Homework 1 (Due Friday, April 20, at 5pm)

1) Kinematics and workspace. Consider the kinematics of the 3-link arm example discussed in class and shown in the figure below. Note that this drawing is not to scale. We wish to consider different possible lengths for each of the three links. For each set of lengths given (sets A and B, below), determine both i) the reachable workspace and ii) the dexterous workspace. Sketch and label each region.

A) L_1=0.8, L_2=0.8, L_3=0.1

B) L_1=1, L_2=2, L_3=0.5

C) Now, assume: L_1=1, and L_2=1.5. Given some arbitrary value for L_3, will some dexterous workspace always exist? If so, explain why. If not, specify the range(s) of values for L_3 for which a dexterous workspace *will* exist.

Recall that the reachable workspace is the set of all points the end effector can reach, while the dexterous workspace is the set of all points the end effect can reach at an arbitrary angle.

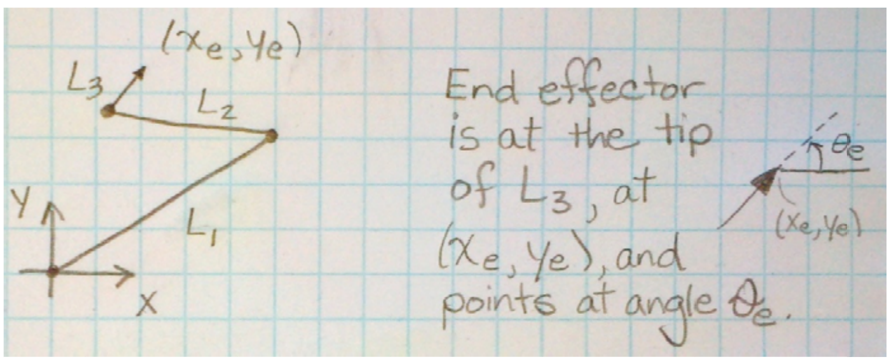

Figure 1. Kinematics of 3-link arm (problem 1).

2) Euler angles. We mentioned Euler angle rotations only briefly in class. This problem is designed to build better intuition about the conventions typically used to specify rotations (orientation) of a rigid body. Rotation angles can be specified either with respect to:

[1] A relative coordinate frame, that is fixed to the rotating body

[2] An absolute coordinate frame, that remains fixed; i.e., a global coordinate frame.

In either case, we require 3 rotations to specify any arbitrary orientation of a rigid body in space. We will refer to case 1 as "Euler angle rotation" and case 2 as "fixed angle rotation".

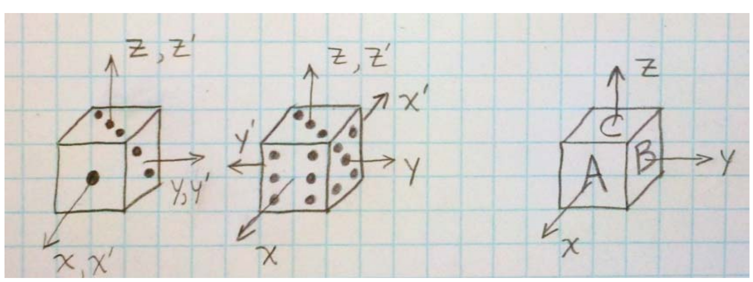

Figure 2. Local coordinate frame definition for dice in problem 2.

A standard American die (e.g., from a set Las Vegas dice) is shown above. The numbers 1 through 6 are arranged such that the numbers on opposite faces always sum to 7: 1 is opposite 6, 2 opposes 5, and 3 is opposite 4. Let us define a relative coordinate frame for a die, as shown at left in Figure 2. When aligned with the global coordinate frame, the x axis points out of side 1, y points out of 2, and z points out of 3, as shown. If we rotate the die at left counter-clockwise by 180 degrees about either z (absolute) or z' (relative), it will appear as shown in the middle figure. If we now rotate the die shown in the middle figure by 90 degrees (again, CCW is conventional) about y (absolute y), then faces A, B, and C in the diagram at right will show 3, 5, and 1, respectively. If we instead rotate the die shown in the middle figure by 90 degrees about the relative axis, y', then A=4, B=5, and C=6.

