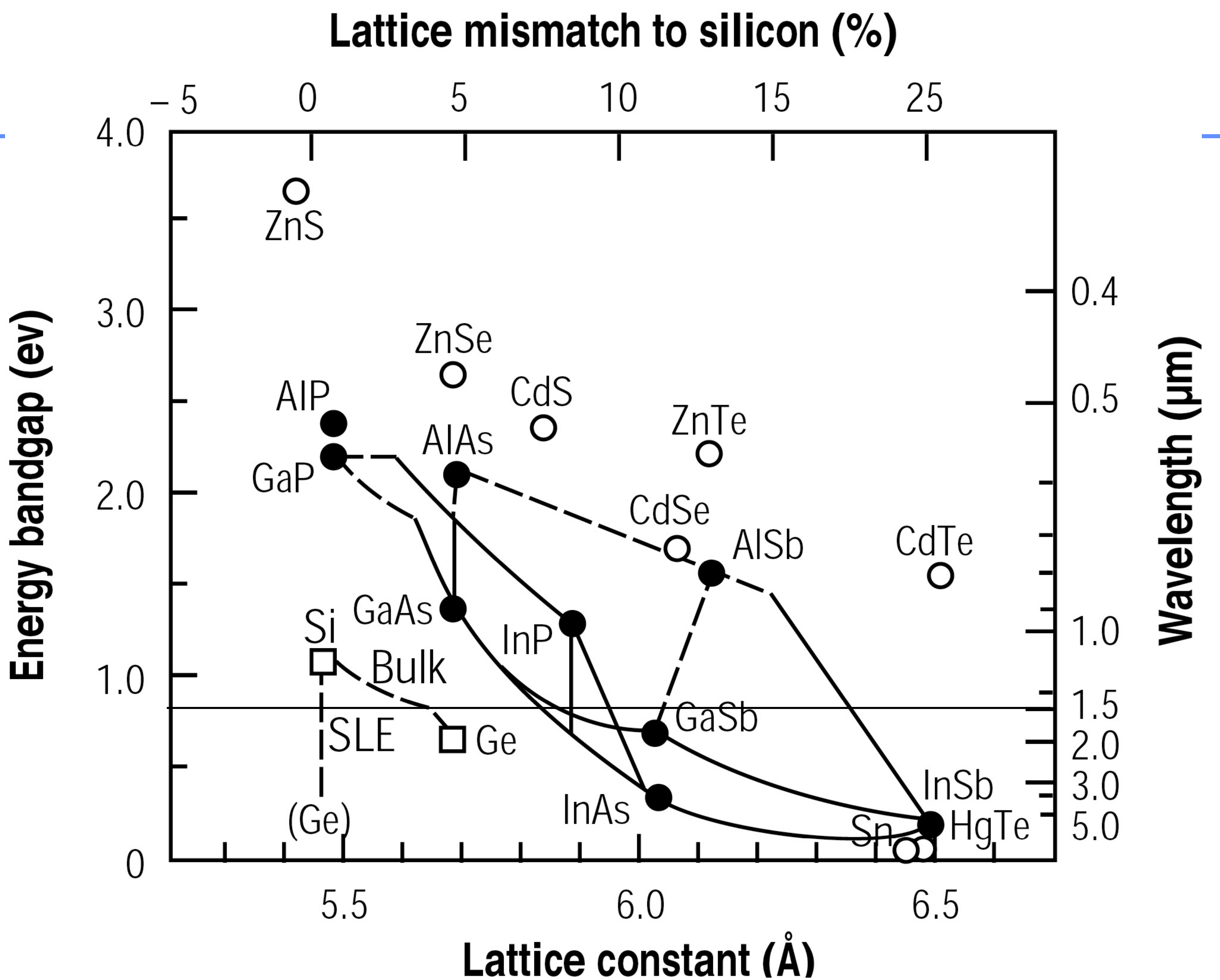
Gain and Absorption

ECE 162C

Lecture #6

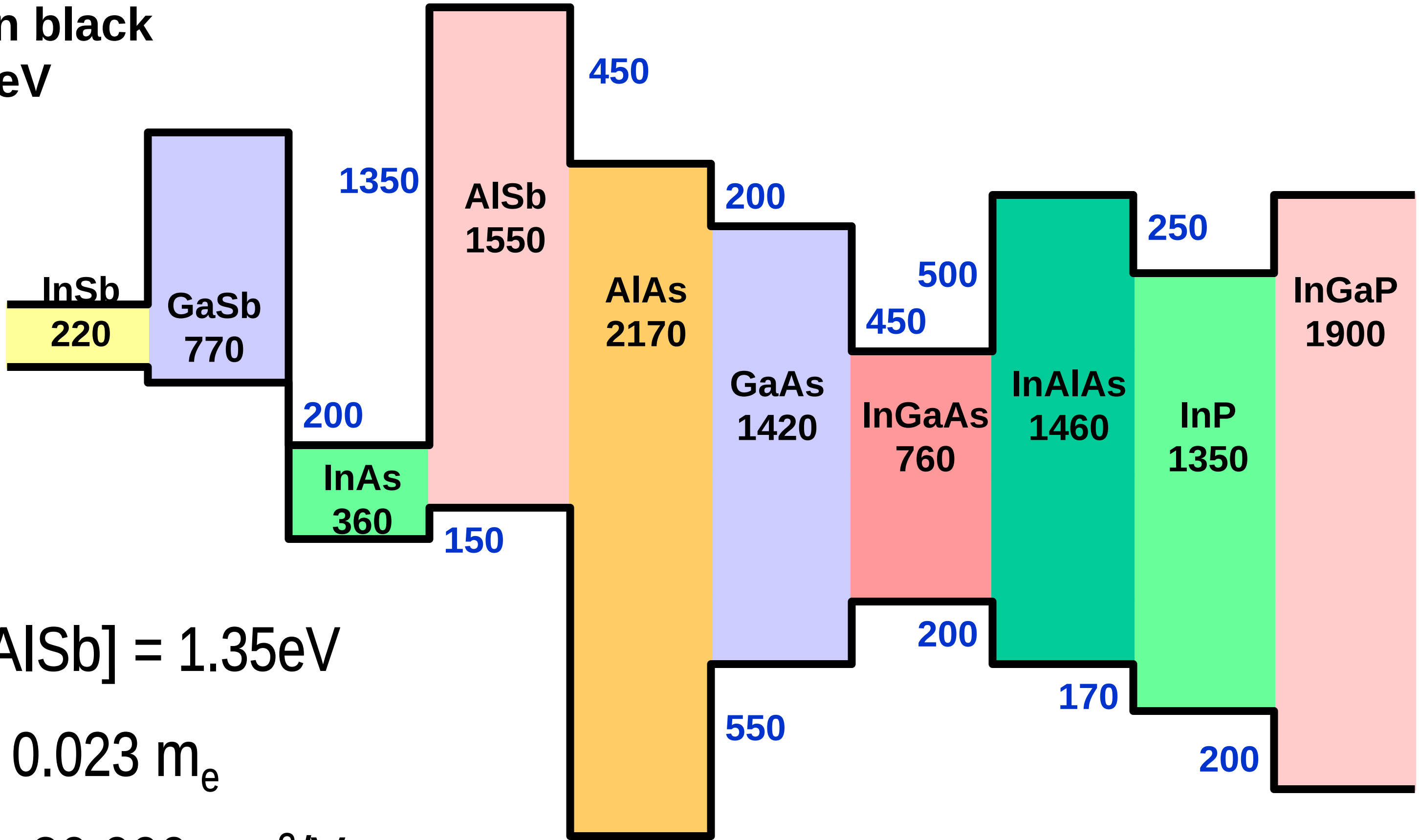
Prof. John Bowers



Lattice mismatch to silicon (%)
– 5
0
5
10
15
25
4.0
3.0
2.0
1.0
0
Energy bandgap (ev)
ZnS
ZnSe
CdS
AlP
AlAs
ZnTe
GaP
CdSe
AlSb
CdTe
GaAs
InP
Si
Bulk
SLE
Ge
(Ge)
GaSb
InAs
InSb
Sn
HgTe
0.4
0.5
1.0
1.5
2.0
3.0
5.0
Wavelength (μm)
5.5
6.0
6.5
Lattice constant (Å)

Bandgap Heaven

- Offsets in blue #s
- Bandgaps in black
- Units are meV



ΔE_C [InAs-AlSb] = 1.35eV

m* [InAs] = 0.023 m_e

InAs RT μ > 30,000 cm^2/Vs

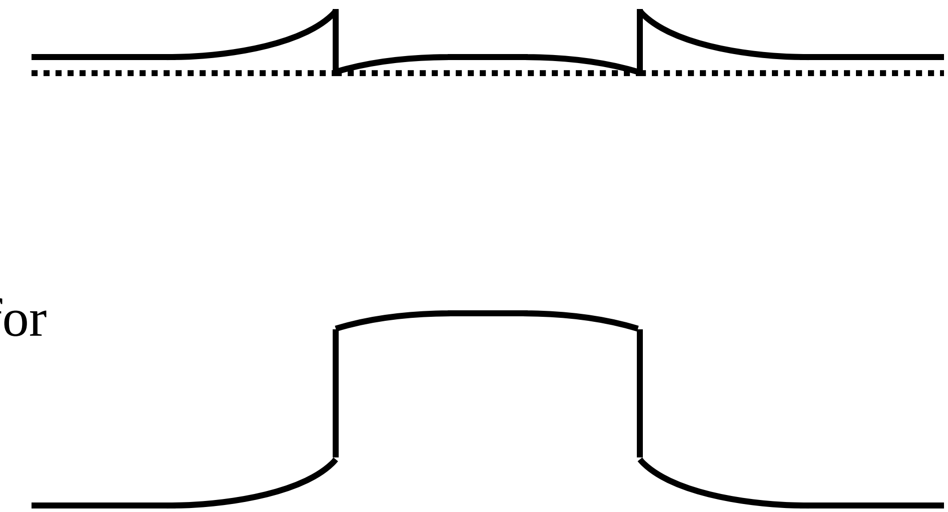
General comments

- If the composition is constant, the bandgap is constant. Hence, the separation of conduction band and valence band are constant.
- If there is no field, the bands are horizontal.
- Use the depletion edge approximation; either the material is depleted of free carriers (and the bands are bent) or there is no field and the bands are flat.
- Depleted doped material has a quadratic bend and linearly increasing field.
- Depleted undoped material has constant electric field and linear band bending.

$$\frac{d^2\Phi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{p - n + N_D - N_A}{\varepsilon}$$

InP/InGaAs/InP Square Well

- N+ InGaAs: E_g=.76 eV
- N+ InP: E_g=1.35 eV
- ΔE_g=.59 eV
- ΔE_c=.20 eV
- ΔE_v=.39 eV
- Adjust Fermi level to account for bias.
- Keep bandgaps constant.



