Homework 1 (Due Friday, October 12, at 5pm)

1) Kinematics and workspace. Consider the kinematics of the 3-link arm example discussed in class and shown in the figure below. Note that this drawing is not to scale. We wish to consider different possible lengths for each of the three links. For parts A) and B) below, determine i) the reachable workspace for (x_2,y_2), ii) the reachable workspace for (x_e, y_e) and ii) the dexterous workspace for (x_e, y_e). Accurately sketch and label each region, so that the dimensions (radii) of the regions are shown clearly.

A) $L_1=0.6$, $L_2=0.6$, $L_3=0.2$

B) $L_1=1$, $L_2=2$, $L_3=0.4$

C) Now, assume: $L_1=1.2$, and $L_2=1$. Given some arbitrary value for L_3, will some dexterous workspace always exist? If so, explain why. If not, specify the range(s) of values for L_3 for which a dexterous workspace *will* exist.

Recall that the reachable workspace is the set of all points the end effector can reach, while the dexterous workspace is the set of all points the end effect can reach at an arbitrary angle.

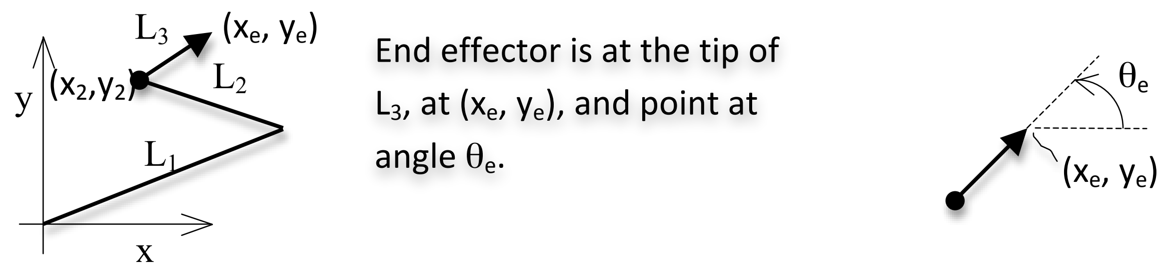

Figure 1. Kinematics of 3-link arm (problem 1).

2) Euler angles. We mentioned Euler angle rotations only briefly in class. This problem is designed to build better intuition about the conventions typically used to specify rotations (orientation) of a rigid body. Rotation angles can be specified either with respect to:

[1] A relative coordinate frame, that is fixed to the rotating body

[2] An absolute coordinate frame, that remains fixed; i.e., a global coordinate frame.

In either case, we require 3 rotations to specify any arbitrary orientation of a rigid body in space. We will refer to case 1 as "Euler angle rotation" and case 2 as "fixed angle rotation".

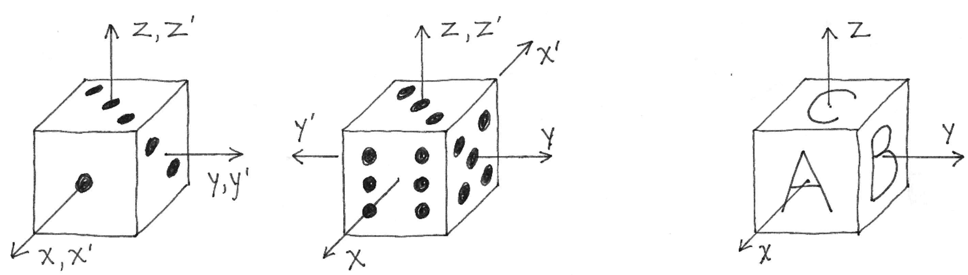

Figure 2. Local coordinate frame definition for dice in problem 2.

A standard American die (e.g., from a set Las Vegas dice) is shown above. The numbers 1 through 6 are arranged such that the numbers on opposite faces always sum to 7: 1 is opposite 6, 2 opposes 5, and 3 is opposite 4. Let us define a relative coordinate frame for a die, as shown at left in Figure 2. When aligned with the global coordinate frame, the x axis points out of side 1, y points out of 2, and z points out of 3, as shown. If we rotate the die at left counter-clockwise by 180 degrees about either z (absolute) or z' (relative), it will appear as shown in the middle figure. If we now rotate the die shown in the middle figure by 90 degrees (again, CCW is conventional) about y (absolute y), then faces A, B, and C in the diagram at right will show 3, 5, and 1, respectively. If we instead rotate the die shown in the middle figure by 90 degrees about the relative axis, y', then A=4, B=5, and C=6.