For each case below, begin with the configuration A=1, B=2, C=3. Determine the new orientation (A=?, B=?, C=?) that results from performing the following rotations of the die:

A) An Euler angle rotation in the order z'-y'-x', by angles -90°, +180°, +90°.

B) A fixed angle rotation in the order x-y-z, by angles +90°, +180°, -90°.

C) An Euler angle rotation in the order y'-z'-y' by angles +180°, +90°, -90°

D) A fixed angle rotation in the order y-z-y by angles -90°, +90°, +180°.

As described in pages 49-53 in Spong, performing a set of rotations in relative coordinates results in the same configuration as performing these rotations in the reverse order in absolute coordinates. ***Note that this would mean your answers to A) and B) should therefore be identical. Similarly, you can check that answer C) and D) are the same, as well.***

Note: It is probably wise to sketch the "intermediate" orientation above, rather than doing all rotations "in your head"!

3) Degrees of Freedom (DOF). In the figure below, a 2-link robot arm is mounted to a rolling base. The system therefore has 3 degrees of freedom: 1 prismatic joint (x) and 2 rotational joints (θ_1 and θ_2). As in problem 1, assume the end effector is a small tool at the tip of the last (most distal to the cart) link. The position and orientation of the end effector can be written as:

$$(x_e, y_e, \theta_e)$$

where all three degrees of freedom are with respect to the absolute coordinate frame (indicated by the dashed lines in the figure).

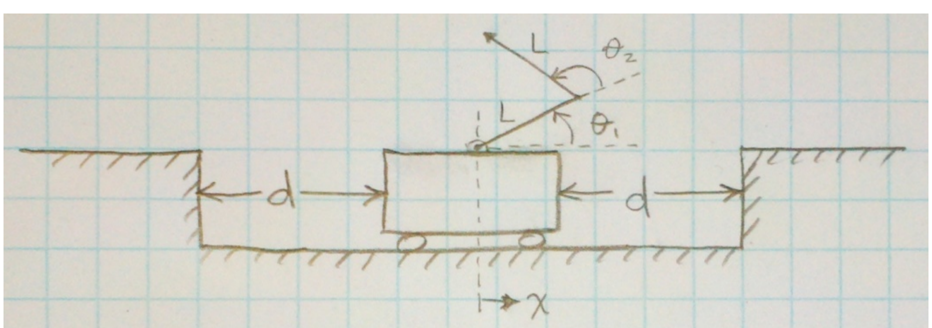

Figure 3. 3DOF robot system for problem 3.

Assume allowable motions are limited to: $0 \le \theta_1 \le 180^\circ$, $y_e \ge 0$, $d = 1.2$ (m), $L = 1$ (m).

A) Sketch the reachable workspace for the end effector. Label carefully.

B) Along the line $x_e = 0$, what is the range of values possible for θ_e, as a function of y_e? You may answer either with an analytic expression (calculated by hand) or by submitting MATLAB code and a plot of the bounds on θ_e and y_e varies.

4) Singularities. Below is a figure of a mechanism constrained to a single degree of freedom, x, at the output. The relationship between θ and x is:

$$x = 2Lcos\theta$$

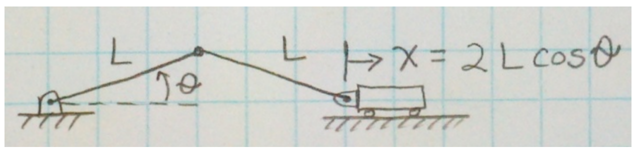

Figure 4. 1 DOF mechanism for problem 4.

A) Derive the relationship between $\dot{x}$ and $\dot{\theta}$.
B) Assume $x = 0$, and we desire $\dot{x} = -5$ (m/s). What must $\dot{\theta}$ be to achieve this?
C) Assume $x = L$, and we desire $\dot{x} = -5$ (m/s). What must $\dot{\theta}$ be to achieve this?
D) Assume $x = 1.9L$, and we desire $\dot{x} = -5$ (m/s). What must $\dot{\theta}$ be to achieve this?
E) Assume $x = 2.0L$, and we desire $\dot{x} = -5$ (m/s). What must $\dot{\theta}$ be to achieve this?

