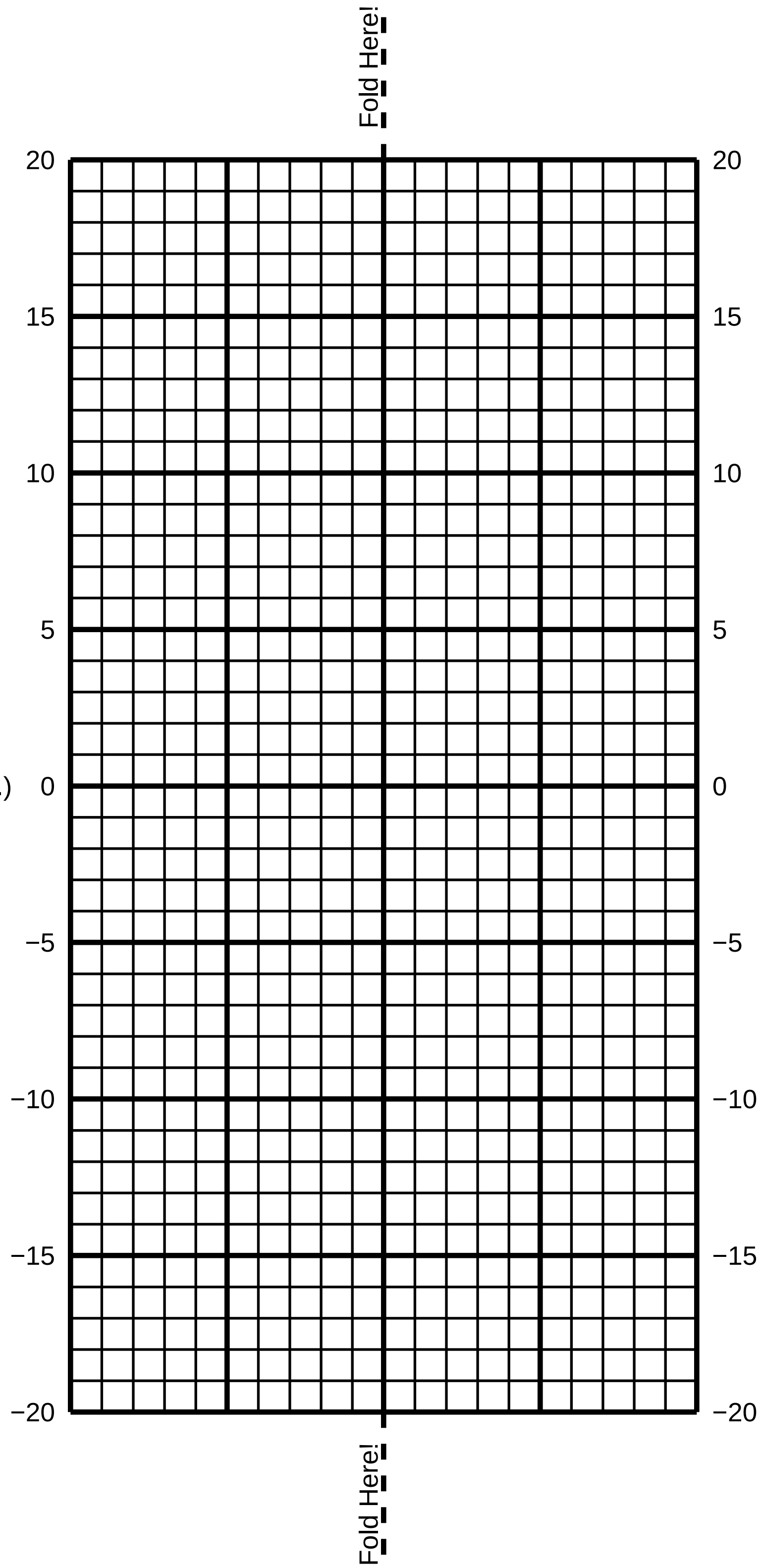
TEAR OFF THIS TOP SHEET FROM MIDTERM

Unstaple this sheet, FOLD, and use as a STRAIGHT EDGE, a SCALE (ruler) to mark grids and/or a RIGHT ANGLE tool, to help draw perpendicular lines to find ICRs.

