Name________________________________

Tear off and save the ID number below, to look up your grade later…

ECE 179d

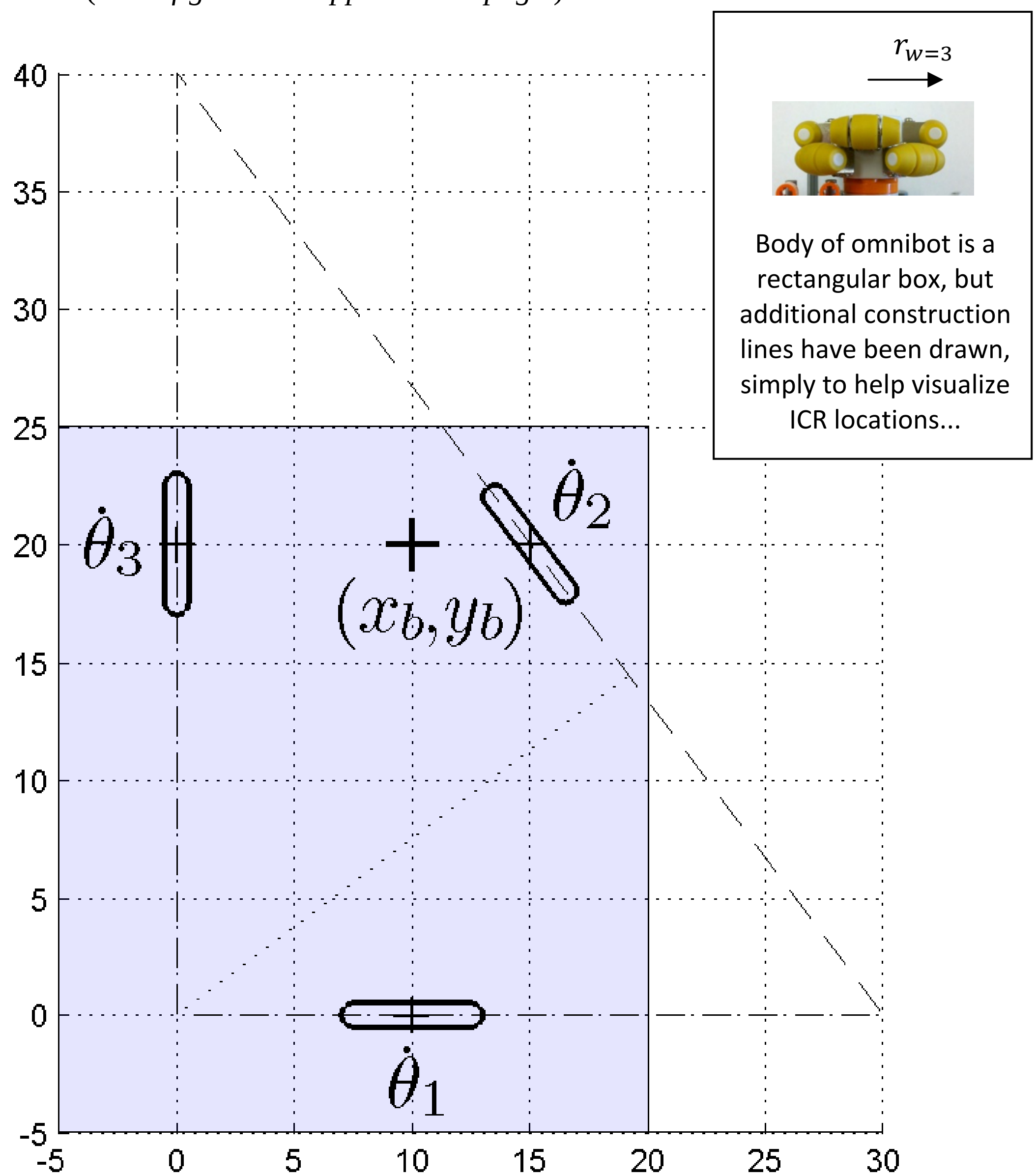
Final Exam

- Extra pages are provided at back of exam, if needed. **Turn in all work!**
- You are allowed one (1) single-sided sheet of notes.
- No calculators or other computing devices are allowed.
- If you are ever stuck on how to start a problem, just SKIP IT and come back later if you have time!

