Omnibot Velocity Kinematics

- We seek an orientation-dependent Jacobian relating wheel velocities, $\dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$, to center of body (COB) velocities,

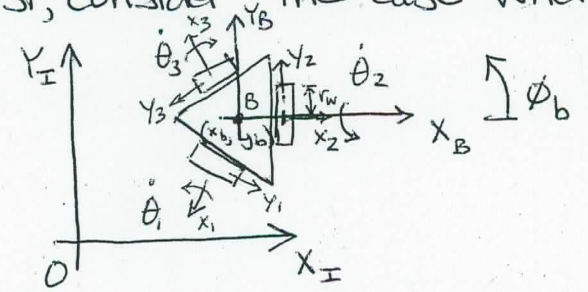
$$\dot{\xi} = \dot{X} = \begin{bmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\phi}_b \end{bmatrix}.$$

$$\dot{X} = \dot{\xi} = \underbrace{J(\phi_b)}\,\dot{q}$$

Here, J is a function of ϕ_b.

- Note the matrix J defines a set of equations that add contributions from $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$ linearly. Thus, we can solve for elements in the 3x3 J matrix intuitively... (one-at-a-time)

First, consider the case where $\boxed{\phi_b = 0}$:



For $\dot{x}_b = \dot{y}_b = 0$,
$\dot{\phi}_b \neq 0$

This requires (by symmetry) equal velocities by each of the 3 wheels:

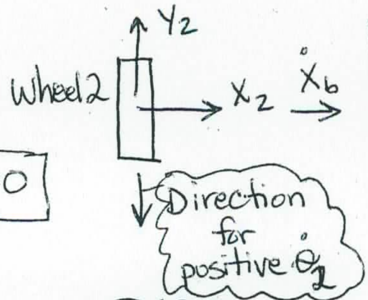
$$L\dot{\phi}_b = -r_w\dot{\theta}_1$$

$$\therefore \dot{\theta}_1 = \frac{-L}{r_w}\dot{\phi}_b$$

$$\boxed{\begin{aligned} \dot{\theta}_1 &= \frac{-L}{r_w}\dot{\phi}_b \\ \dot{\theta}_2 &= \frac{-L}{r_w}\dot{\phi}_b \\ \dot{\theta}_3 &= \frac{-L}{r_w}\dot{\phi}_b \end{aligned}}$$

Now again enforce $\phi_b = 0$ ...

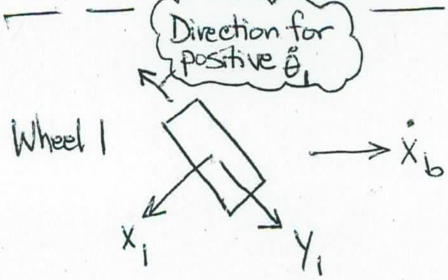
$$\boxed{\begin{array}{c}\text{For } \dot{y}_b = \dot{\phi}_b = 0 \\ \dot{x}_b \neq 0\end{array}}$$



$\boxed{\alpha_2 = 0}$

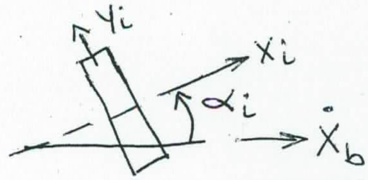
$$\boxed{\dot{\theta}_2 = 0}$$

Because there is zero velocity in the $\vec{y}_2$ direction.



$$\boxed{r_w \dot{\theta}_1 = +\sin\alpha_1 \cdot \dot{x}_b}$$

Because:



$$\dot{x}_i = (\cos\alpha_i)\dot{x}_b$$

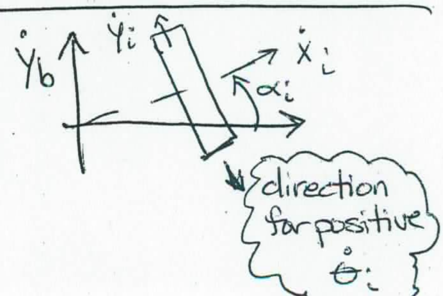
$$\dot{y}_i = (-\sin\alpha_i)\dot{x}_b$$

and: $\dot{y}_i = -r_w \dot{\theta}_i$

For $i = 1, 2, 3$ $\xrightarrow{\text{All wheels}}$ $\therefore \boxed{\dot{\theta}_i = \left(\frac{\sin\alpha_i}{r_w}\right)\dot{x}_b}$