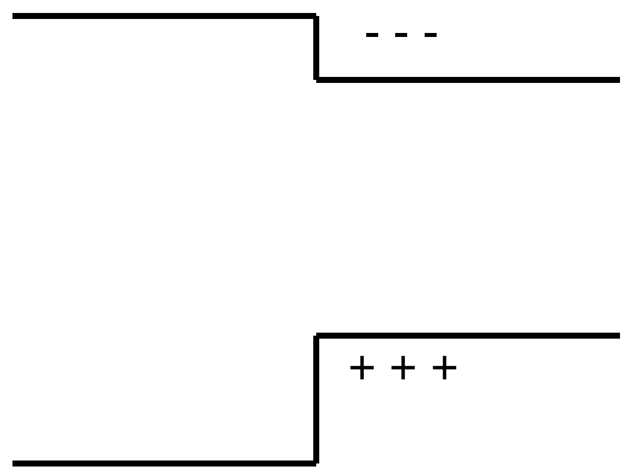
Calculate the absorption bandedge assuming a 100 Angstrom quantum well and m_e^*=0.1 m_e and m_h^*=1 m_e

Calculate the absorption bandedge assuming a 100 Angstrom quantum dot and m_e^*=0.1 m_e and m_h^*=1 m_e

Check the validity of your assumptions.

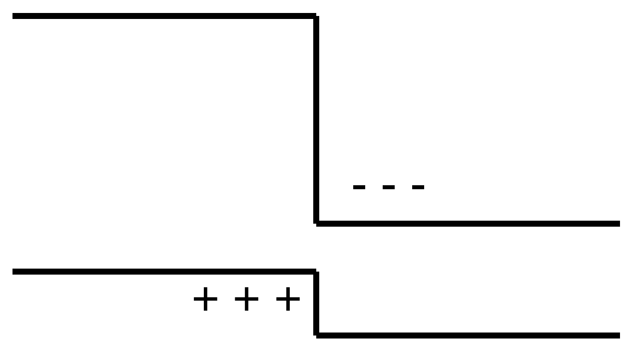
Three types of heterojunctions

- Type I (Straddling)
  - Free electrons and holes reside in the same region of space.
  - Examples: AlAs/GaAs, InP/GaInAs



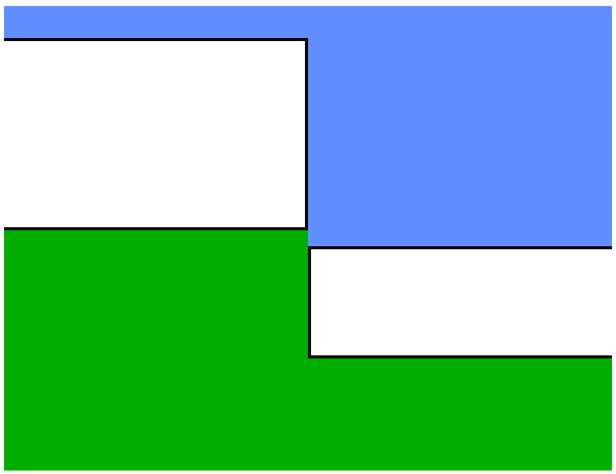
Three types of heterojunctions

- Type I (Straddling lineup)
- Type II (Staggered lineup)
  - □ ΔE_c and ΔE_c have the same sign.
  - – Free electrons and holes reside in different regions of space.
  - – Example: AlSb/InAs



Three types of heterojunctions

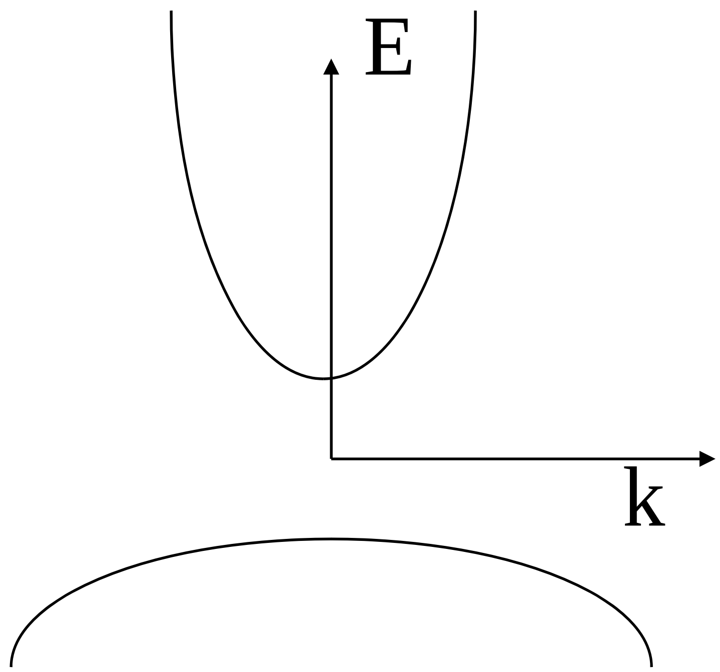
- Type I (Straddling lineup)
- Type II (Staggered lineup)
- Type III (Broken Gap Lineup)
  - □ ΔE_c and ΔE_c have the same sign.
  - – Free electrons and holes reside in different regions of space.
  - – Example: GaSb/InAs