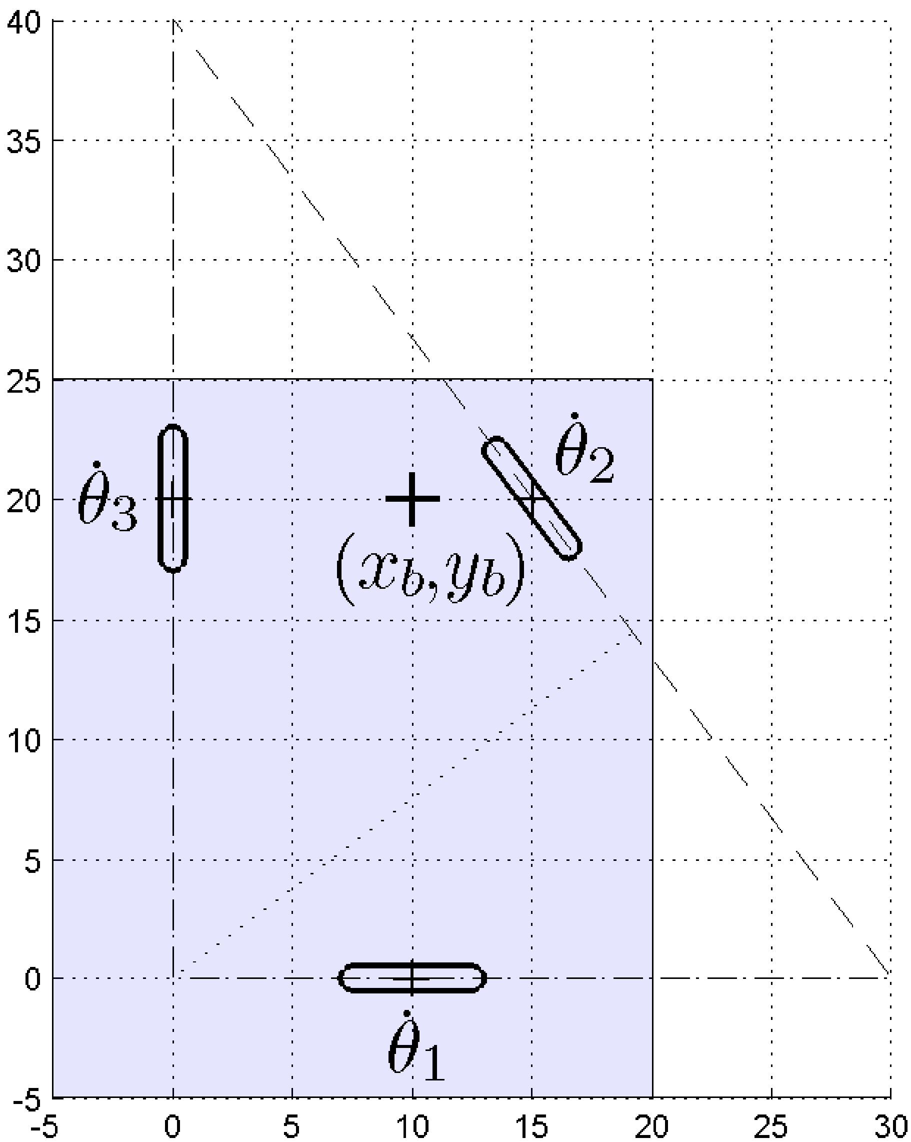
Good Luck!

Problem 1 – Below is a figure of an omnibot. The three angles $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$, are the rotations of the powered (motor-driven) axes. Each angle is defined such that a POSITIVE rotation in any $\dot{\theta}_n$ results in a POSITIVE (i.e., counter-clockwise) rotation in the body of the omnibot, $\dot{\phi}_b$. There are also freely-spinning rollers that allow for free rotation perpendicular to each driven axis. The radius for each of the primary (driven) wheels is $r_w = 3$, as drawn to scale below.

a) Derive the 3x3 Jacobian matrix relating wheel velocities, $\dot{q}_w$, to the translational and angular velocities of the point (x_b,y_b) on the omnibot below. Recall for a robot that $\dot{\xi}_b = J\dot{q}_w$, where, $\xi_b = \begin{bmatrix} x_b \\ y_b \\ \phi_b \end{bmatrix}$ gives the position and orientation of the point (x_b,y_b), and $q_w = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$ gives the angular rotations of the wheels. (*Same figure also appears next page*.)