Now w/ $\phi_b = 0$ ...

$$\boxed{\begin{array}{c}\text{For } \dot{x}_b = \dot{\phi}_b = 0 \\ \dot{y}_b \neq 0\end{array}}$$



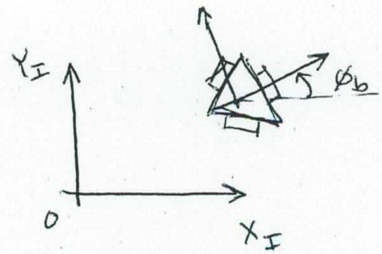
$\dot{y}_i = (\cos\alpha_i)\dot{y}_b$

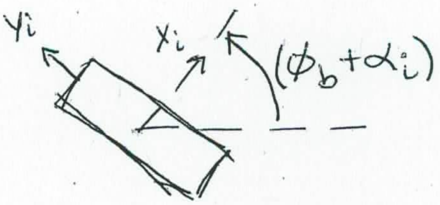
and $\rightarrow$ $\dot{y}_i = -r_w \dot{\theta}_i$

$$\boxed{\dot{\theta}_i = \left(\frac{-\cos\alpha_i}{r_w}\right)\dot{y}_b}$$

Now, when $\phi_b \neq 0$,

- Same math applies, except angle to wheel i is now "$\phi_b + \alpha_i$", instead of just α_i:





- Combining all the linearly-independent terms:

$$\dot{\theta}_i = \left(\frac{\sin(\phi_b+\alpha_i)}{r_w}\right)\mathring{x}_b + \left(\frac{-\cos(\phi_b+\alpha_i)}{r_w}\right)\mathring{y}_b + \left(\frac{-L}{r_w}\right)\mathring{\phi}_b$$

So,

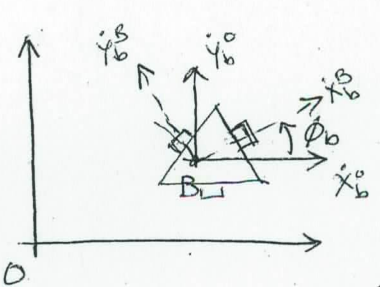
$$\underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} \frac{\sin(\phi_b+\alpha_1)}{r_w}, & \frac{-\cos(\phi_b+\alpha_1)}{r_w}, & \frac{-L}{r_w} \\ \frac{\sin(\phi_b+\alpha_2)}{r_w}, & \frac{-\cos(\phi_b+\alpha_2)}{r_w}, & \frac{-L}{r_w} \\ \frac{\sin(\phi_b+\alpha_3)}{r_w}, & \frac{-\cos(\phi_b+\alpha_3)}{r_w}, & \frac{-L}{r_w} \end{bmatrix}}_{(J^{-1})} \underbrace{\begin{bmatrix} \mathring{x}_b \\ \mathring{y}_b \\ \mathring{\phi}_b \end{bmatrix}}_{\xi} = \tilde{M}\,\xi$$

° means wrt inertial frame definition of $\vec{x}$ & $\vec{y}$...

(Let's call this 3x3 matrix "$\tilde{M}$"...)

$$\therefore \boxed{J = \tilde{M}^{-1}}$$

• We can <u>also</u> solve for $\tilde{M}$ is two steps, by relating ① $\dot{q}$ to $\dot{X}_b^o$ and then relating ② $\dot{X}_b^o$ to $\dot{X}_b^B$



$$R = \begin{bmatrix} \cos\phi_b & \sin\phi_b & 0 \\ -\sin\phi_b & \cos\phi_b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② $$\dot{X}_b^B = R\dot{X}_b^o \Leftrightarrow \begin{cases} \dot{x}_b^B = \cos\phi_b \dot{x}_b^o + \sin\phi_b \dot{y}_b^o \\ \dot{y}_b^B = -\sin\phi_b \dot{x}_b^o + \cos\phi_b \dot{y}_b^o \\ \dot{\phi}_b^B = \dot{\phi}_b^o \end{cases}$$

We already found previously that:

B means wrt rotated coords of the body

① $$\underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} \frac{\sin(\alpha_1)}{r_w}, & \frac{-\cos(\alpha_1)}{r_w}, & \frac{-L}{r_w} \\ \frac{\sin(\alpha_2)}{r_w}, & \frac{-\cos(\alpha_2)}{r_w}, & \frac{-L}{r_w} \\ \frac{\sin(\alpha_3)}{r_w}, & \frac{-\cos(\alpha_3)}{r_w}, & \frac{-L}{r_w} \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \dot{x}_b^B \\ \dot{y}_b^B \\ \dot{\phi}_b^B \end{bmatrix}}_{\dot{X}_b^B}$$

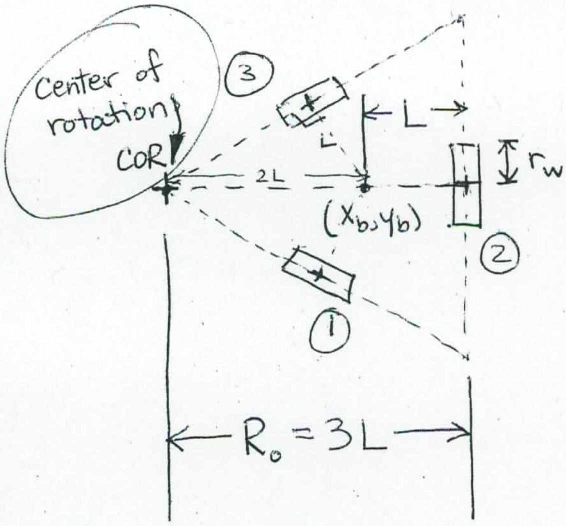
1 & 2

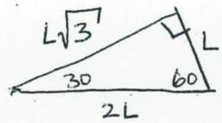
$$\left.\begin{aligned} \therefore\ \dot{q} &= MR\dot{X}_b^o \\ \dot{q} &= \underbrace{\tilde{M}\dot{X}_b^o} \end{aligned}\right\} \therefore \boxed{\tilde{M} = MR}$$

from page 3 result...

Siegwart Eqn 3.19 uses "MR" form...

- Finally, we can find J directly by holding 2 of 3 $\dot{\theta}_i$ velocities at zero and moving the third:





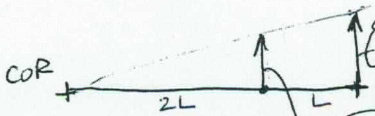
e.g. $\dot{\theta}_1 = \dot{\theta}_3 = 0$

Only $\dot{\theta}_2$ allowed.

Then, $\dot{x}_b = 0$

$$\dot{y}_b = \left(\frac{-2}{3}\right) r_w \dot{\theta}_2$$

$$\dot{\phi}_b = \left(\frac{-r_w}{L}\right)\dot{\theta}_2$$



$\dot{y}_2 = -r_w \dot{\theta}_2$

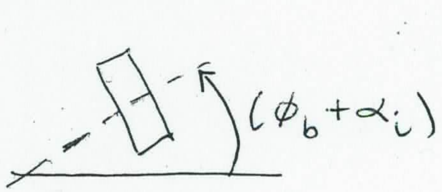
$\dot{y}_b = \frac{2}{3}\dot{y}_2 = \frac{-2}{3}\dot{\theta}_2$

$\boxed{\text{For arbitrary } \alpha_i \text{ \& } \phi_b} \rightarrow$

$$\dot{x}_b = \sin(\phi_b + \alpha_i)\left(\frac{2}{3}\right) r_w \dot{\theta}_i$$

$$\dot{y}_b = -\cos(\phi_b + \alpha_i)\left(\frac{2}{3}\right) r_w \dot{\theta}_i$$

$$\dot{\phi}_b = \frac{-r_w}{3L}\dot{\theta}_i$$



$$\begin{bmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\phi}_b \end{bmatrix} = \dot{\xi} = J\dot{q} = \underbrace{\begin{bmatrix} \sin(\phi_b+\alpha_1)\frac{2r_w}{3}, & \sin(\phi_b+\alpha_2)\frac{2r_w}{3}, & \sin(\phi_b+\alpha_3)\frac{2r_w}{3} \\ -\cos(\phi_b+\alpha_1)\frac{2r_w}{3}, & -\cos(\phi_b+\alpha_2)\frac{2r_w}{3}, & -\cos(\phi_b+\alpha_3)\frac{2r_w}{3} \\ \frac{-r_w}{3L}, & \frac{-r_w}{3L}, & \frac{-r_w}{3L} \end{bmatrix}}_{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

wrt global origin