Homework 4

<u>Problem 1</u> – Lagrangian equations of motion: Rolling can in a hollow cylinder.

a) For the system shown below, how many independent and complete generalized coordinates are required?

b) Derive the equation(s) of motion of the rolling can (with mass m_i and inertia J_i). Assume there are no applied forces or loss terms. (i.e., assume the dynamics include only conservative forces.) Also assume the can rolls inside the (fixed) outer cylinder *without slipping*.

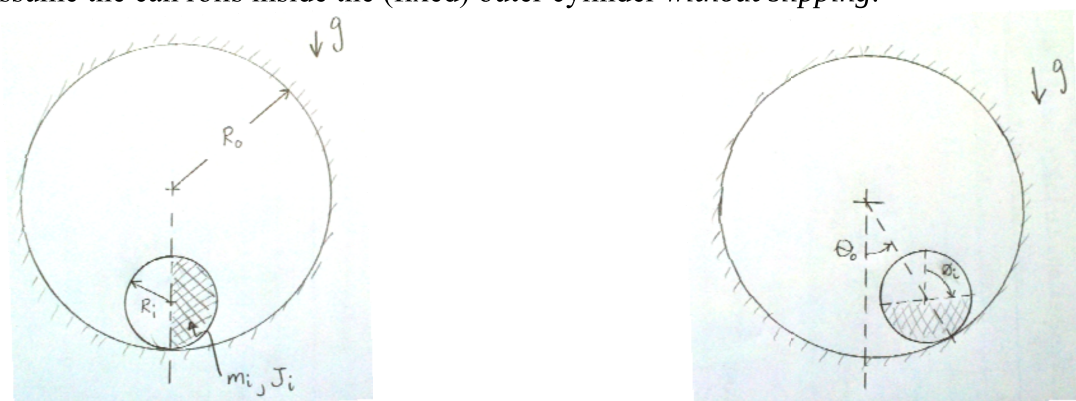


Fig. 2 – The outer perimeter is fixed, and a can rolls freely (without slipping) inside, always touching the perimeter. Definitions for the angle to the point of contact, θ_o, and the angle of rotation of the can, θ_i, both of which are measured with respect to an absolute coordinate system, are shown. Shading on the can is only intended to illustrate orientation more clearly: the center of mass is at the center of the can, and the can has mass m_i and inertia J_i.

Problem 2 – Lagrangian equations of motion: Seesaw with a cart.

In Fig. 4, assume the only mass is a point mass at the center of the cart.

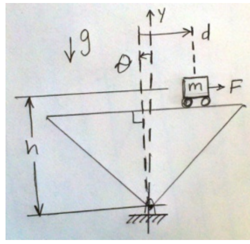


Fig. 4 – Massless seesaw with a (point-mass) cart driven on top.

a) Write expressions for the location (x,y) of the cart, in terms of the geometry shown.

b) Write expressions for the x and y components of velocity of the cart.

c) Derive the equation(s) of motion for the cart. (Assume the only non-conservative force is the input force, F.)

Problem 3 – Systems with compliance. (4^{th}-order dynamics.)

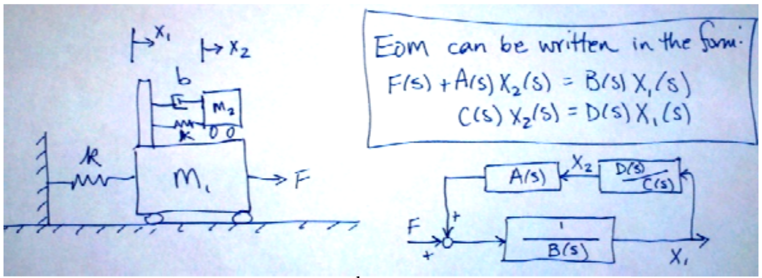
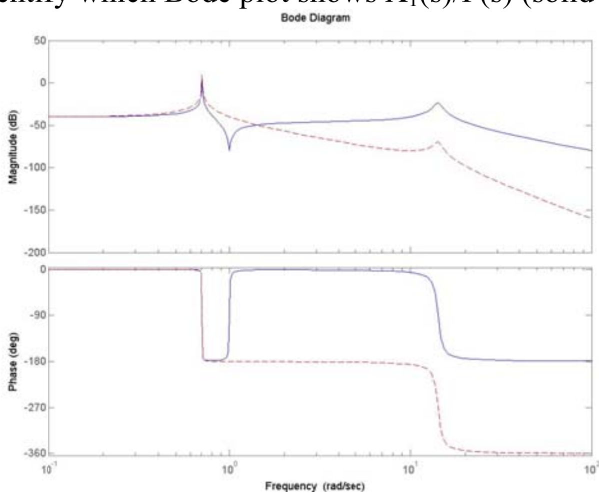