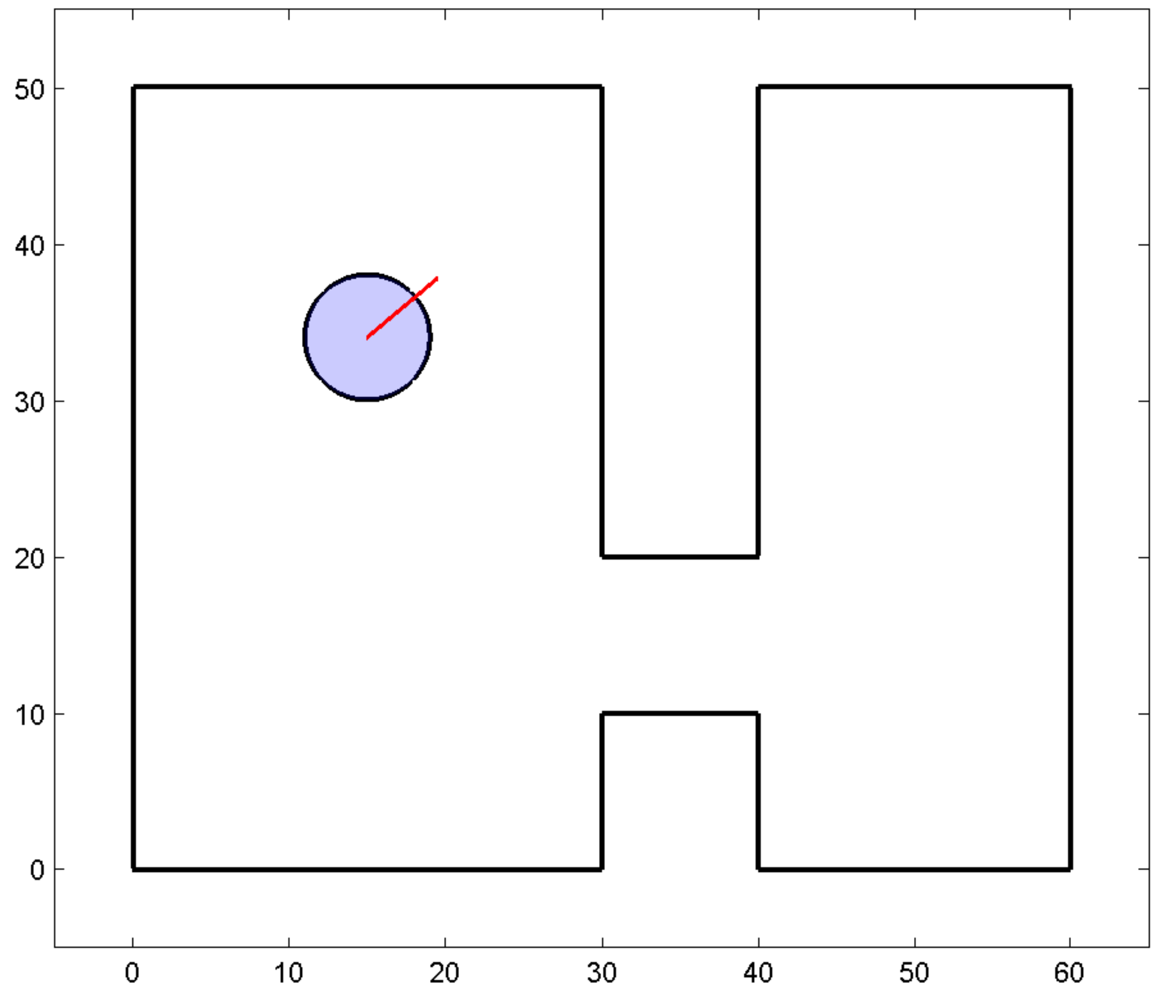


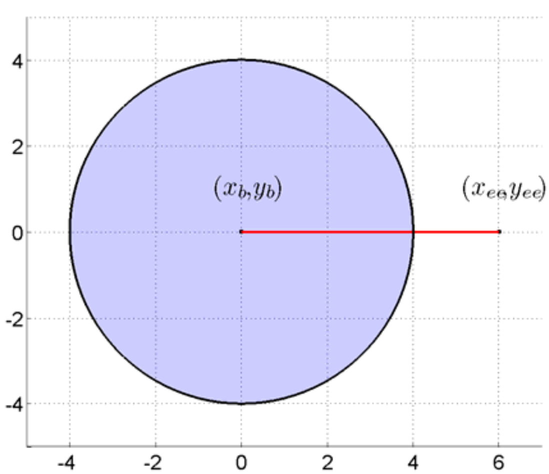
(Problem 1 – continued. The figure below is IDENTICAL in scale to the one on the previous page and is included only for convenience, for graphical calculations.)



