ECE ~~194B~~
ME 179D

~~Homework 4 – Extra Credit Quiz Review~~ ~~Due in class, Tues. May 24~~

*** Practice Quiz from Spring 2011 ***

~~Homework 4 – Extra Credit~~

Solutions

Problem 1 This problem involves the standard, 2-link, planar robot arm we've discussed several times in class. Figure 1 shows a cartoon of an arm, where θ_1 is the absolute angle of link 1, with respect to the x axis, and where θ_2 is the relative angle of link 1, with respect to θ_1. (Both angles are measured in the counter-clockwise direction.) Lengths are intentionally NOT drawn to scale!

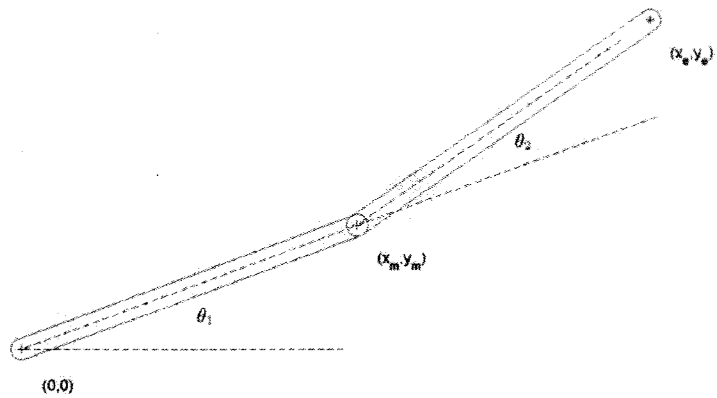


Fig. 1: *Link lengths, L_1 and L_2, are **not** drawn to scale, and angles do not correspond to the solution configuration!*

Say you have a 2-link arm that is in a geometric configuration such that the Jacobian is:

$$J = \begin{bmatrix} -3.5 & -1.5 \\ 2 & 3 \end{bmatrix}$$

a) Sketch the 2-link arm corresponding to this Jacobian (to scale). Label the coordinates of the elbow (x_m, y_m) and of the end effector (x_e, y_e). Recall: $\dot{\xi} = J\dot{q}$. Here, $\xi = \begin{bmatrix} x_e \\ y_e \end{bmatrix}$ and $q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$.

$y_1 = 0, \; x_1 = 0$

$y_{ee} - y_1 = -(-3.5)$

$\therefore y_{ee} = 3.5$

$y_{ee} - y_2 = -(-1.5)$

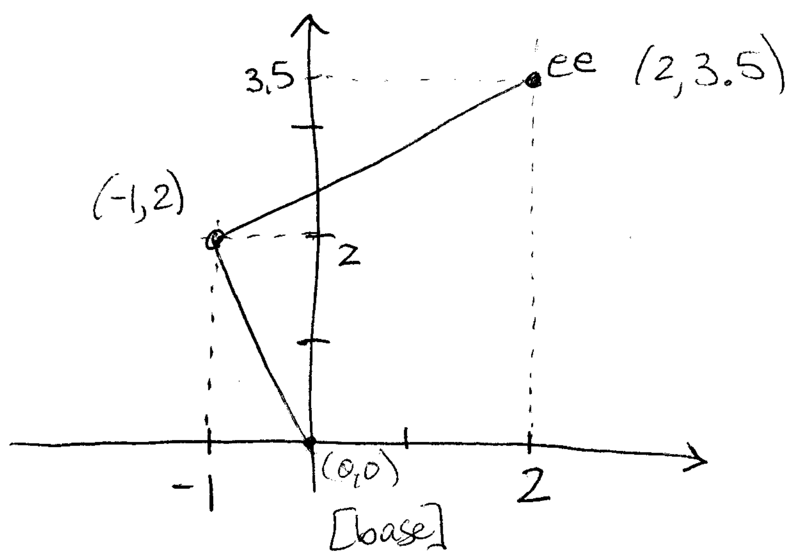
$\therefore y_2 = -1.5 + y_{ee} = 2$

$x_{ee} - x_1 = 2$

$\therefore x_{ee} = 2$

$x_{ee} - x_2 = 3$

$\therefore x_2 = x_{ee} - 3 = -1$



Problem 2 – Lagrangian equations of motion: Rolling can in a hollow cylinder.

