Name $\qquad$
Tear off and save the ID number below, to look up your grade later...

## ECE 179d

## "Midterm" Practice Quiz

- Extra pages are provided at back of exam, if needed. Turn in all work!
- You are allowed one (1) single-sided sheet of notes.
- No calculators or other computing devices are allowed.

Problem 1 - This problem involves the standard, 2-link, planar robot arm we've discussed several times in class.


Fig. 1: Link lengths are drawn above with correct scaling. Coordinates for "elbow" and end effector are labeled.
a) Solve for the Jacobian matrix relating velocities of joints to those of the end effector, based on the given coordinates of the elbow ( $\mathrm{x}_{\mathrm{m}}=0.8, \mathrm{y}_{\mathrm{m}}=0.6$ ) and end effector ( $\mathrm{x}_{\mathrm{e}}=0.8, \mathrm{y}_{\mathrm{e}}=1.6$ ).
Recall that $\dot{\xi}_{e}=J \dot{q}_{a}$, and also $\tau_{a}=J^{T} F_{e}$. Here, $\xi=\left[\begin{array}{l}x_{e} \\ y_{e} \\ \phi_{e}\end{array}\right]$ and $q=\left[\begin{array}{l}\theta_{1} \\ \theta_{2}\end{array}\right] . \phi_{e}=\theta_{1}+\theta_{2}$.

Problem 2 - Below (at left) is a diagram of the wheel lay-out for a different proposed omnibot design. The arrows show the instantaneous direction of motion for each wheel when it powered to rotate with position angular velocity. Wheels 1,2 , and 3 are located (as illustrated) at the following ( $\mathrm{x}, \mathrm{y}$ ) locations: Wheel 1: $(3,0)$, Wheel 2: $(4,4)$, Wheel 3: $(0,3)$.


a) For part a, only, assume the wheel are traditional, "no-slip" wheels. Is there an instantaneous center of rotation (ICR) for all three wheels? If so, what are its ( $\mathrm{x}, \mathrm{y}$ ) coordinates, and if not, why not?
b) Now assume all three wheels are omni-directional wheels, with free rollers at 90 degrees with respect to the primary axis, as in lab and as shown above at right. Now, sketch and solve for the coordinately of each of the 3 ICR's obtained when powering only one wheel while allow the other two to rotate only in their "free roller" (sideways) directions.
i) Powering only wheel 1 ? $\mathrm{X}_{\mathrm{ICR}}=$ $\qquad$ , $\mathrm{Y}_{\mathrm{ICR}}=$ $\qquad$
ii) Powering only wheel 2? $\mathrm{X}_{\mathrm{ICR}}=$ $\qquad$ , $\mathrm{Y}_{\mathrm{ICR}}=$ $\qquad$
iii) Powering only wheel 3 ? $\mathrm{X}_{\mathrm{ICR}}=$ $\qquad$ , $\mathrm{Y}_{\mathrm{ICR}}=$ $\qquad$
c) For this some configuration (shown again below, for convenience), solve for either the Jacobian, $J$, or its inverse. (...But specify which you are solving for!) Leave the wheel radius, $\mathrm{r}_{\mathrm{w}}$, as a variable.


Recall, $\left[\begin{array}{l}\dot{x}_{b} \\ \dot{y}_{b} \\ \dot{\phi}_{b}\end{array}\right]=\dot{\xi}=J \dot{q}=J\left[\begin{array}{l}\dot{q}_{1} \\ \dot{q}_{2} \\ \dot{q}_{3}\end{array}\right]$, with the inverse relationship being $\dot{q}=J^{-1} \dot{\xi}$, where $\mathrm{q}_{\mathrm{n}}$ is the angle of rotation of wheel $n$ above its main, powered axis. HINT: Once you know the ICR for a given wheel, this problem becomes remarkably analogous to the multi-link arm Jacobian problem!

Problem 3 - Shown below is the wheel geometry for a 2-wheeled "bicycle", drawn to scale:

$$
\theta=30^{\circ}, \quad d=1 \text { meter. }
$$

Each wheel can roll along its longer axis and turn on its contact point, but no slip is allowed perpendicular (sideways) to the rolling direction of a wheel.


Fig. 3 - Bicycle wheel configuration.
a) On the figure above, sketch the location of the instantaneous center of rotation (ICR).
b) What is the distance from the ICR to the center of wheel B?
c) Assume that in this configuration, wheel A rolls at speed $\alpha$ and wheel B rolls at speed $\beta$. What is the ratio, $N=\frac{\alpha}{\beta}$, of the two wheel speeds?

Problem 4 - Rotational systems with gear ratios. (This problem should be easy...I hope!)


Fig. 5a - A motor driving a mechanical impedance.

For the system shown in Figure 5, all impedance elements are labeled. There is a small gear at the motor with radius $R_{m}$ and which drives a larger gear with radius $R_{L}$, on the load end. (Each angle is positive when measured counter-clockwise, as shown.) Parameter values are:
$J_{m}=0.1\left(\mathrm{~kg}^{*} \mathrm{~m}^{\wedge} 2\right), J_{L}=0.5\left(\mathrm{~kg}^{*} \mathrm{~m}^{\wedge} 2\right), k=48\left(\mathrm{~N}^{*} \mathrm{~m} / \mathrm{rad}\right), R_{m}=1(\mathrm{~cm}), R_{L}=5(\mathrm{~cm})$
a) What is the relationship (transfer function) between $\theta_{m}$ and $\theta_{L}$ ?
b) How many degrees of freedom (required generalized coordinates) does the system have?
c) What are the poles of the system?
d) For extra credit (and perhaps to help in modeling), complete the block diagram (next page...).

4d) (...continued.) EXTRA CREDIT:
To fully describe the dynamic system, determine what expression should go into each each of the 5 blank spaces (number \#1 through \#5) in the block diagram below. For \#5, you simply need either $\mathrm{a}-$ sign or $\mathrm{a}+$ sign.


Figure 5b: Block diagram to be completed, for extra credit.

5e) Extra credit: Describe briefly the quantity in each of the boxes, \#1-\#4. (e.g., "a mechanical impedance", or $1 / Z(s)$ [inverse impedance], or "gear ratio", etc...)
(Blank page for extra work.)
Be sure to LABEL WHICH PROBLEM YOU ARE WORKING ON!
(Blank page for extra work.)
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