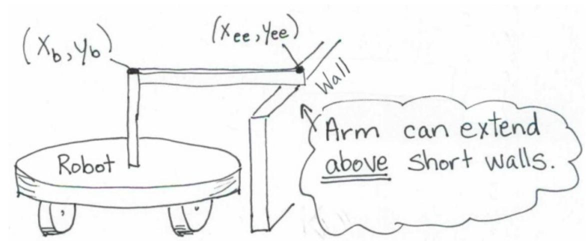
Problem 2 – Below is an overhead image of a circular, wheeled robot. The robot has two differential drive wheels, plus a third (freely rotating) omni-wheel. It is confined by short walls: the body may not go outside the thick borders shown, but the arm itself can.

a) Sketch and clearly label the reachable workspace for the end effector, (x_{ee}, y_{ee}).
b) Sketch and clearly label the dexterous workspace for the end effector.

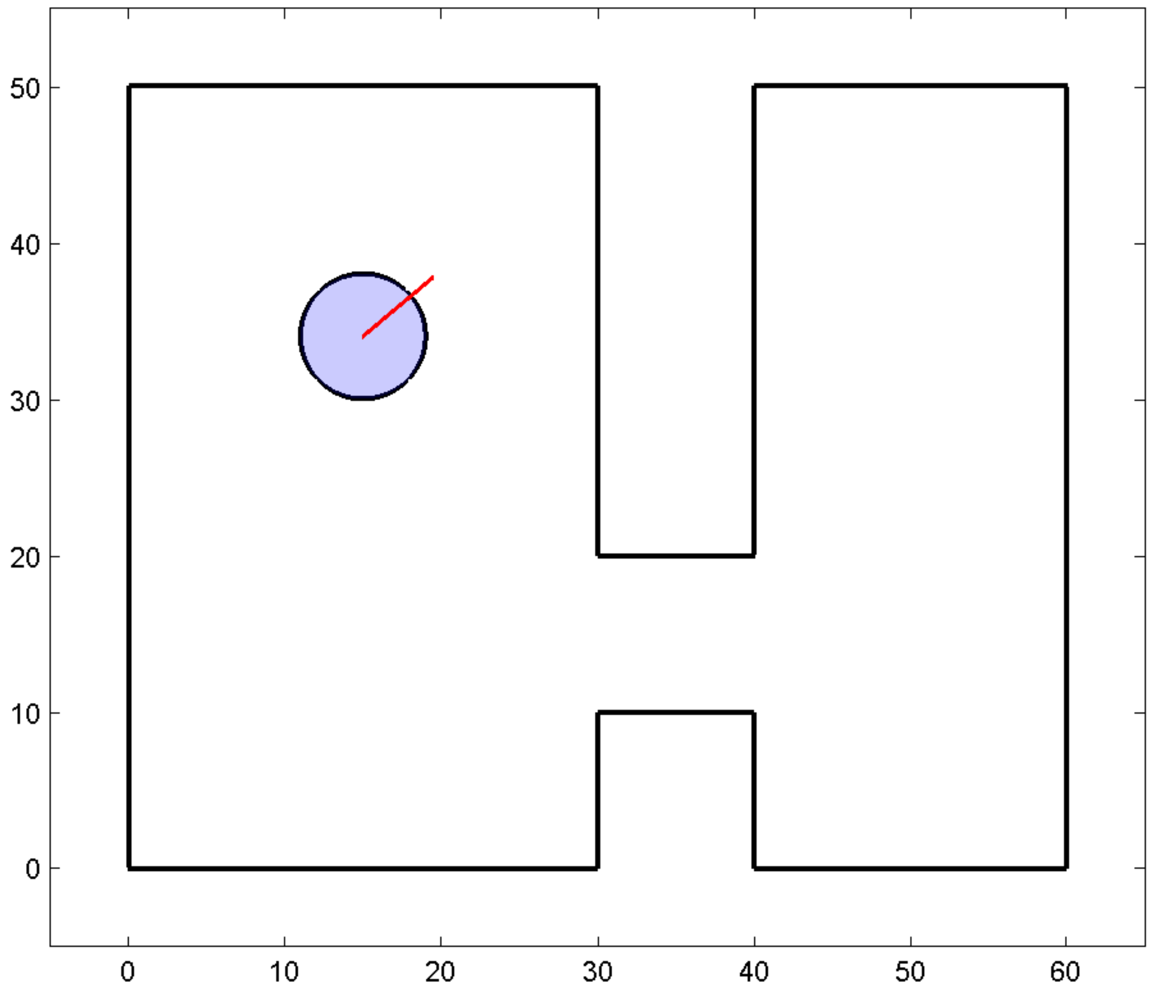




At left is a close-up of the robot. The chassis (body) has a radius of 4, and the length of the arm is 6.