For each case below, begin with the configuration A=1, B=2, C=3. Determine the new orientation (A=?, B=?, C=?) that results from performing the following rotations of the die:

A) An Euler angle rotation in the order z',y',z', by angles -90°, +90°, +90°.

B) A fixed angle rotation in the order z,y,z, by angles +90°, +90°, -90°.

C) An Euler angle rotation in the order y',z',x' by angles +45°, +90°, +45°

D) A fixed angle rotation in the order x,z,y by angles +45°, +90°, +45°.

As described in pages 49-53 in Spong, performing a set of rotations in relative coordinates results in the same configuration as performing these rotations in the reverse order in absolute coordinates. ***Note that this would mean your answers to A) and B) should therefore be identical. Similarly, you can check that answer C) and D) are the same, as well.***
Note: It is probably wise to sketch the "intermediate" orientations above, rather than doing all rotations "in your head"!

3) Ball Toss Trajectory.
In Lab 2, you will design a controller to toss a ping pong ball into a cup. In this problem, we will look at some issues in trajectory planning.

Once the ping-pong ball has been thrown and it traveling through free space, we will assume that it follows a "ballistic trajectory": constant forward velocity, and constant downward acceleration due to gravity. (This neglects air resistance or other effects, to simplify the physics in a "toy example"…) For a given initial ball velocity, a 45 degree angle (so that the x and y components of velocity are equal) maximizes the distance in x that the ball will travel before falling to its initial y position (i.e., the take-off y height, when the ball was released), as depicted in Figure 3.

Now, assume that $\dot{\theta}_1$ and $\dot{\theta}_2$ each have the same maximum possible velocity magnitude, $\dot{\theta}_{max}$. Our goal in this problem is to MAXIMIZE the distance we can throw a ball.

a) Solve for the set of values for θ_1 and θ_2 that will result in a 45-degree initial trajectory for the ball (heading in the positive x direction).

b) Assume $\dot{\theta}_{max} = 10$ (rad/s), L_1=0.15 (m), and L_2=0.11 (m). Solve for the initial x (and y) velocities of the ping-pong ball, at the moment when the angles are at the values from part (a) [which is the "time of release" for the ball at its 45-degrees trajectory].

c) Sketch (by hand) a SMOOTH, feasible trajectory for the ball JUST PRIOR to its release, and show that this is always within the "reachable workspace" of the arm by also shading in this workspace region. There are two important issues here:
   1. The ball must always be within the reachable workspace of the 2-link arm.
   2. The final velocity must be at +45 degrees.

   Here, the trajectory should be a smooth (curvy) path that stays in the feasible workspace for the arm and has a slope of 45 deg at its end. Use your answer from part a to be sure the geometry of the take-off point is correct, within the feasible workspace.

Recall from lecture that the geometric distances shown (A, B, C, and D) map to values in the Jacobian, J. This should give you intuition about how to MAXIMIZE the magnitude of dx/dt and dy/dt.

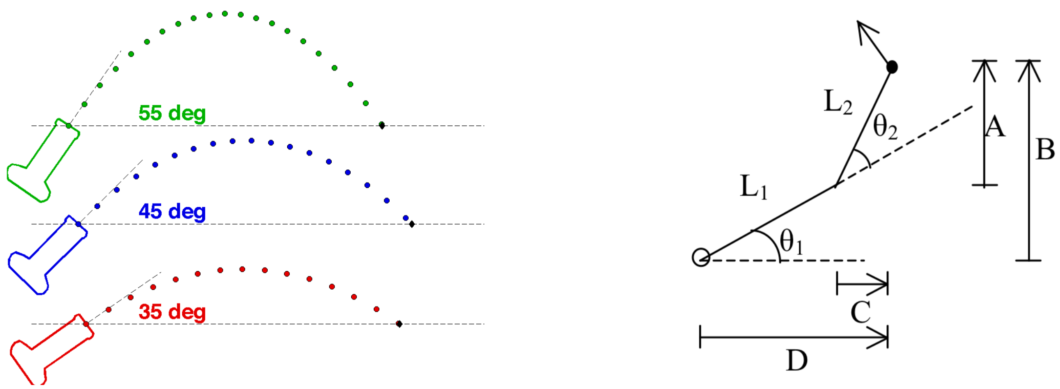

Figure 3. Ballistic trajectory (left) and 2-link arm (right) for problem 3.