Configuration E is known as a "singularity". It is a configuration in which it becomes impossible to move in a particular location: that is, note that $\dot{x} > 0$ is impossible at $x = 2.0L$. Singularities can be "dangerous" places to operate, because (often) they are configurations near which the robot's input velocity (here, $\dot{\theta}$) must "blow up", unbounded, to achieve a finite output velocity at the end effector. This phenomenon explains (in part) why humanoid robots often ***avoid*** operating with legs "fully extended", near a singularity.

5) Work and Mechanical Impedance. Conservative vs. non-conservative forces.

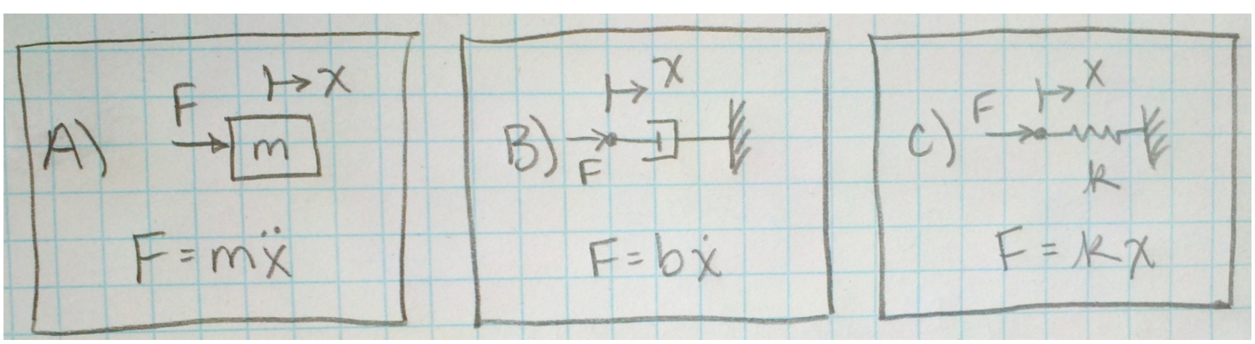

Figure 5. Mass, viscous damping, and spring impedance elements (problem 5).

Masses (or inertia) and springs are conservative impedances, while damping is non-conservative: a dashpot (mechanical resistance) and an electrical resistor both dissipate energy. For each of the 3 mechanical impedances in Figure 5 (A, B, and C), calculate the work done (i.e., the integral of force times velocity) by moving point "p" from the state ($x = 0$, $\dot{x}$=0) to the state ($x = 5$ (m), $\dot{x}$=4 (m/s)) in each of the following two ways:

I. 2 seconds of constant acceleration at $\ddot{x} = 1.0$ (m/s^2), then 1 second at $\ddot{x} = 2.0$ (m/s^2).

II. 2 second at $\ddot{x} = 2.0$ (m/s^2), then 0.25 seconds at $\ddot{x} = 0$ (m/s^2).

D) Do your results make sense, given that damping is non-conservative, while inertia and stiffness are conservative impedance elements? (Hint: which are "path dependent"?)

6) Analogous mechanical and electrical impedances as "circuit elements". Figure 6 shows a 4th-order (translational mechanical) system (at right) and an analogous circuit structure (at left).