(Unfold and use to trace grid lines.)



Name_________________________________

Tear off and save the ID number below, to look up your grade later…

ECE 179d

"Midterm" Quiz

- Extra pages are provided at back of exam, if needed. **Turn in all work!**
- You are allowed one (1) single-sided sheet of notes.
- No calculators or other computing devices are allowed.

Good Luck!

<u>Problem 1</u> – Recall for a multi-link arm that $\dot{\xi}_e = J\dot{q}_a$, and also $\tau_a = J^T F_e$.

Here, $\xi_e = \begin{bmatrix} x_e \\ y_e \\ \phi_e \end{bmatrix}$ gives the position and orientation of the end effector (at end-most tip of the last link), and $q_a = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$ gives the *relative* angles of the links, such that $\phi_e = \theta_1 + \theta_2 + \theta_3$.

a) For a particular 3-link arm in a particular configuration, the Jacobian is as follows:

$J = \begin{bmatrix} -2 & -1 & 2 \\ 5 & 8 & 3 \\ 1 & 1 & 1 \end{bmatrix}$. **Sketch and <u>clearly label</u> the corresponding 3-link arm geometry.**

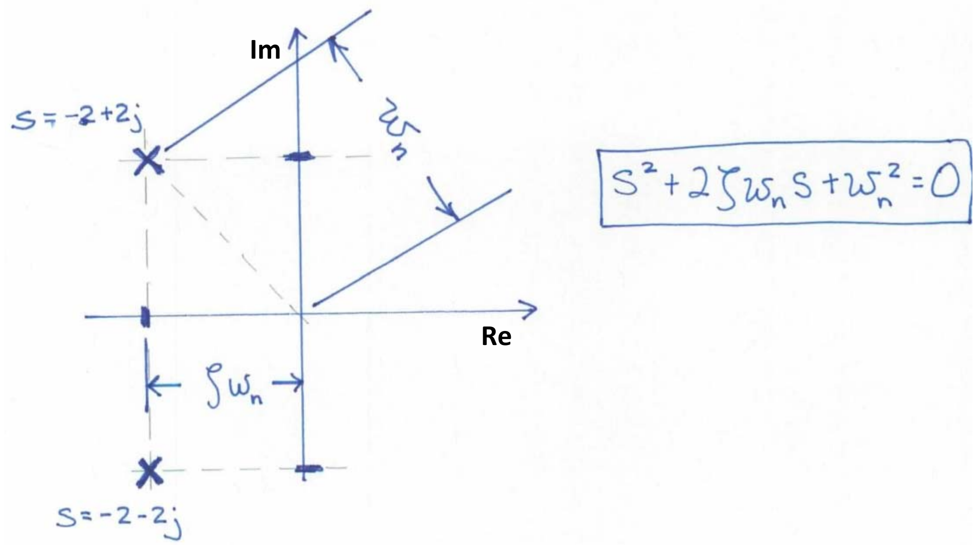
Hint: *It is probably easiest <u>first to calculate the location of the end effector</u> and later to fill in intermediate joints.* Assume the base of the first (q_1) link is at (x=0, y=0).

Clearly label the **COORDINATES OF THE END POINTS** of ALL 3 links in your sketch.
(The next page is BLANK to provide more space for needed calculations.)

<u>(Problem 1</u> – continued. Use additional space below, if needed.)

Problem 2 – Control Law Partitioning for single-input single-output (SISO) systems.

a) Solve for the characteristic equation ($s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$) for a system with complex poles at $s = -2 \pm 2j$, as shown. (The picture of poles on the complex s-plane below may be helpful.)