4) Singularities. Below is a figure of a mechanism constrained to a single degree of freedom, x, at the output. The relationship between θ and *x* is:

$$x = 2L\cos\theta$$

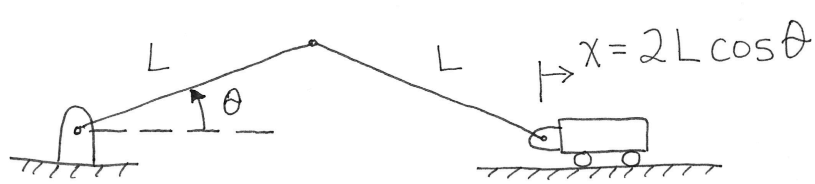

Figure 4. 1 DOF mechanism for problem 4.

A) Derive the relationship between $\dot{x}$ and $\dot{\theta}$.
B) Assume $x = 0$, and we desire $\dot{x}$ = -4 (m/s). What must $\dot{\theta}$ be to achieve this?
C) Assume $x = L$, and we desire $\dot{x}$ = -4 (m/s). What must $\dot{\theta}$ be to achieve this?
D) Assume $x = 1.9L$, and we desire $\dot{x}$ = -4 (m/s). What must $\dot{\theta}$ be to achieve this?
E) Assume $x = 2L$, and we desire $\dot{x}$ = -4 (m/s). What must $\dot{\theta}$ be to achieve this?

Configuration E is known as a "singularity". It is a configuration in which it becomes impossible to move in a particular location: that is, note that $\dot{x} > 0$ is impossible at $x = 2L$. Singularities can be "dangerous" places to operate, because (often) they are configurations near which the robot's input velocity (here, $\dot{\theta}$) must "blow up", unbounded, to achieve a finite output velocity at the end effector. This phenomenon explains (in part) why humanoid robots often ***avoid*** operating with legs "fully extended", near a singularity.

5) Analogous mechanical and electrical impedances as "circuit elements". Figure 5 shows a 4^{th}-order (translational mechanical) system (at right) and an analogous circuit structure (at left). ("4^{th}-order" means, for one thing, that the denominator of the transfer functions will be a 4^{th}-order one…)