Bands

- Direct bandgap: Minimum of conduction band and maximum of valence band occur at the same point in k space, typically k=0 (defined as Γ).

$$E = \frac{\hbar^2 k^2}{2m_e}$$



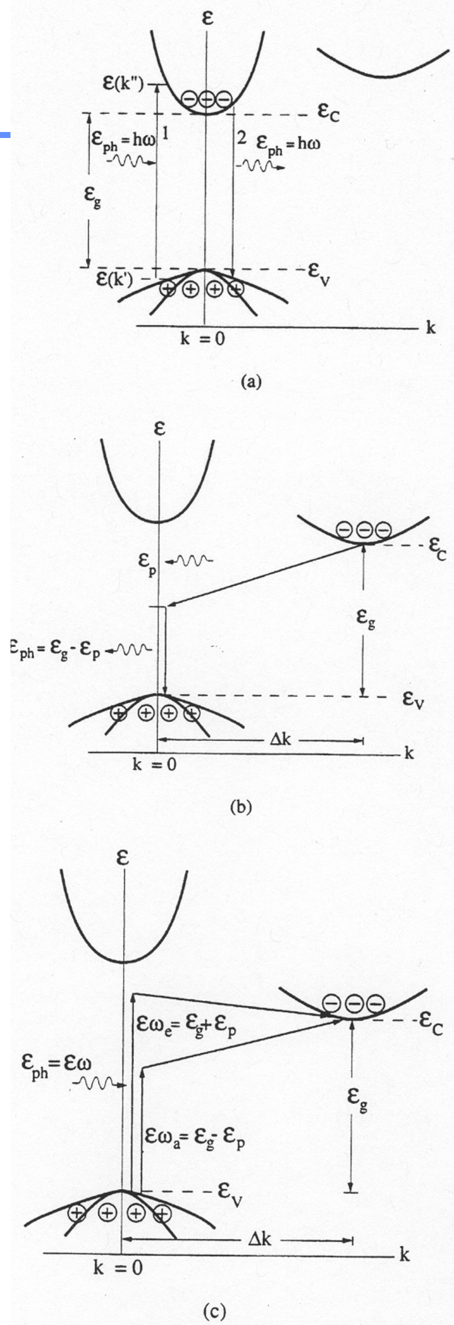
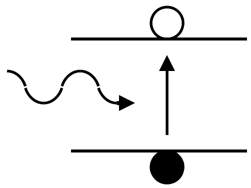


Figure 3.2 Illustration of band-to-band of absorption and recombination processes in (a) direct bandgap semiconductor and (b) and (c) indirect bandgap semiconductor.

Si: Indirect gap.

Hence, a phonon is required to conserve momentum. Less likely to occur. Lower gain and absorption (except at higher energies).

