Homework 5

Problem 6 – Due 5pm Friday, Nov. 30, along with Problems 2-5 of Homework 4. Near its upright equilibrium, the linearized EOMs for the inverted pendulum (IP) robot are:

$$(J_w + R_w^2(m_b + m_w))\ddot{\phi}_w + (R_w L m_b)\ddot{\theta}_b + B\dot{\phi}_w - B\dot{\theta}_b = K_m u \quad \text{(Equation 1a)}$$

$$(R_w L m_b)\ddot{\phi}_w + (J_b + L^2 m_b)\ddot{\theta}_b - B\dot{\phi}_w + B\dot{\theta}_b - m_b g L\theta_b = -K_m u \quad \text{(Equation 2a)}$$

where u is the input to the motor, and K_m is simply a scalar constant. For simplicity, we can write these equations by lumping the coefficients for the acceleration terms as shown below:

$$M_{11}\ddot{\phi}_w + M_{12}\ddot{\theta}_b + B\dot{\phi}_w - B\dot{\theta}_b = K_m u \quad \text{(Equation 1b)}$$

$$M_{12}\ddot{\phi}_w + M_{22}\ddot{\theta}_b - B\dot{\phi}_w + B\dot{\theta}_b - m_b g L\theta_b = -K_m u \quad \text{(Equation 2b)}$$

Equations 1b and 2b can also be represented by two transfer functions of the form below:

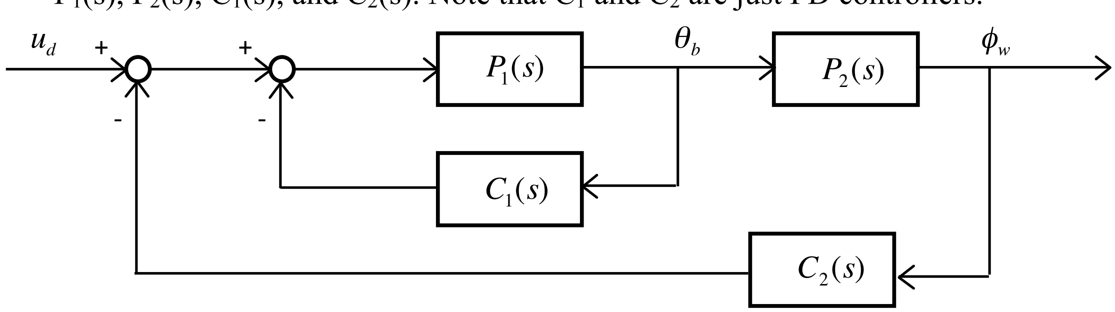
$$\frac{\theta_b(s)}{U(s)} = \frac{as^2}{s^4 + bs^3 + cs^2 + ds} \quad \text{(Equation 3)}$$

$$\boxed{\frac{\phi_w(s)}{\theta_b(s)} = \frac{es^2 + f}{as^2}} \quad \text{(Equation 4)}$$

a) Show work to demonstrate that Eqs. 1b and 2b result in the transfer functions in Eqs. 3 and 4, and also solve for all of the coefficients in Eqs. 3 and 4 (a,b,c,d,e,f) in terms of the constants in Eqs. 1b and 2b. Hint: First, convert the equations to the Laplace ("s") domain. Then, note that Equations 1b and 2b can be used together to eliminate any one of the three variables u, θ_b, or ϕ_w, leaving a relationship between the remaining two.

b) Solve for the transfer function $\frac{\phi_w(s)}{U(s)}$ in terms of the coefficients in Eqs 3 and 4. (Easy!)

c) For a state space regulator, the control law is $u_f = -Kx = -K_1x_1 - K_2x_2 - K_3x_3 - K_4x_4$. Our state vector is $X = [\phi_w \quad \theta_b \quad \dot{\phi}_w \quad \dot{\theta}_b]^T$. The block diagram below shows this controller, with an additional disturbance input, u_d, intentionally included, so that we can calculate a closed-loop transfer function from u_d to either ϕ_w or θ_b. Fill in the blanks for $P_1(s)$, $P_2(s)$, $C_1(s)$, and $C_2(s)$. Note that C_1 and C_2 are just PD controllers.



d) Now, solve for the two closed-loop transfer function, $\frac{\phi_w(s)}{U_d(s)}$ and $\frac{\theta_b(s)}{U_d(s)}$, in terms of the control gains K_1, K_2, K_3, and K_4 and coefficients a, b, c, d, e, and f.

e) As discussed in class, this system is controllable, meaning all four poles can be set arbitrarily. Show that this is so, because one can pick the four coefficients of the forth-order characteristic polynomial of both closed-loop transfer functions from part "d)" above arbitrarily, and (correspondingly) set the roots of the polynomial.

f) Solve for K_1, K_2, K_3, and K_4 to set the poles of the closed-loop system at s=-1, s=-7, s=-7, and s=-100 (rad/sec). You must show work to receive any credit. Use the parameters from Prelab 5, which are given again below.

This entire problem is largely tutorial in nature and should be generally straightforward to solve, although some of the algebra may get messy.

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Jw = 1.6e-05;  % (kg*m^2) Wheel inertia
Rw = 0.0310; % (meters)  Radius of wheel
L = 0.0950; % (meters)  Length from wheel to body mass.
mb = 0.5910; % (kg) body mass
mw = 0.034; % (kg) combined mass of BOTH wheels
Jb = 0.0019; % (kg*m^2) Body inertia
b = 0.062; % damping factor (approximate!); "b" and "B" are just the same here.
g = 9.81;  % (m/s^2) gravity
Nmotors=2;  % number of motors used
Klego=2;   % A scaling parameter for motor output (approx)
Km = Nmotors*b/Klego ; % Motor effort constant; Km = 0.062 (approx)
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