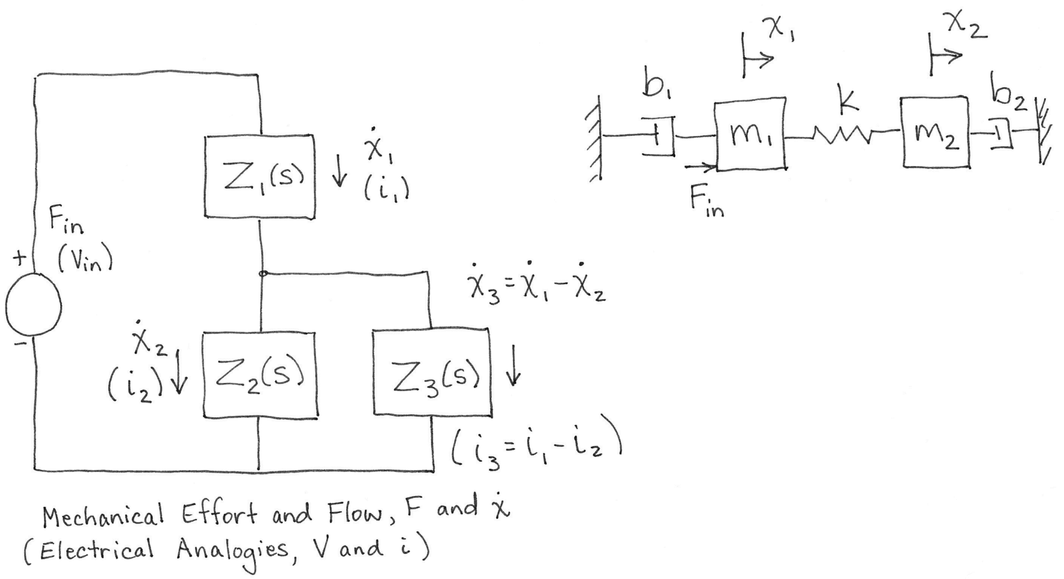

Figure 5. Mechanical impedances in a circuit (problem 5).

A) Use the definitions given in class for mechanical impedance, $Z_m(s) = \dfrac{F(s)}{v(s)} = \dfrac{F(s)}{sX(s)}$, and for electrical impedance, $Z_e(s) = \dfrac{V(s)}{I(s)}$, to solve for $Z_1(s)$, $Z_2(s)$, and $Z_3(s)$ in the circuit diagram.

B) Solve for the transfer functions $\dfrac{X_1(s)}{F(s)}$ and $\dfrac{X_2(s)}{F(s)}$.

6) Reflected inertia and mechanical impedance.

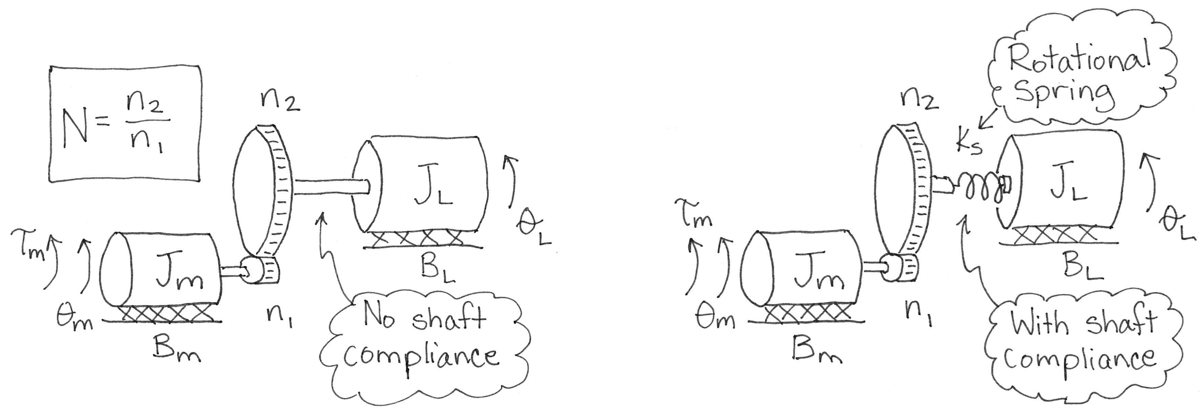

Figure 6. Motor, transmission, and lab (problem 6).

Figure 6 shows a system with a motor driving a load. In the lefthand diagram, $\dot{\theta}_m$ (motor velocity) and $\dot{\theta}_L$ (load velocity) are simply related through the gear ratio: $\dot{\theta}_m = -N\dot{\theta}_L$. In the righthand diagram, there is now a spring element, with stiffness k_g, between the larger gear and the load inertia, J_L. This spring models the compliance that is sometimes a noticeable factor in real transmission systems, and it changes the dynamics from a 2nd-order system to a 4th-order system.

Note, the lefthand system is one we have already considered in class (Lecture 4). Also, if you look carefully, you should notice that the righthand system is (intentionally) very similar to the translational mechanical system depicted in problem 5. (So in a way, you are solving the same problem again, but through a different perspective.)

A) Solve for the transfer functions $\frac{\theta_m(s)}{\tau(s)}$ and $\frac{\theta_L(s)}{\tau(s)}$ for the system with <u>no</u> shaft compliance (that is, in the limit as k_g become infinitely stiff).

B) Solve for the transfer functions $\frac{\theta_m(s)}{\tau(s)}$ and $\frac{\theta_L(s)}{\tau(s)}$ for the system <u>with</u> shaft compliance.