Fig. 5 - a 4^{th}-order system.

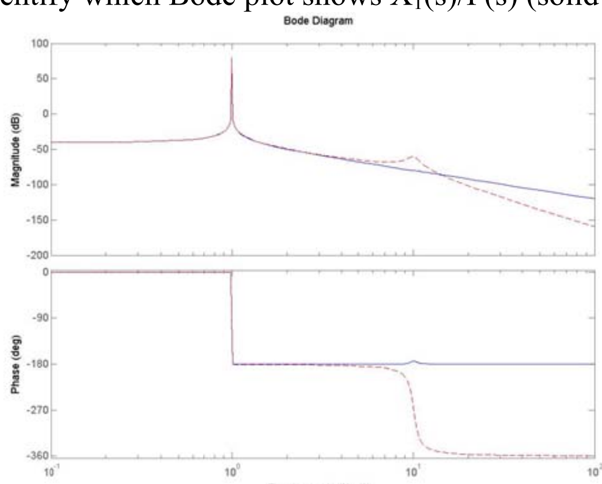
Assume k=100 N/m. Below are two sets of Bode plots. Your job is to determine which plot (solid vs dashed) is for $X_1(s)/F(s)$; note that the other plot is for $X_2(s)/F(s)$. Also estimate (roughly) the magnitude of each mass in set of plots (a and b).

a) Identify which Bode plot shows $X_1(s)/F(s)$ (solid or dashed?). Is $m_1<m_2$? Estimate m_1 and m_2.



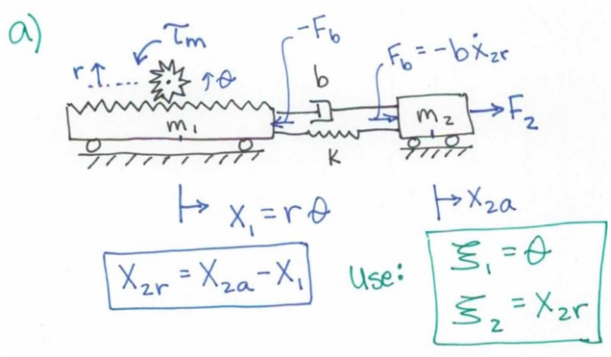
(Bode plots above are for "a".)

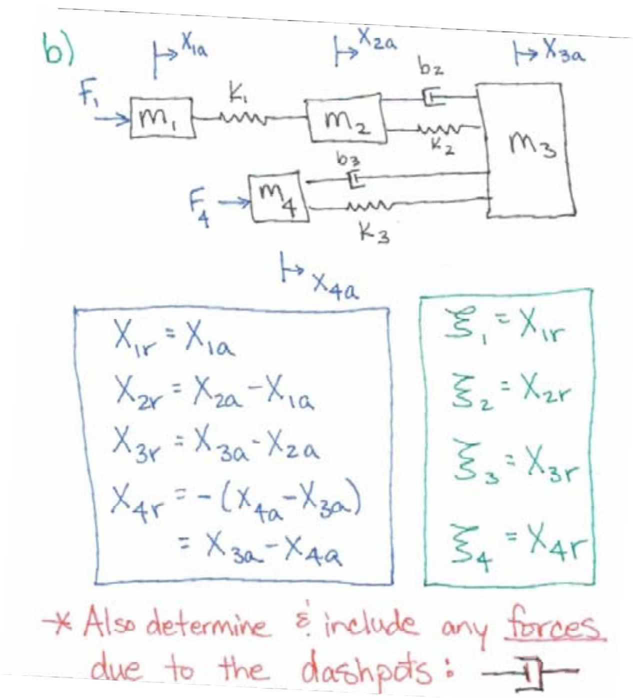
b) Identify which Bode plot shows $X_1(s)/F(s)$ (solid or dashed?). Is $m_1<m_2$? Estimate m_1 and m_2.

