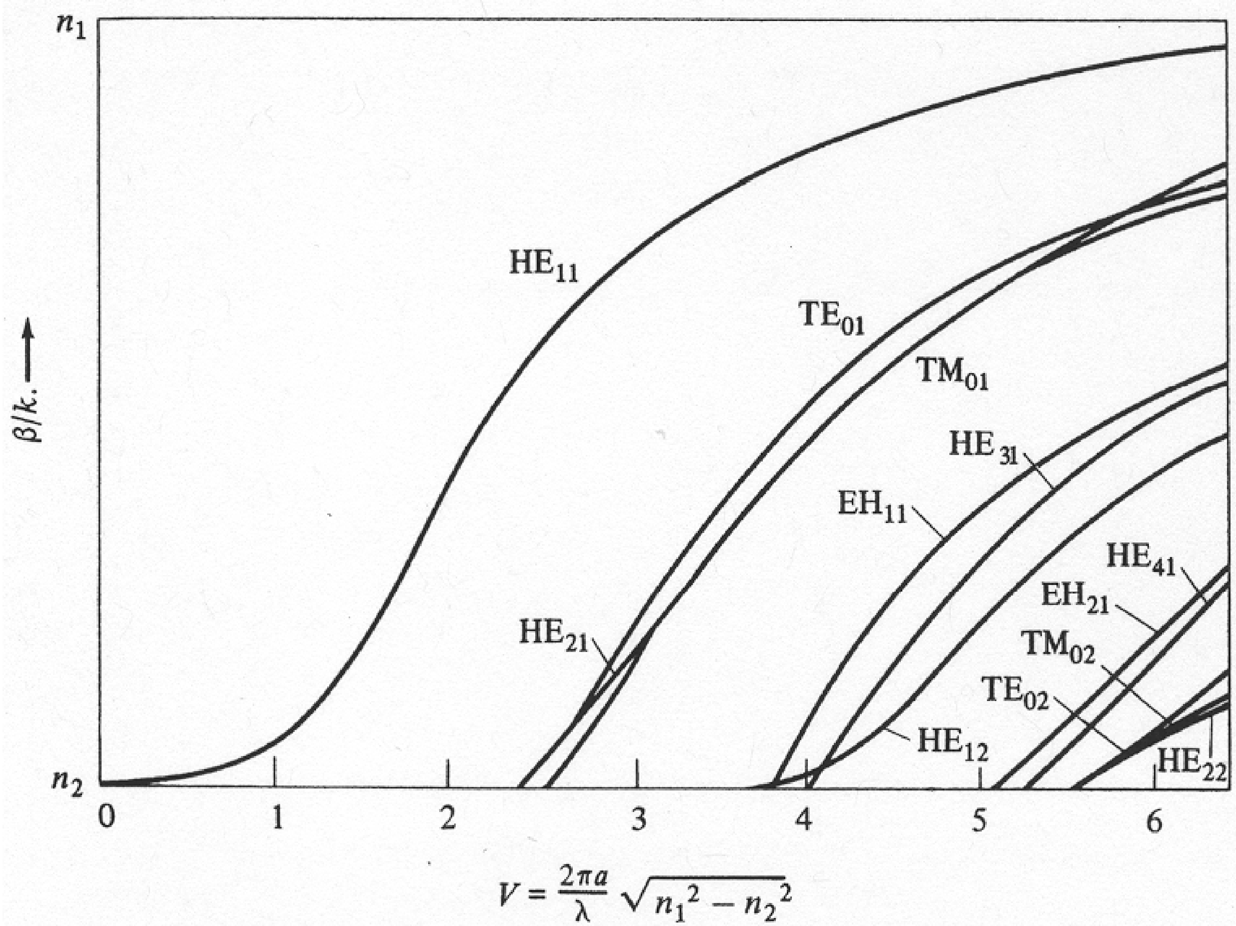


Figure 3-5 Normalized propagation constant as a function of V parameter for a few of the lowest-order modes of a step-index waveguide [4].

For n1-n2<<n1, LP approximation is valid.