a) For the system shown below, how many independent and complete generalized coordinates are required? Assume no slip: Only one G.C. (either ϕ_i or θ_o)

b) Derive the equation(s) of motion of the rolling can (with mass m_i and inertia J_i). Assume there are no applied forces or loss terms. (i.e., assume the dynamics include only conservative forces.) Also assume the can rolls inside the (fixed) outer cylinder *without slipping*.

Fig. 2 – The outer perimeter is fixed, and a can rolls freely (without slipping) inside, always touching the perimeter. Definitions for the angle to the point of contact, θ_o, and the angle of rotation of the can, θ_i, both of which are measured with respect to an absolute coordinate system, are shown. Shading on the can is only intended to illustrate orientation more clearly: the center of mass is at the center of the can, and the can has mass m_i and inertia J_i.

$$T^* = \frac{1}{2}mv^2 + \frac{1}{2}J\dot{\phi}_i^2$$

$$= \frac{1}{2}m_i\left((R_o-R_i)\dot{\theta}_o\right)^2 + \frac{1}{2}J_i\dot{\phi}_i^2$$

$$V = m_i g(R_o-R_i)(1-\cos\theta_o)$$

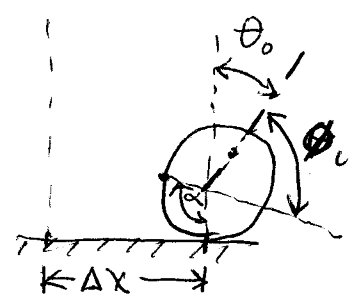
stretch out picture...

Geometry (no slip) requires that:

$$\begin{cases} \Delta x = R_o \cdot \Delta\theta_o \\ \Delta x = R_i \cdot (\Delta\phi_i + \Delta\theta_o) \end{cases}$$

$$\therefore R_o\theta_o = R_i\phi_i + R_i\theta_o$$

$$(R_o-R_i)\theta_o = R_i\phi_i \longrightarrow \boxed{\phi_i = \left(\frac{R_o-R_i}{R_i}\right)\theta_o}$$



Angle that has rolled is "$\alpha = \theta_o + \phi_i$"...

either is ok!

① $$\mathcal{L} = T^* - V = \frac{1}{2}\left[m_i(R_o-R_i)^2 + J_i\left(\frac{R_o-R_i}{R_i}\right)^2\right]\dot{\theta}_o^2 + \ldots$$

$$\ldots m_i g(R_o-R_i)\cos\theta_o$$

② or: $$\mathcal{L} = \frac{1}{2}(m_iR_i^2 + J_i)\dot{\phi}_i^2 + m_i g(R_o-R_i)\cos\left(\frac{R_o}{R_o-R_i}\phi_i\right)$$

① $$\boxed{\left(m_i(R_o-R_i)^2 + J_i\left(\frac{R_o-R_i}{R_i}\right)^2\right)\ddot{\theta}_o + m_i g(R_o-R_i)\sin\theta_o = 0}$$ OR... $$\boxed{(m_iR_i^2+J_i)\ddot{\phi}_i + m_i g(R_o-R_i)\sin\left(\frac{R_i}{R_o-R_i}\phi_i\right)}$$

Either is OK. Depends on choice of Generalized Coordinate.

Problem 3 – Shown below is the wheel geometry for a 3-wheeled omnibot. The driven directions of motion for each wheel are labeled *u*, while the "free-wheeling" directions are each labeled *v*. The arrows show the positive direction for each, and (for simplicity) we'll assume *u* and *v* are the linear (not rotational) instantaneous velocities at the center of each wheel hub (due to rolling).