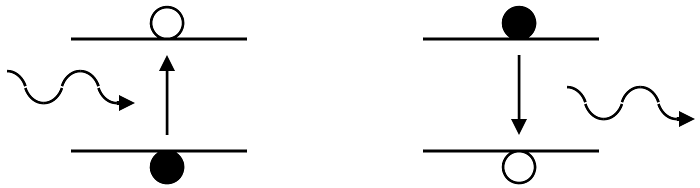
Electronic transitions



R_{12}

- R_{12}: absorption of a photon

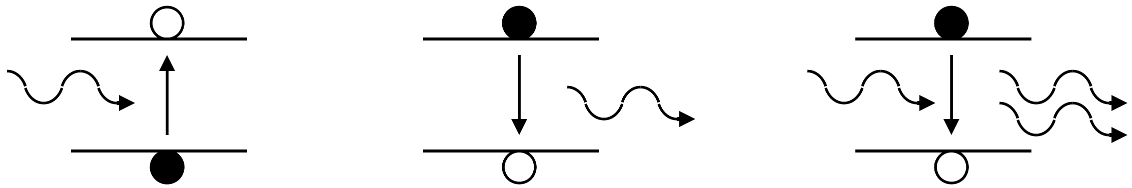
Electronic transitions



- R_{sp}

- R_{12}: absorption of a photon
- R_{sp}: spontaneous emission of a photon

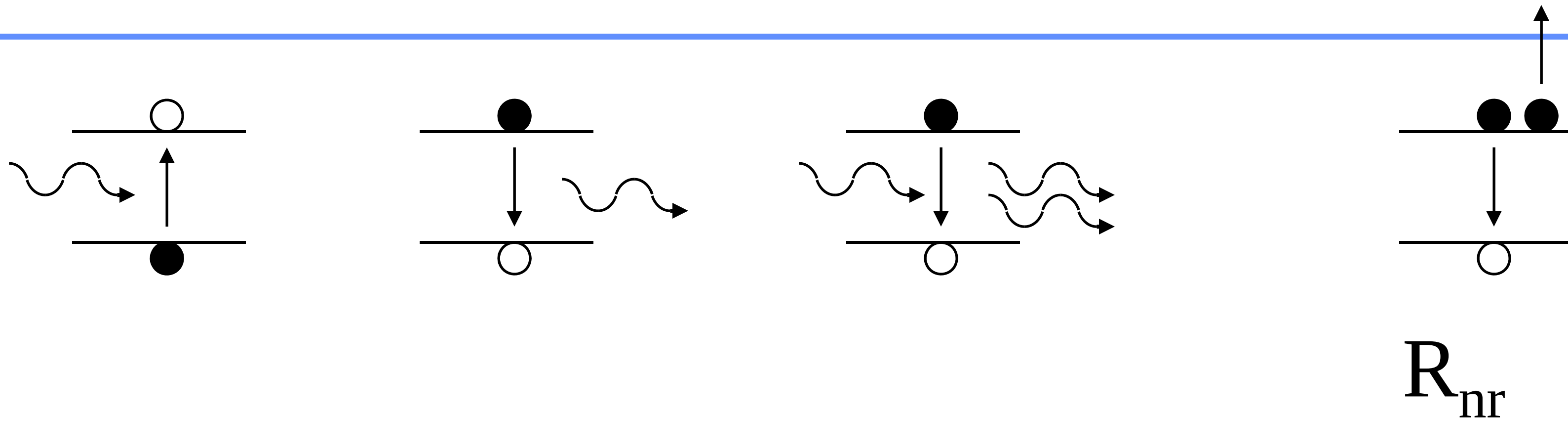
Electronic transitions



- R_{21}

- R_{12}: absorption of a photon
- R_{sp}: spontaneous emission of a photon
- R_{21}: stimulated emission of a photon

Electronic transitions



- R_{12}: absorption of a photon
- R_{sp}: spontaneous emission of a photon
- R_{21}: stimulated emission of a photon
- R_{nr}: nonradiative recombination (Auger, trap, etc.)

Rate Equations

$$\frac{dN}{dt} = G - R$$

N is the electron density (assumed equal to hole density)

G is the generation rate of electrons

R is the total recombination rate

Rate Equations

$$\frac{dN}{dt} = G - R$$

$$G = \frac{\eta_i I}{qV}$$

$$R = R_{sp} + R_{nr} + R_i + R_{st}$$

$$R = BN^2 + AN + CN^3 + R_{st}$$

$$R \approx \frac{N}{\tau}$$

N is the electron density (assumed equal to hole density)

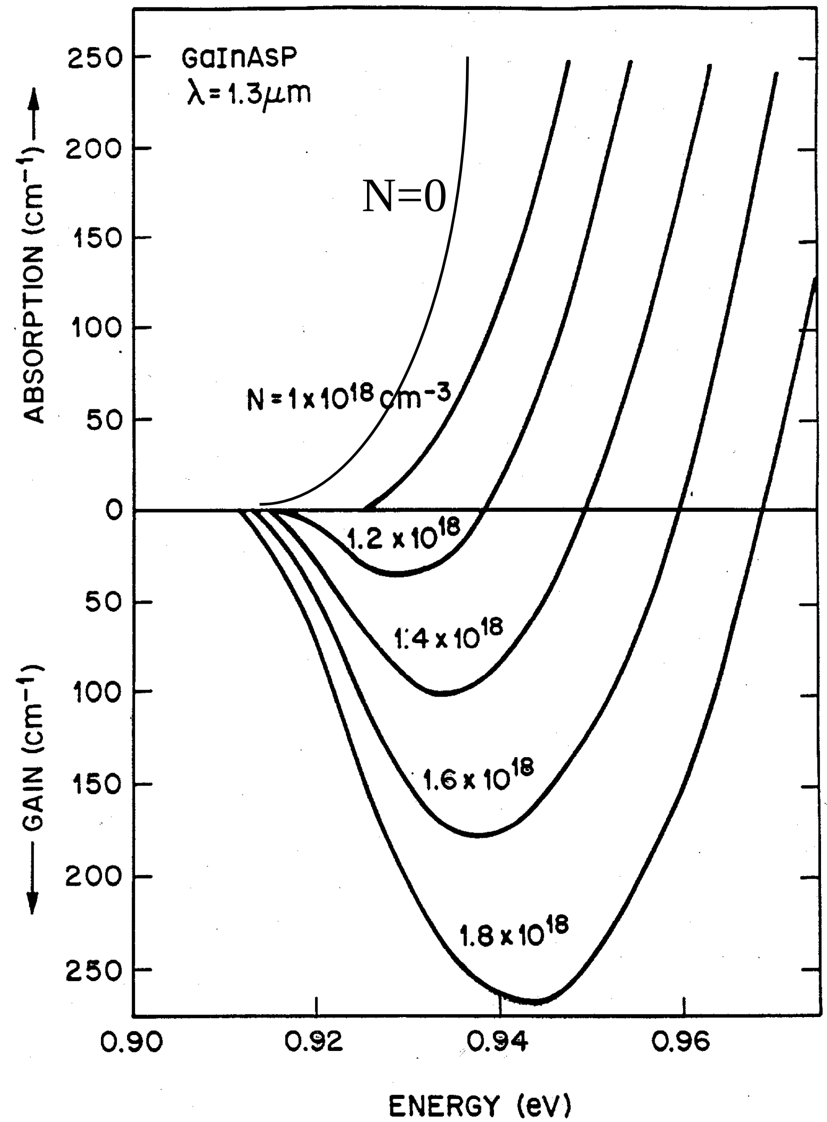
G is the generation rate of electrons

R is the total recombination rate

Material Gain

Calculated gain curves for InGaAsP/InP laser operating at 1.3μm

- Gain peak moves to shorter wavelengths with higher pumping
- Higher differential gain for wavelengths shorter than the gain peak