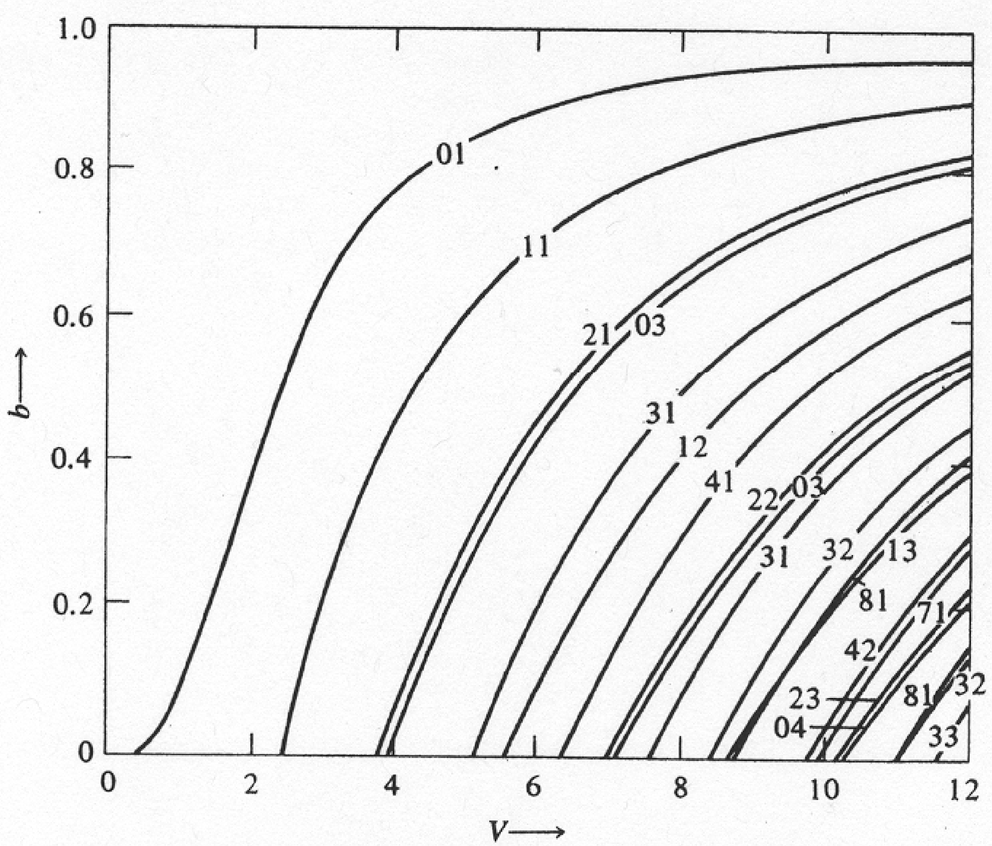


Figure 3-6 Normalized propagation constant b as function of normalized frequency V for the guided modes of the optical fiber, $b = (\beta/k_c - n_2)/(n_1 - n_2)$. (After Reference [5].)