(**Problem 2**, continued. – Below is a COPY of the room, for scrap sketching.)



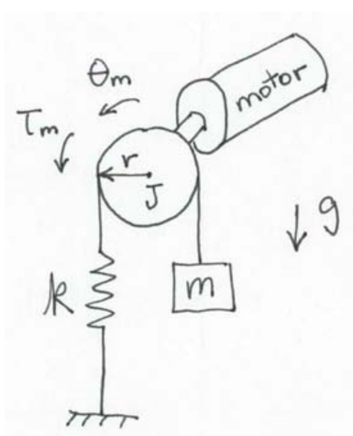
Problem 3 – Motor-pulley system.
Below is a sketch of a motor attached directly to a pulley. The inertia of the pulley and the rotating armature of the motor is (combined) labeled as "J". The pulley has radius "r". When the pulley rotates, it displaces the spring, "k", and moves a mass, "m". There is gravity, as shown.

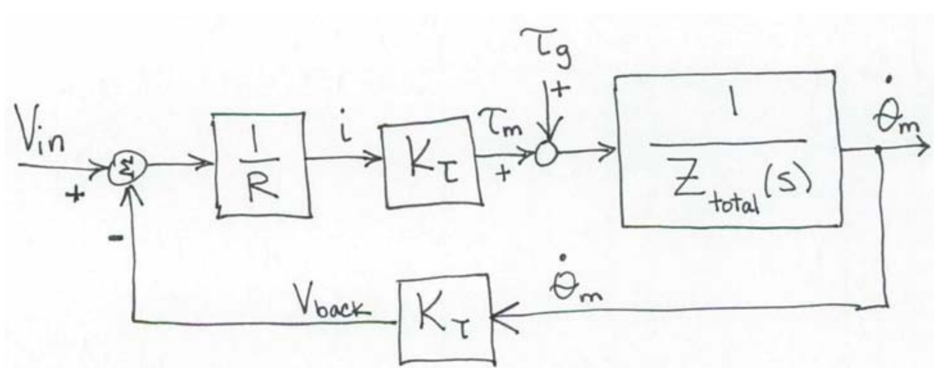
Complete the block diagram shown by solving for each of the following:

a) Find the total mechanical impedance, $Z_{total}(s)$, of the motor-pulley system.
b) Find the torque due to gravity, τ_g.
c) Find the transfer function from voltage input, V_{in}, to motor velocity output, $\dot{\theta}_m$.
(As always, set all other inputs to zero when finding an input-to-output transfer function.)

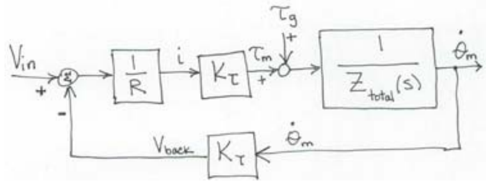
One way to do both parts "a)" and "b)" is to use the **Lagrangian** approach to find the equation of motion, using $\xi = \theta$ as the generalized coordinate. (There is only one degree of freedom here.) You should find an expression that includes terms in theta and its derivatives, the motor torque, and one additional term (which is torque due to gravity, τ_g).

It is also possible to solve for a and b using a more direct approach, however. You may use whatever methods you wish, so long as you show work.





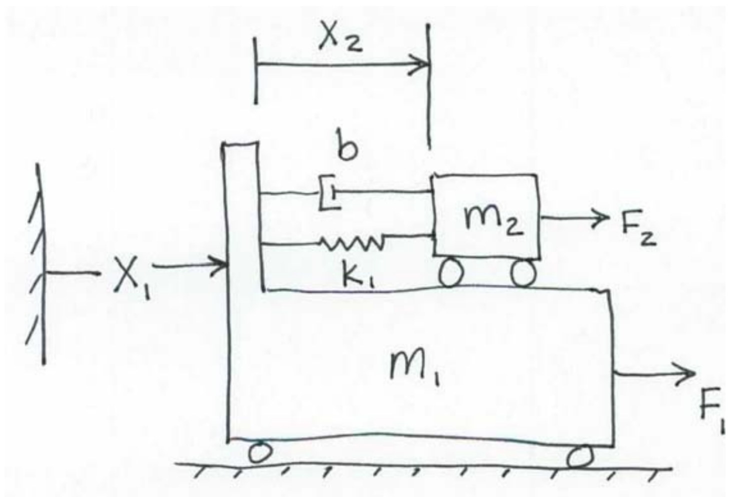
(**Problem 3** – additional space below. Here is the block diagram again, too.)



