1

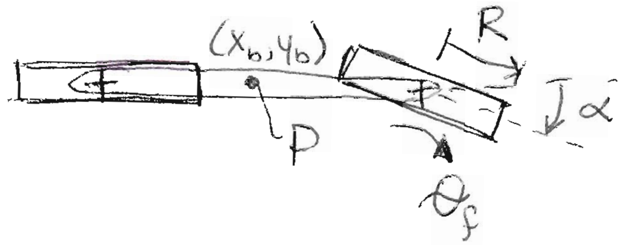


Fig. above shows phi (body angle) for a different config.

a) Sketch instantaneous center of rotation (ICR).

b) Write expressions for $\dot{x}_b$, $\dot{y}_b$ and $\dot{\phi}_b$ for the configuration above. Either:

Expression should be a function of front wheel velocity, dtheta_f/dt, for geometry shown in the lefthand fig.

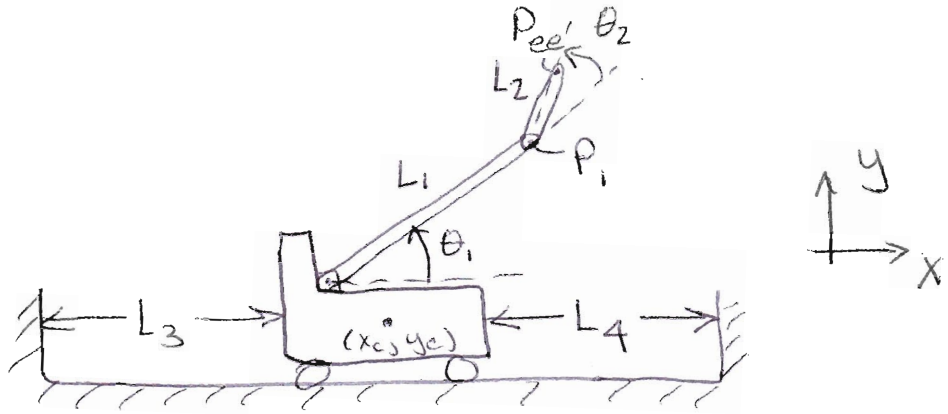
i. label needed dimensions

OR

ii. measure needed lengths by hand.

(HINT: Think of point P as it rotates about the ICR. Recall how Δy & Δx appear in the Jacobian...)

(2) The cart can roll such that $-L_3 \leq x_c \leq L_4$, $y_c = 0$, $0 \leq \theta_1 \leq \frac{\pi}{2}$ and $-\frac{\pi}{2} \leq \theta_2 \leq \frac{\pi}{2}$ for the mounted arm, with link lengths L_1 & L_2, as shown.



a) Sketch the reachable workspace for end effector (at Pee).

b) Sketch the reachable workspace for P_1 (where L_1 & L_2 links meet).

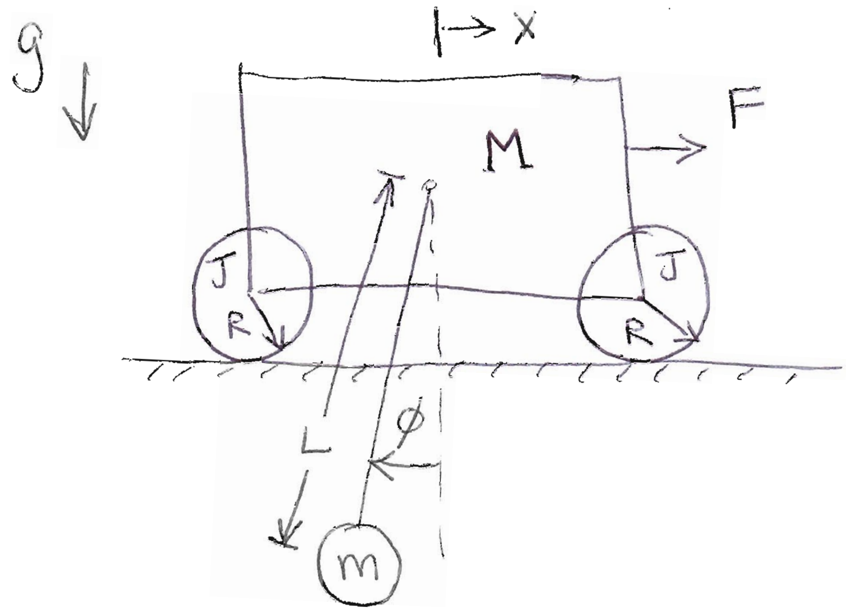
c) Sketch the dexterous workspace for Pee, where $\theta_1 + \theta_2$ can be <u>any</u> angle.

d) OK, that is a trick, since

$$-\frac{\pi}{2} \leq (\theta_1 + \theta_2) \leq \pi.$$

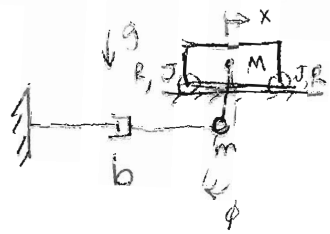
- Assume $0 \leq \theta_1 \leq \frac{\pi}{2}$ and θ_2 can be <u>any</u> angle. <u>Now</u> sketch the dexterous WS.