Single mode cut off: V=2.405

Degenerate Modes LP_{11}

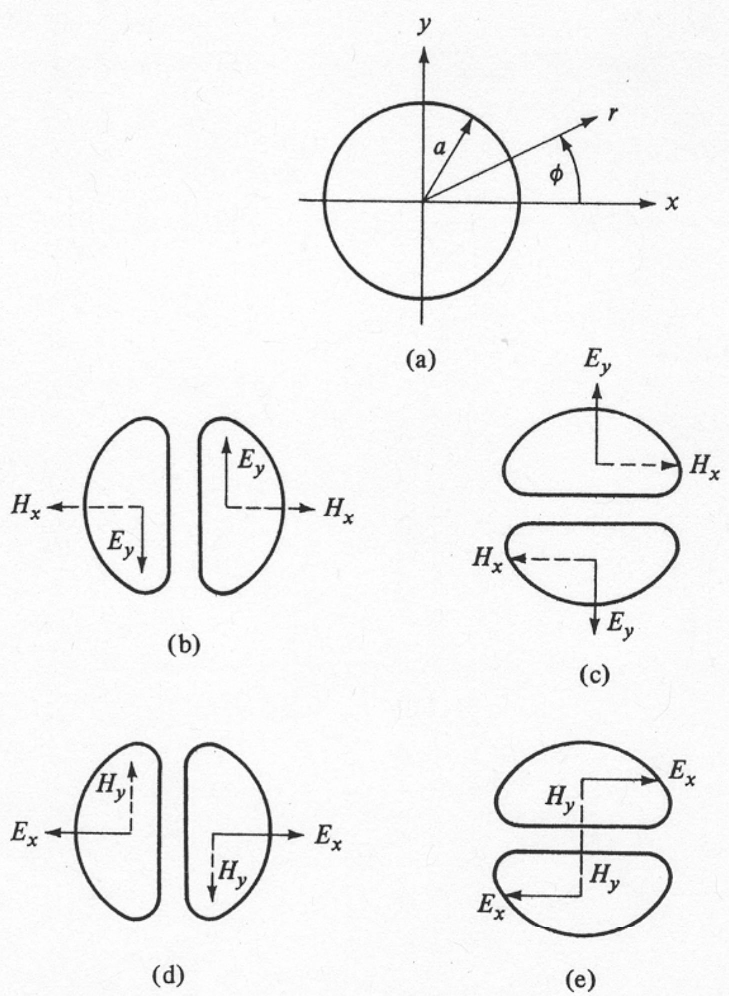


Figure 3-8 Sketch of the fiber cross section and the four possible distributions of LP_{11}.