Problem 4 – Solve for the equations of motion, using the Lagrangian approach.

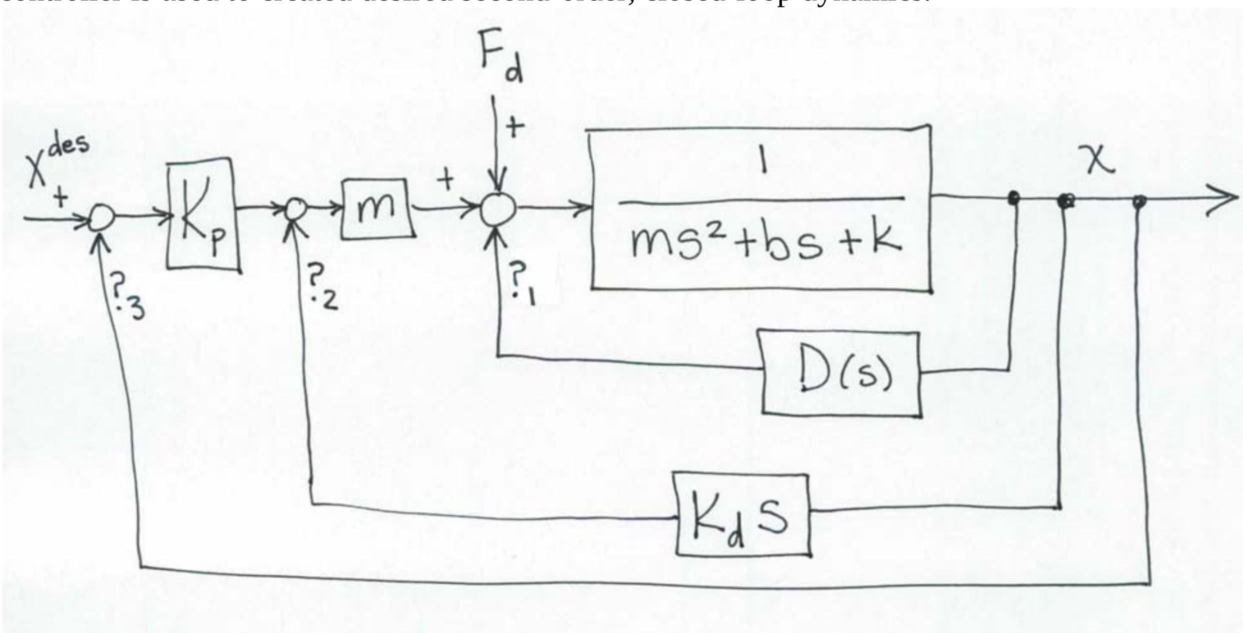
Use coordinates $\xi_1 = x_1$ and $\xi_2 = x_2$, as illustrated below. Note that x_1 is an absolute coordinate, while ***x_2* is measured relative to *x_1***. Clearly indicate the non-conservative forces, too.

Both F1 and F2 are EXTERNALLY APPLIED forces (not forces between one mass and the other).



(Problem 4 – extra space for work, below.)

Problem 5 – Inner loop / Outer loop control.
Below is a block diagram for the control strategy discussed several times in class in which an inner loop is used to "cancel out" the natural dynamics of a plant, while an outer-loop PD controller is used to created desired second-order, closed-loop dynamics.



a) Solve for D(s), to cancel dynamics with the inner loop.

b) Clearly label whether each of the 3 "?" symbols represents a plus sign (+) or a minus sign (-).

c) Solve for K_p and Kd such that the closed-loop poles are underdamped, with natural frequency 5 (rad/s) and damping ratio 0.5.

(Blank page for extra work.)
Be sure to LABEL WHICH PROBLEM YOU ARE WORKING ON!

(Blank page for extra work.)
Be sure to LABEL WHICH PROBLEM YOU ARE WORKING ON!