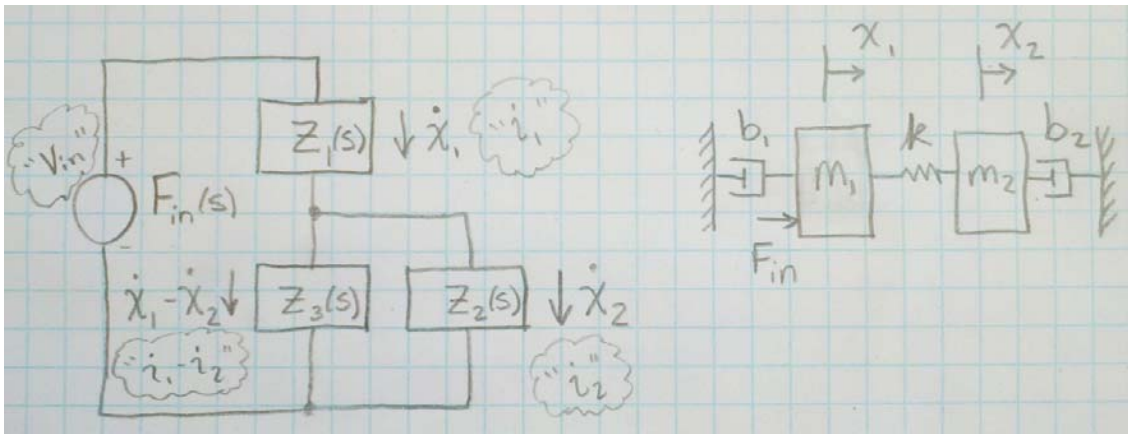

Figure 6. Mechanical impedances in a circuit (problem 6).

A) Use the definitions given in class for mechanical impedance, $Z_m(s) = \frac{F(s)}{v(s)} = \frac{F(s)}{sX(s)}$, and for electrical impedance, $Z_e(s) = \frac{V(s)}{I(s)}$, to solve for, $Z_1(s)$, $Z_2(s)$, and $Z_3(s)$ in the circuit diagram.

B) Solve for the transfer functions $\frac{X_1(s)}{F(s)}$ and $\frac{X_2(s)}{F(s)}$.

7) Reflected inertia and mechanical impedance.

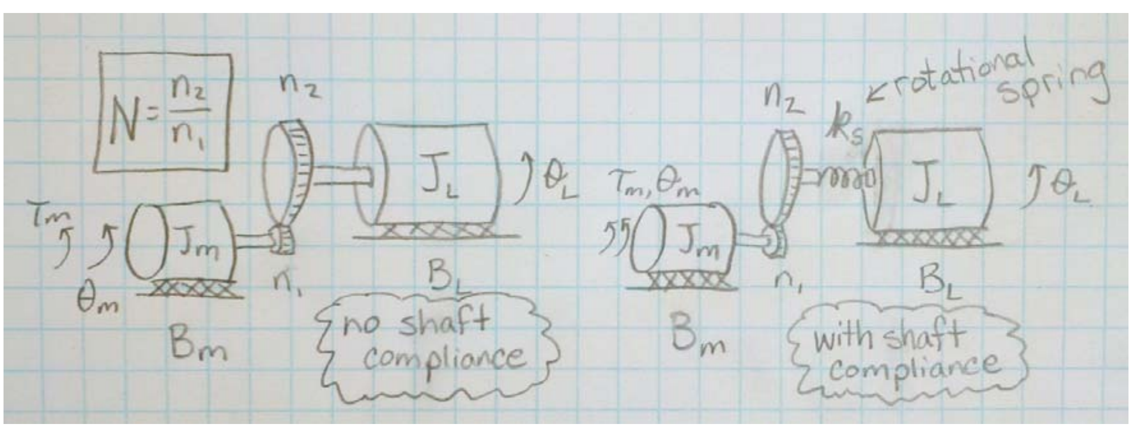

Figure 7. Motor, transmission, and lab (problem 7).

Figure 7 shows a system with a motor driving a load. In the lefthand diagram, $\dot{\theta}_m$ (motor velocity) and $\dot{\theta}_L$ (load velocity) are simply related through the gear ratio: $\dot{\theta}_m = N\dot{\theta}_L$. In the righthand diagram, there is now a spring element, with stiffness k_g, between the larger gear and the load inertia, J_L. This spring models the compliance that is sometimes a noticeable factor in real transmission systems, and it changes the dynamics from a 2nd-order system to a 4th-order system.

Note, the lefthand system is one we have already considered in class (Lecture 4). Also, if you look carefully, you should notice that the righthand system is (intentionally) very similar to the translational mechanical system depicted in problem 6.

A) Solve for the transfer functions $\frac{\theta_m(s)}{\tau(s)}$ and $\frac{\theta_L(s)}{\tau(s)}$ for the system with <u>no</u> shaft compliance (that is, in the limit as k_g become infinitely stiff).

B) Solve for the transfer functions $\frac{\theta_m(s)}{\tau(s)}$ and $\frac{\theta_L(s)}{\tau(s)}$ for the system <u>with</u> shaft compliance.