Modes as a function of V parameter

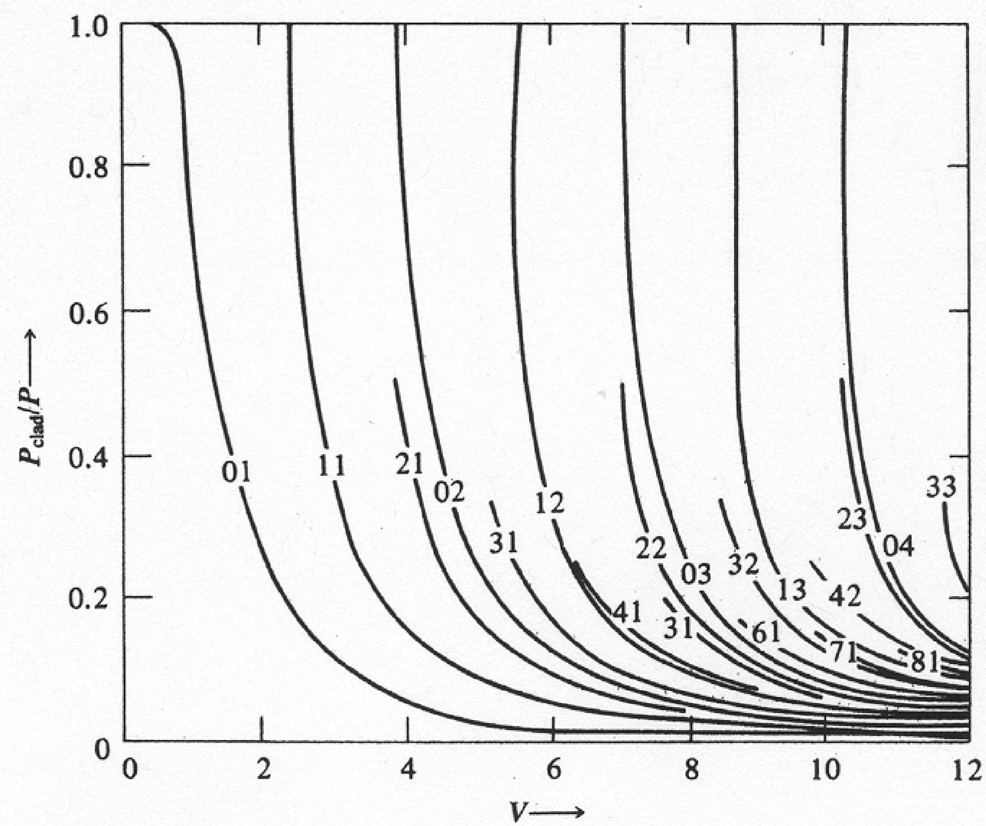
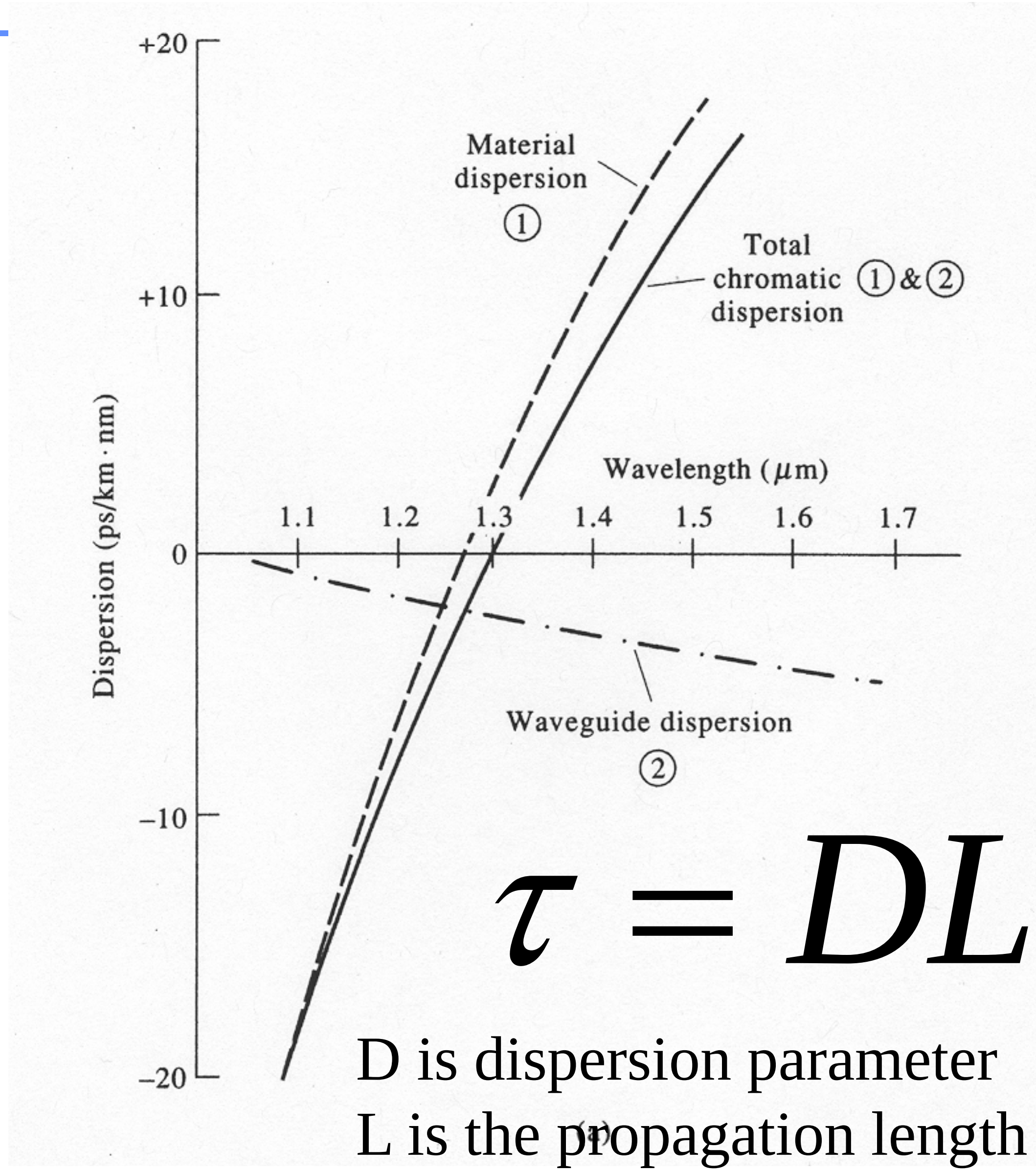


Figure 3-9 Fractional power contained in the cladding as a function of the frequency parameter V. (After Reference [5].)