N.K. Dutta, J. Appl. Phys., **51**, 6095 (1980)

Gain

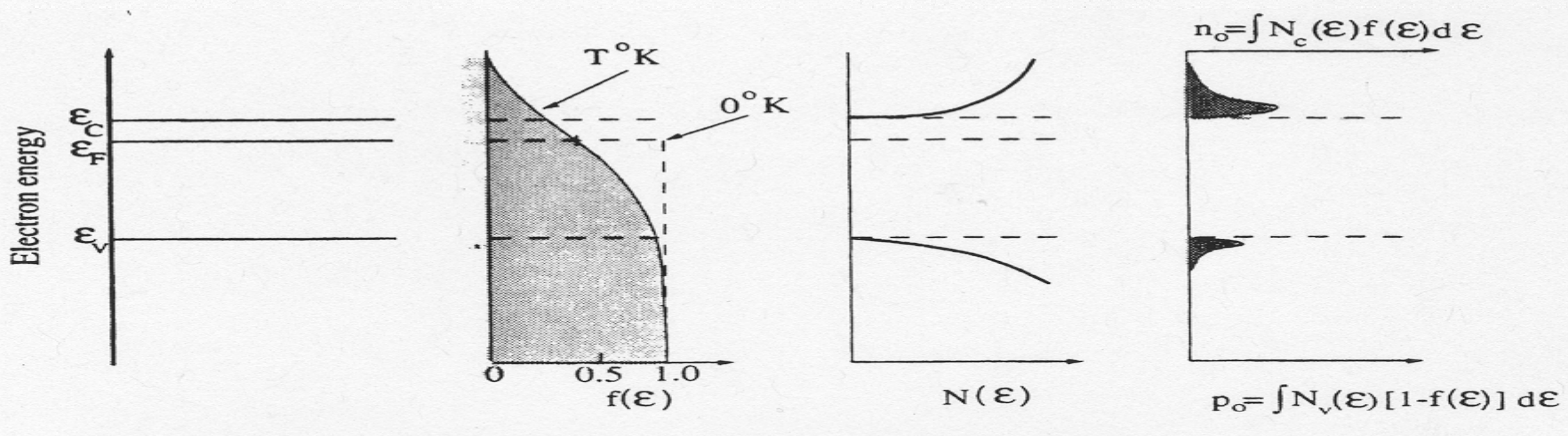


Figure 2.18 Distribution functions, density-of-states functions, and carrier distributions (in energy) in an n-type nondegenerate semiconductor.

High gain requires

1) upper level full (f~1)

2) lower level empty (f~0)

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

Carrier Injection

- In equilibrium,

$$pn = n_i^2$$

- Under forward bias,

$$pn >> n_i^2$$

- Under reverse bias,

$$pn << n_i^2$$

Carrier Injection

- In equilibrium,

$$pn = n_i^2 \qquad E_{Fn} = E_{Fp} = E_F$$

- Under forward bias,

$$pn >> n_i^2 \qquad E_{Fn} - E_{Fp} > 0$$

- Under reverse bias,

$$pn << n_i^2 \qquad E_{Fn} - E_{Fp} < 0$$

Quasi Fermi Levels

$$n = n_i e^{(E_{Fn} - E_i)/kT}$$

$$p = n_i e^{-(E_{Fp} - E_i)/kT}$$

$$pn = n_i^2 e^{(E_{Fn} - E_{Fp})/kT}$$

Quasi Fermi Levels

$$n = n_i e^{(E_{Fn} - E_i)/kT}$$

$$p = n_i e^{-(E_{Fp} - E_i)/kT}$$

$$pn = n_i^2 e^{(E_{Fn} - E_{Fp})/kT}$$

- Gain occurs when

$$g(\hbar\omega) > 0 \quad when \quad E_{Fn} - E_{Fp} > \hbar\omega$$

Optical Gain in Semiconductors

Gain between two levels depends on:

- Carrier density, i.e. level of inversion

$$\frac{R_{21}}{R_{12}} = \frac{f_2(1-f_1)}{f_1(1-f_2)} = e^{\left(\Delta E_f - E_{21}\right)/kT}$$

- Reduced density of states

$$\frac{1}{\rho_r} = \frac{1}{\rho_c} + \frac{1}{\rho_v}$$

- Transition matrix element $|M|^2$

Gain : Reduced Density of States

The optical gain is proportional to the reduced density of state at the transition energy:

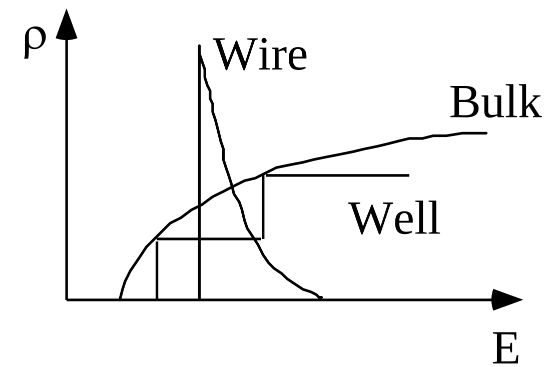
$$g(\omega) \propto (f_2 - f_1)\rho_r(\hbar\omega)$$