(3) a) Derive the EOM for the system below, using the Lagrangian approach.



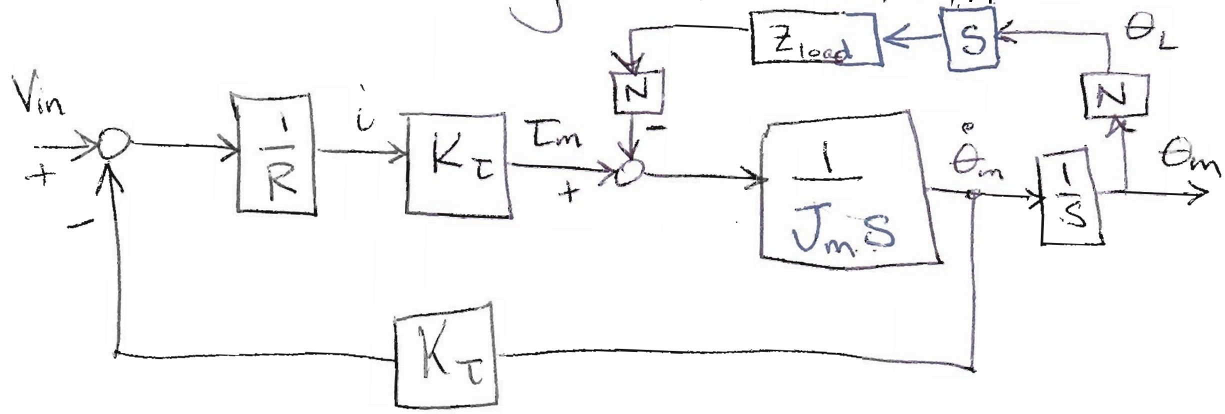
- Each wheel has radius R & inertia J. Assume no slip.
- Cart has mass M.
- Pendulum has length L & point mass m.
- There is gravity, as shown.
- Input F is applied to the cart.

b) Repeat when there is a damper between pendulum mass m and ground!



For example, imagine air resistance is $-b\dot{x}_m$, where $x_m = x_M - L\sin\phi$

(4) a) Reduce the following block diagram:



- Here Z_{load} is a <u>mechanical impedance</u> that is connected to a voltage-controlled motor with gear ratio.

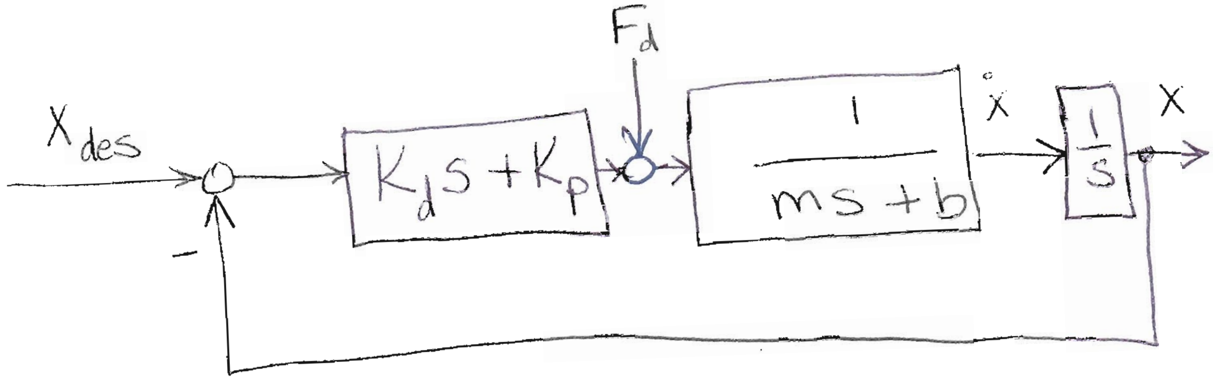
b) Solve for an impedance, Z_{load}, such that the entire system has second-order dynamics with $\omega_n = 10$ rad/sec and $\zeta = .2$

(natural freq.) (damping ratio)

* Use the following values, to make math easy:
$K_\tau = 2 \left(\frac{Nm}{Amp}\right)$, $R = 1$ (Ohm), $J_m = 1$ (kg-m²), $N = 10$.

c) Is your solution for Z_{load} unique? Or are there multiple solutions?

(5)



a) Solve for the closed-loop transfer function from X_{des} to X. (Hint, any other input, here F_d, is zero when calculating a "transfer function".)

b) What is the steady-state error for a unit step input in X_{des}? ($F_d = 0$)

c) What is the transfer function from F_d to X?

d) What is the steady-state value (error) for X given a unit step in F_d? ($X_{des} = 0$)

e) What is required for the poles to be stable? (Assume $m>0$, $b>0$. What values can K_d & K_p have?)

f) What are the zeros of $\frac{X(s)}{X_{des}(s)}$? (roots of the numerator)