At cutoff, all the power is in the cladding.

Dispersion



$$\tau = DL\sigma$$

D is dispersion parameter
L is the propagation length
σ is the spectral width

Dispersion (sum of material and waveguide dispersion)

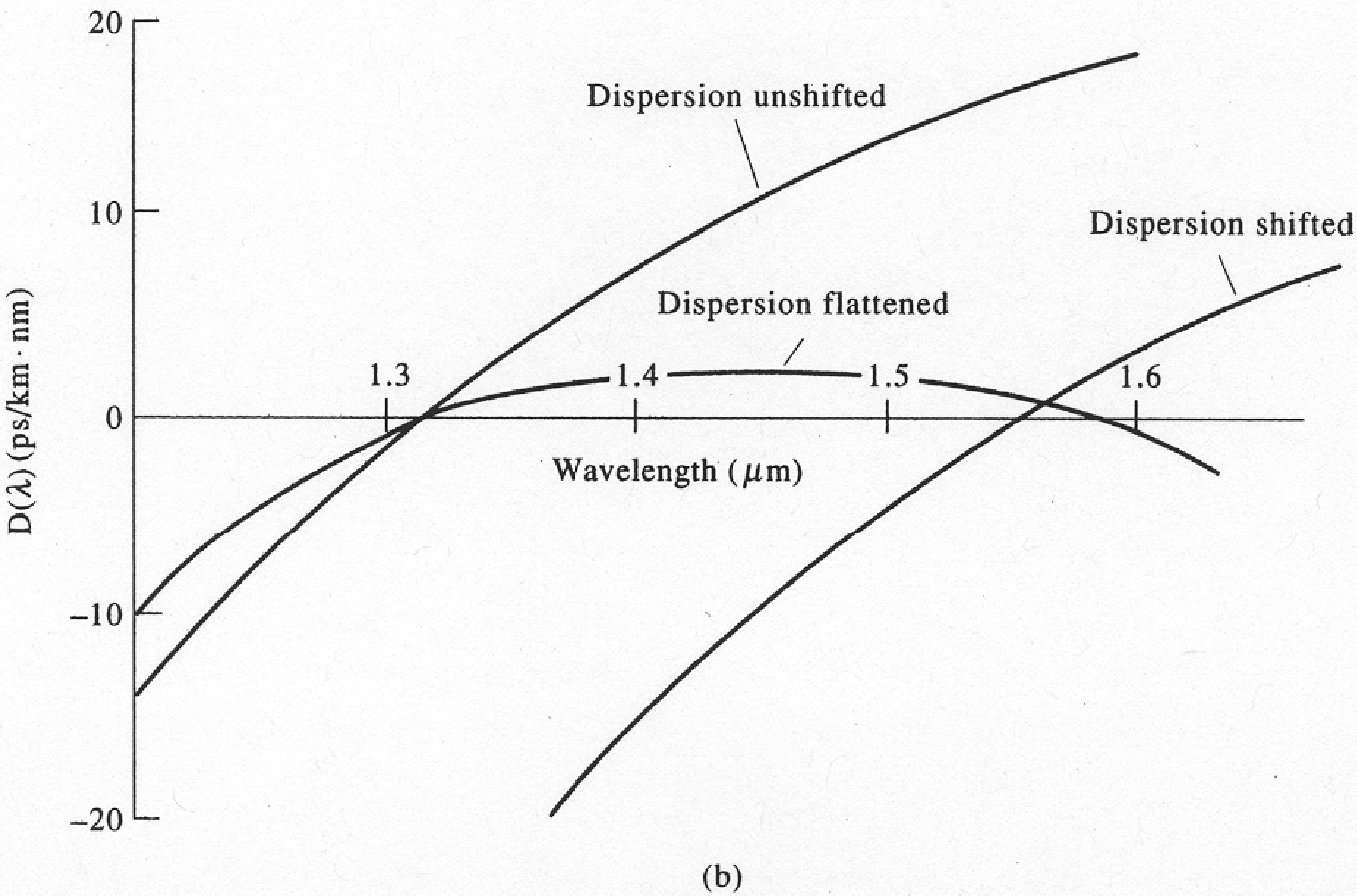


Figure 3-10 Group velocity disperion of (a) dispersion-unshifted 1.3 μm fiber and (b) dispersion- flattened and dispersion-shifted fibers. (After Reference [1].)