Assume the driven velocities are: $u_a = u_b = 0$, and $u_c = 1$ (m/s).

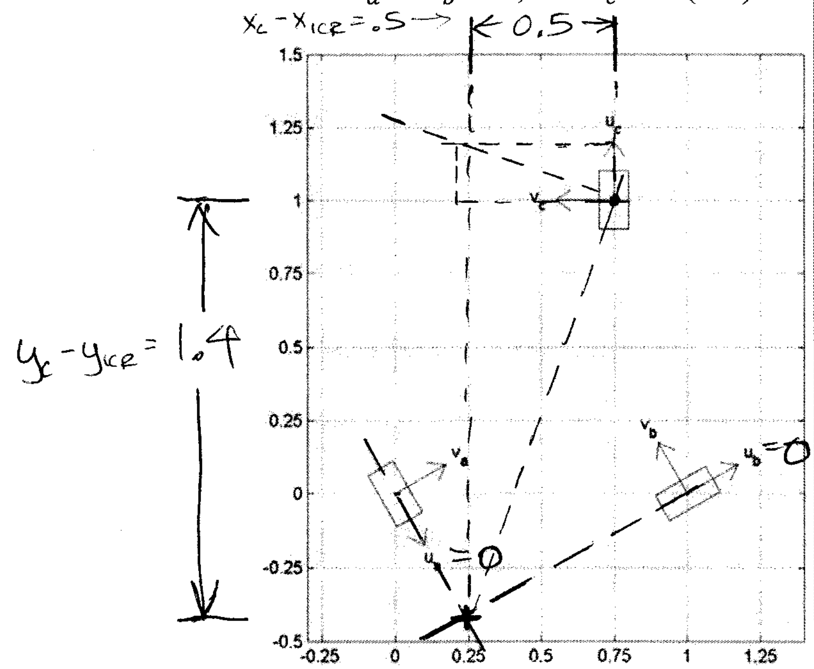


Fig. 3 – Omnibot wheel configuration. (Units are meters.)

Note to F2012 class: Each omniwheel at left has its free-rolling roller axis orthogonal to the driven wheel axis, as in Lab 3. This year, we have ALSO discussed the equations when the roller axis is along some "diagonal" axis, instead. BE PREPARED FOR EITHER CASE.

a) At what (*x*,*y*) location is the instantaneous center of rotation (ICR)? (Sketching appropriate lines on Figure 3 may be helpful!)

ICR is at around $\begin{cases} x = 0.25 \\ y = -0.4 \end{cases}$

b) Solve for the free-wheeling velocities at each wheel, v_a, v_b, and v_c.

$$\frac{v_c}{u_c} = \frac{1.4}{0.5}, \quad \therefore \boxed{v_c = 2.8\ (m/s)} \Leftarrow \begin{cases} \dot{x}_c = -(y_c - y_{ICR})\dot{\phi}_b \\ \dot{y}_c = +(x_c - x_{ICR})\dot{\phi}_b \end{cases}$$

$$\dot{\phi}_b = \frac{\dot{y}_c}{x_c - x_{ICR}} = \frac{u_c}{x_c - x_{ICR}} = \frac{1}{(.75 - .25)} = \frac{1}{.5} = 2.0 \quad \begin{cases} \& \ v_c = -\dot{x}_c \\ u_c = +\dot{y}_c \end{cases}, \text{ as drawn.}$$

$$\boxed{\begin{aligned} v_b &= R_b \dot{\phi}_b \cong 0.875 * 2.0 = 1.75\ (m/s) \\ v_a &= -R_a \dot{\phi}_b \cong -0.45 * 2.0 = -0.9\ (m/s) \end{aligned}}$$

Problem 4 – Lagrangian equations of motion: Seesaw with a cart.

In Fig. 4, assume the only mass is a point mass at the center of the cart.