For a simple model of a single-link arm, a control torque, $\tau,$ is to be used to regulate the angle (i.e., keep it near zero). Assume the dynamics of the system can be modeled as follows:

$$mL^2\ddot{\theta} = -B\dot{\theta} - mgLsin\theta + \tau$$

b) Solve for a control law (i.e., an equation for control torque tau) for this system to cancel the natural system dynamics and to produce a closed-loop system that behaves like a linear, second-order dynamic system with poles at:

$$s = -2 \pm 2j$$

Problem 3 – The bike-trailer system.

The goal of this problem is to find the instantaneous center of rotation (ICR) for each of the 4 wheels in a system consisting of a bicycle pulling a two-wheeled trailer.

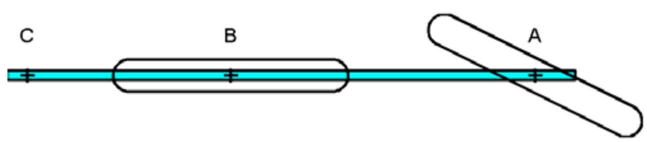
a) For the **bicycle alone, shown below**, points A, B, and C all exist on a single, rigid frame.

i) Sketch the ICR for the bicycle, given the wheel positions and orientations shown.

ii) Sketch the instantaneous directions of motion (velocity vectors) at points A, B, and C.

iii) List, from lowest to highest, the instantaneous speed at points A, B, and C. E.g., **if** A were the slowest-moving of the three points and C were the fast point, then you would list:

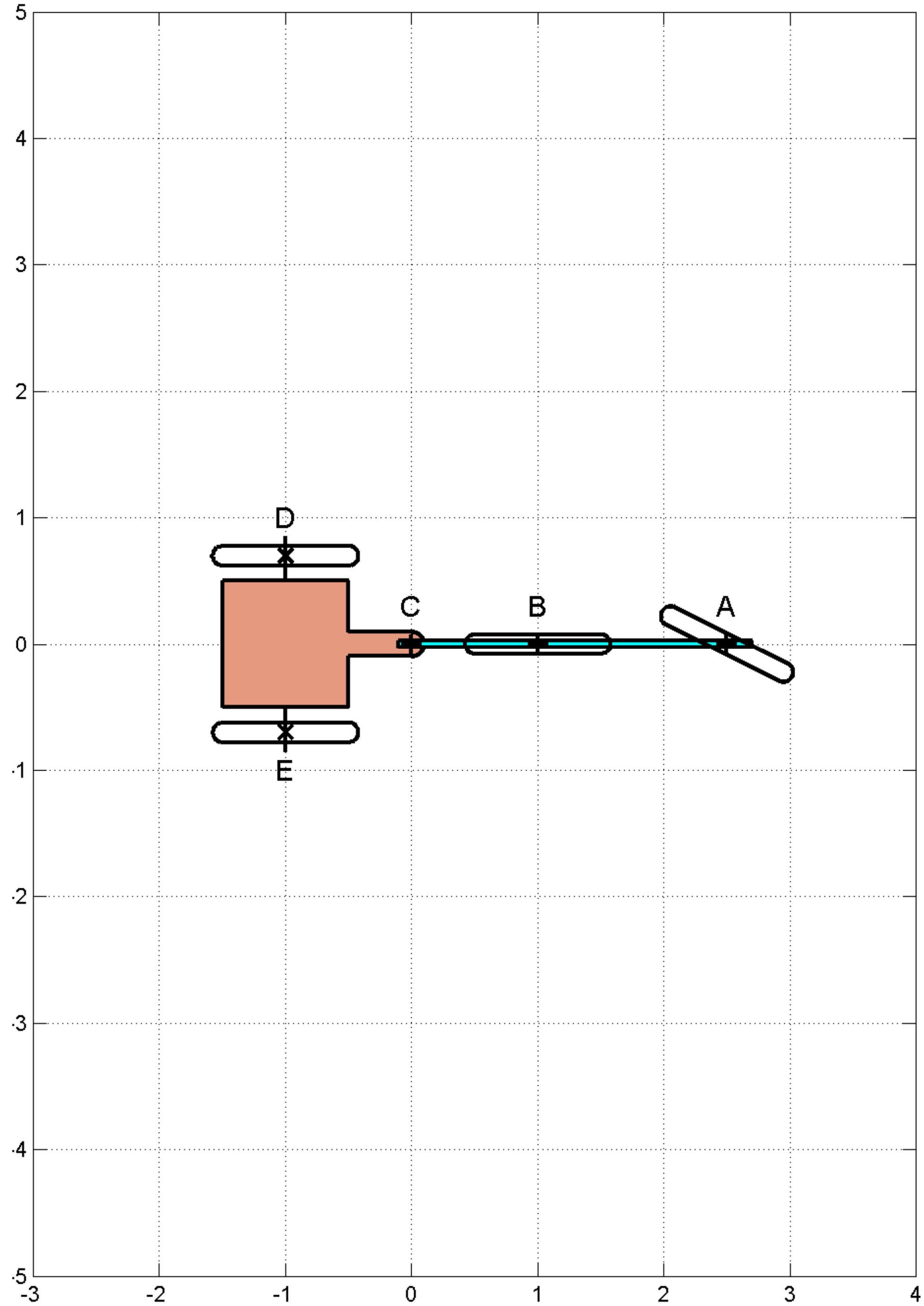
$$v_A < v_B < v_C.$$



3.b) Below is the full 4-wheeled system: the bicycle is attached to a 2-wheeled trailer at point C. Points A, B, and C are part of the bicycle subsystem. Points C, D, and E are part of the trailer subsystem. Note that point C is a pivot connection between the bicycle and trailer, and so its instantaneous velocity must be compatible with the location of the ICR for the bicycle rigid-body subsystem and with the trailer rigid-body subsystem.

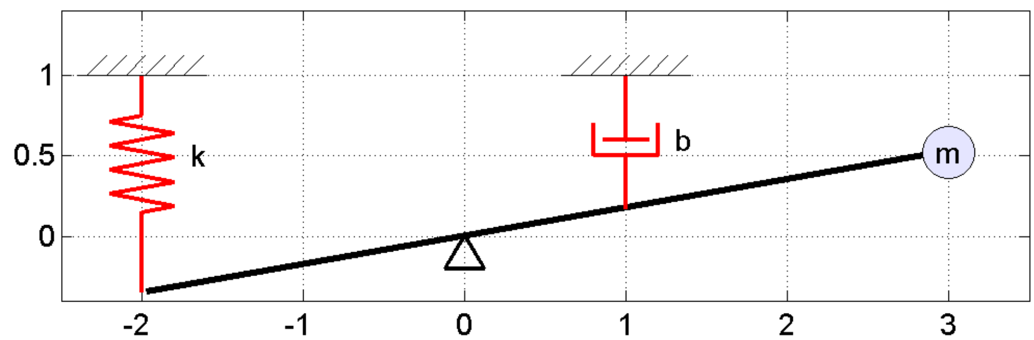
i) Sketch the instantaneous directions of motion at points C, D, and E on the figure below.

ii) Sketch **and clearly label** the ICR (instantaneous center of rotation) for each of the two rigid bodies: the bicycle and the trailer.



Problem 4 –Reflected Impedances: Mechanical elements connected via gear ratios.
For the see-saw system below, a rigid, massless bar can pivot about the apex of the small triangle. A spring, damper and mass are connected to the bar, as shown, each at different radii (r_k, r_b, and r_m) from the pivot point. Assume small angle motions (so linearization is valid).

a) Write the characteristic equation for the system, and solve for ω_n (natural frequency) and ζ (damping ratio, zeta), leaving the answer in symbolic form (in terms of k, b, m, r_k, r_b, and r_m).



b) Now, solve for ω_n and ζ numerically, for m=1 (kg), k=900 (N/m), b=36 (N-s/m) and the geometry shown above. (Although you may not know r_k, r_b, and r_m exactly from the plot, you do know their values *relative* to one another, which is all that is needed to solve the problem.)
(*Hint: numbers were picked so math works out "nicely" if you are correct, which may help you catch errors in part a…*)

(Blank page for extra work.)
Be sure to LABEL WHICH PROBLEM YOU ARE WORKING ON!

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