Loss in early optical fibers
(now the O-H peaks around 1.4 μm are small)

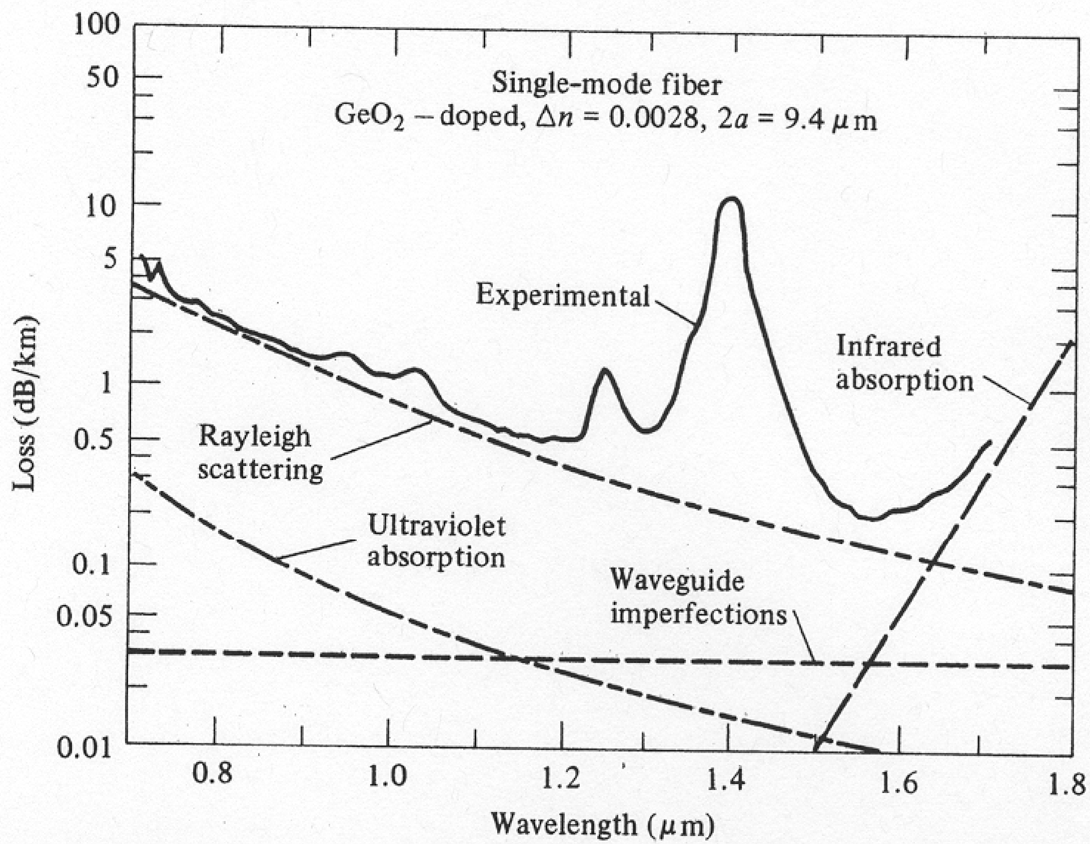


Figure 3-19 Observed loss spectrum of a germanosilicate single-mode fiber. Estimated loss spectra for various intrinsic materials effects and waveguide imperfections are also shown. (From Reference [20].)

Summary

- Single mode condition required for high performance
- Multimode fiber used for low cost
- Dispersion is designable.
- 1.3 micron: zero of dispersion
- 1.55 micron: minimum loss
- Zero dispersion is not good because of nonlinearity

Homework #1

- Read Kasap, Chapter 1
- Problems 1.2, 1.3, 1.7,1.8 due Wednesday, April 9