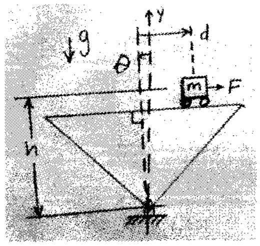


Fig. 4 – Massless seesaw with a (point-mass) cart driven on top.

a) Write expressions for the location (x,y) of the cart, in terms of the geometry shown.

$$x = -h\sin\theta + d\cos\theta$$
$$y = h\cos\theta + d\sin\theta$$

$\boxed{\text{h is constant. } \theta \text{ \& } d \text{ are DOF's.}}$

b) Write expressions for the x and y components of velocity of the cart.

$$\dot{x} = -h\cos\theta\,\dot\theta + \dot d\cos\theta - d\sin\theta\,\dot\theta$$
$$\dot{y} = -h\sin\theta\,\dot\theta + \dot d\sin\theta + d\cos\theta\,\dot\theta$$

c) Derive the equation(s) of motion for the cart. (Assume the only non-conservative force is the input force, F.)

$$T^* = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot x^2+\dot y^2) = \frac{1}{2}m\left[(-h\cos\theta\,\dot\theta + \dot d\cos\theta - d\sin\theta\,\dot\theta)^2 + (-h\sin\theta\,\dot\theta + \dot d\sin\theta + d\cos\theta\,\dot\theta)^2\right]$$

$$V = mgy = mg(h\cos\theta + d\sin\theta)$$

$$\mathcal{L} = T^* - V, \quad ①\ \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\theta}\right) - \frac{\partial\mathcal{L}}{\partial\theta} = 0, \quad ②\ \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot d}\right) - \frac{\partial\mathcal{L}}{\partial d} = F$$

$$T^* = [h^2\cos^2\theta\,\dot\theta^2 + \dot d^2\cos^2\theta + d^2\sin^2\theta\,\dot\theta^2 - 2h\dot d\cos^2\theta\,\dot\theta + 2hd\cos\theta\sin\theta\,\dot\theta^2 - 2\dot d d\cos\theta\sin\theta\,\dot\theta + \ldots$$
$$h^2\sin^2\theta\,\dot\theta^2 + \dot d^2\sin^2\theta + d^2\cos^2\theta\,\dot\theta^2 - 2h\dot d\sin^2\theta\,\dot\theta - 2hd\cos\theta\sin\theta\,\dot\theta^2 + 2\dot d d\cos\theta\sin\theta\,\dot\theta] * \frac{1}{2}m$$

$\boxed{\text{note } \sin^2 + \cos^2 = 1}$

$$\therefore T^* = h^2\dot\theta^2 + \dot d^2 + d^2\dot\theta^2 - 2h\dot d\dot\theta$$

$$\mathcal{L} = T^* - V = h^2\dot\theta^2 + \dot d^2 + d^2\dot\theta^2 - 2h\dot d\dot\theta - mgh\cos\theta - mgd\sin\theta$$

$$①\ \frac{d}{dt}\frac{m}{2}(2h^2\dot\theta - 2d^2\dot\theta - 2h\dot d) - (mgh\sin\theta - mgd\cos\theta) = 0$$

$$\boxed{mh^2\ddot\theta + md^2\ddot\theta + 2md\dot d\dot\theta - mh\ddot d - mgh\sin\theta + mgd\cos\theta = 0} \leftarrow \text{EOM 1}$$

$$②\ \frac{d}{dt}(2\dot d - 2h\dot\theta) - (2d\dot\theta^2 - mg\sin\theta) = F$$

$$\boxed{m\ddot d - mh\ddot\theta - md\dot\theta^2 + mg\sin\theta = F} \leftarrow \text{EOM 2}$$

Problem 5 – Systems with compliance. (4^{th}-order dynamics.)

*Note: Spring also exists to a fixed wall, k_W, as shown.