The reduced density of states:

$$\rho_r(E) = \frac{\sqrt{E}}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}$$ (Bulk)

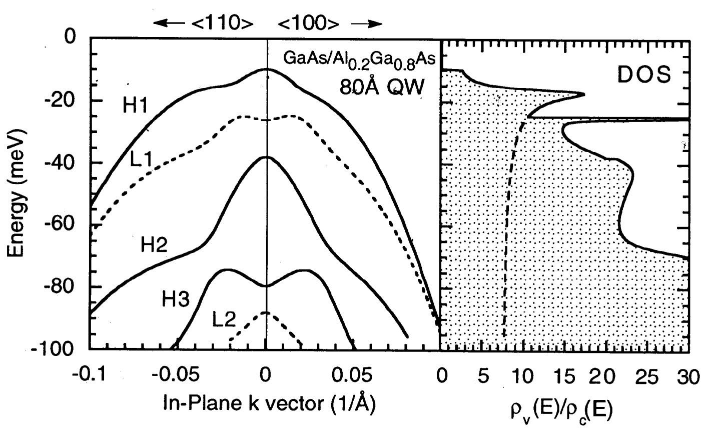
$$\rho_r(E) = \frac{m}{2\pi\hbar^2}\frac{1}{L_z}$$ (Well)

$$\rho_r(E) = \frac{\sqrt{2m}}{\hbar}\frac{1}{2\pi L_x L_y\sqrt{E}}$$ (Wire)

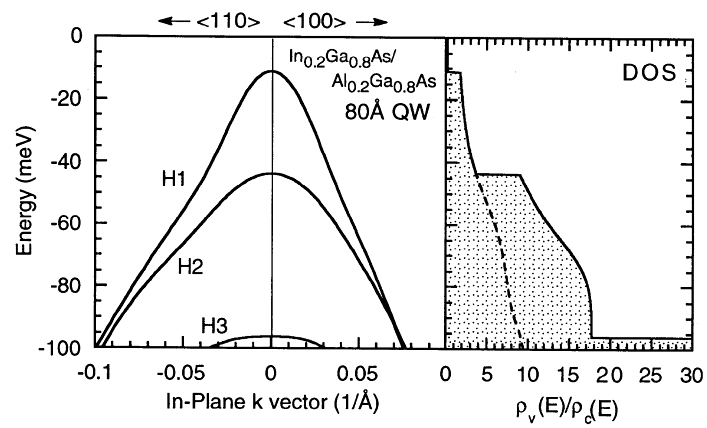


Gain : Strained QW

Compressive strain increases the energy gap between the heavy hole and the light hole subbands. This means fewer carriers in light hole band.



Lattice matched QW



Compressive strain

L.A. Coldren and S.W. Corzine, "Diode Lasers and Photonic Integrated Circuit", Wiley (1995)

Gain

$$N_p = N_p(0)e^{gz}$$

$$g = \frac{1}{N_p}\frac{dN_p}{dz} = \frac{1}{v_g N_p}\frac{dN_p}{dt} = \frac{1}{v_g N_p}(R_{21} - R_{12})$$

Gain

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$$R_{21} = R_r f_2(1 - f_1)$$

$$R_{12} = R_r f_1(1 - f_2)$$

$$R_{21} - R_{12} = R_r(f_2 - f_1)$$

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$$Fermi's Golden Rule$$

$$R_r = \frac{2\pi}{\hbar}|M_{21}|^2 \rho_r(E_{21})$$

$$g = \frac{1}{v_g N_p}\frac{2\pi}{\hbar}|M_{21}|^2 \rho_r(E_{21})(f_2 - f_1)$$

Lineshape function

- To get total gain, integrate over all possible states that contribute to the gain (within a lineshape function).

$$g(\hbar\omega) = \int g_{21} L(\hbar\omega - E_{21}) dE_{21}$$

$$L(\hbar\omega - E_{21}) = \frac{1}{\pi} \frac{\hbar / \tau_{in}}{(\hbar / \tau_{in})^2 + (\hbar\omega - E_{21})^2}$$

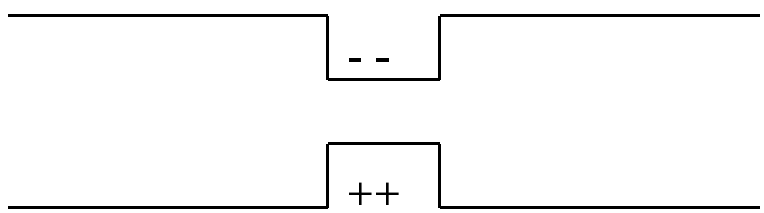
- This Lorentzian lineshape is common, but not the most accurate.

Lasing threshold

- Lasing occurs when the round trip gain equals the loss in the cavity.
- Not all of the mode sees gain, but only the fraction that overlaps with the gain region. Hence the modal gain is related to the material gain by an effective confinement factor Γ

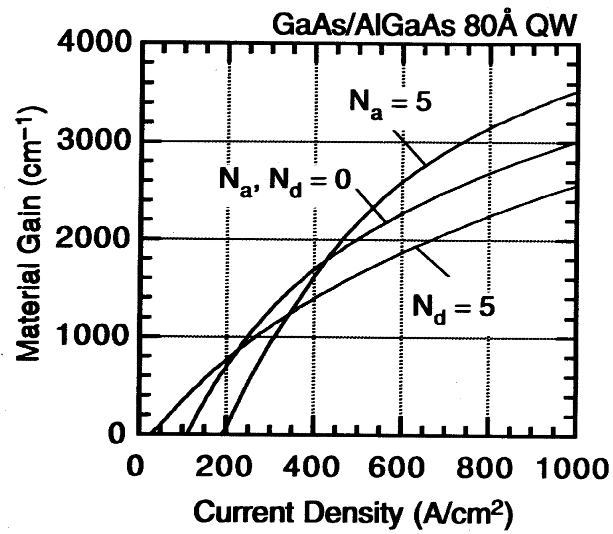
Double Heterostructure Lasers (Kroemer)

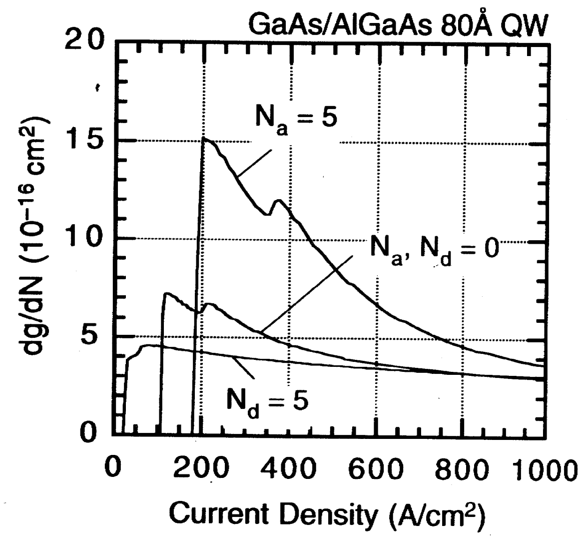
- Carriers diffuse away so it is difficult to get high gain
- A method of confining the carriers to a region in space is necessary
- Double heterostructure (proposed in 1964 but not implemented until 1968, which led to the first cw lasers).



Gain : Doping

p-doping in the active region increases the differential gain, at the expense of increased threshold current density





L.A. Coldren and S.W. Corzine, "Diode Lasers and Photonic Integrated Circuit", Wiley (1995)

Rate Equations

Neglecting the phase of the optical field, the length dependence of the carrier and photon densities, and the modal dependence; the rate equations for the averaged photon and carrier densities become:

$$\frac{dS}{dt} = \frac{\Gamma v_g a\left(N - N_{tr}\right)}{1 + \varepsilon S} S - \frac{S}{\tau_p} + \frac{\beta \Gamma N}{\tau_n}$$

$$\frac{dN}{dt} = \eta_i \frac{I}{qV} - \frac{v_g a(N - N_{tr})}{1 + \varepsilon S} S - \frac{N}{\tau_n}$$

Gain

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$$g = \frac{1}{v_g N_p}\frac{2\pi}{\hbar}\left|M_{21}\right|^2 \rho_r(E_{21})(f_2 - f_1)$$

Lineshape function

- To get total gain, integrate over all possible states that contribute to the gain (within a lineshape function).

$$g(\hbar\omega) = \int g_{21} L(\hbar\omega - E_{21}) dE_{21}$$

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- This Lorentzian lineshape is common, but not the most accurate.

Lasing threshold

- Lasing occurs when the round trip gain equals the loss in the cavity.
- Not all of the mode sees gain, but only the fraction that overlaps with the gain region. Hence the modal gain is related to the material gain by an effective confinement factor Γ

Lasing threshold

r_1 is the amplitude reflection coefficient.

R_1 is the power reflection coefficient

R is the average power reflection coefficient

$$R_1 = r_1^2$$

$$R = \sqrt{R_1 R_2} = r_1 r_2$$

$$R_1 R_2 e^{(\Gamma g_{th} - \alpha_i)2L} = 1$$

$$\Gamma g_{th} = \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}$$

$$\Gamma g_{th} = \alpha_i + \alpha_m = \alpha_T$$

$$\alpha_m = \frac{1}{2L} \ln \frac{1}{R_1 R_2} = \frac{1}{L} \ln \frac{1}{R}$$

$R = 0.32$ (typical for InGaAsP or GaAs Lasers)

Round trip phase

- Round trip phase must be a multiple of pi.
- The round trip cavity length must be a multiple of the wavelength

$$m\lambda = 2nL$$

- The spacing between modes is

$$\Delta\lambda = \frac{\lambda^2}{2n_g L}$$

Laser Structure
Current Confinement

- Dielectric layers, e.g. SiO_2, SiN_x, polyimide or oxidized AlGaAs
  - Very low capacitance
  - Possible reliability problem
  - Poor thermal characteristics
- Reverse biased p-n junctions
  - Good high power and high T capability
  - Large depletion capacitance
- Larger bandgap homojunctions
  - Simple fabrication
  - Leakage current and diffusion capacitance are high
- Semi-insulating semiconductor regions