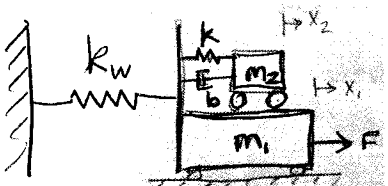
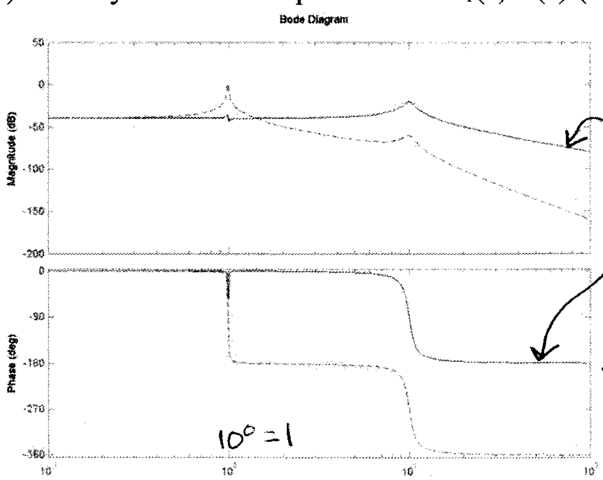


Fig. 5 - a 4^{th}-order system.

Assume k=100 N/m. Below are two sets of Bode plots. Your job is to determine which plot (solid vs dashed) is for $X_1(s)/F(s)$; note that the other plot is for $X_2(s)/F(s)$. Also estimate (roughly) the magnitude of each mass in set of plots (a and b).

a) Identify which Bode plot shows $X_1(s)/F(s)$ (solid or dashed?). Is $m_1<m_2$? Estimate m_1 and m_2.



$10^0 = 1$

-180°

(Bode plots above are for "a".)

zero freq

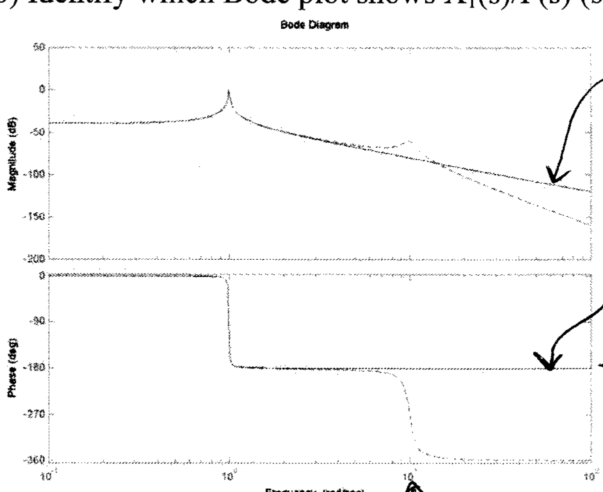
$\frac{X_2}{F}$ has a larger phase drop, due to pair of zeros for $\frac{X_1}{F}$.

$\therefore \frac{X_1(s)}{F(s)}$ is SOLID line.

Zero due to $\sqrt{\frac{k}{m_2}}$ is at around $1\ \frac{rad}{sec}$, $\therefore \boxed{m_2 \approx 100}$.

$\sqrt{\frac{k}{m_1}} \approx 10$, $\therefore \boxed{m_1 \approx 1}$ $\boxed{m_1 < m_2}$

b) Identify which Bode plot shows $X_1(s)/F(s)$ (solid or dashed?). Is $m_1<m_2$? Estimate m_1 and m_2.



-180°

(Bode plots above are for "b".)

zero freq.

As before, $\frac{X_1}{F}$ is <u>SOLID</u>.

$\sqrt{\frac{k}{m_2}}$ is zero freq @ $10\ \frac{rad}{sec}$,

$\boxed{m_2 \approx 1}$ $\boxed{m_2 < m_1}$

$\boxed{m_1 \approx 100}$