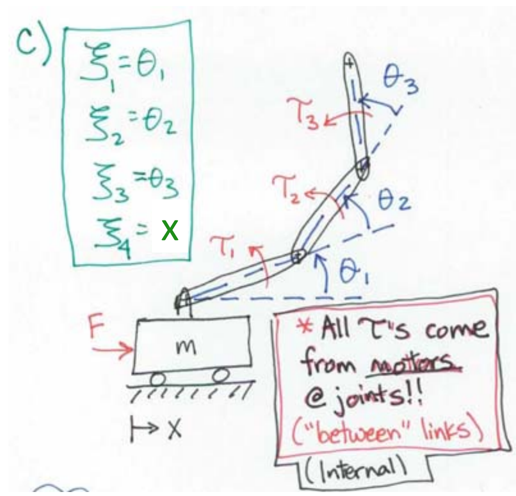


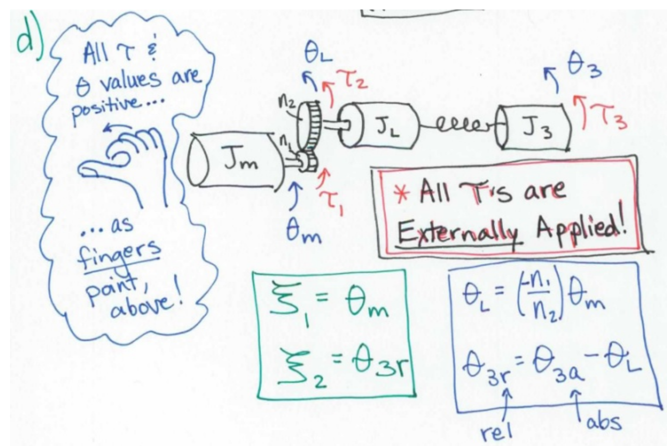
(Bode plots above are for "b".)

Problem 4 – For each system and corresponding definition of generalized coordinates (GC's), identify the non-conservative forces ("big Xi", Ξ_i) associated with each GC, necessary in using the Lagrangian approach to develop equations of motion.









In **a**: non-conservative forces and torques include:

tau_m, F2, and Fb (due to dashpot, applied as equal and opposite forces on each mass).

In **b**: non-conservative forces and torques include:

tau1, tau2, tau3, and F.

In **c**: non-conservative forces and torques include:

F1, F4, and any damping forces (similar to part a) due to both b2 and b3.

In **d**: non-conservative forces and torques include:

tau1, tau2, and tau3. n1 and n2 are the numbers of teeth on the gears, as shown.

Note, when theta_m velocity is positive, theta_L velocity is negative!

Problem 5 – For the two-link system shown,

a) Write the Jacobian, J, and Jacobian transpose, J^T.

Assume we will use *relative* generalized coordinates, as shown, where theta1 is absolute but theta2 is relative to theta1.

b) Write the non-conservative forces, Ξ_1 and Ξ_2. Each should involve a mathematical expression that includes some subset of actuator torques, *tau1* and *tau2*, and the externally-applied "disturbance" forces, F_x and Fy, at the end effector.

c) Comment on how the Jacobian (or its transpose) describes the effect the disturbance forces will have on each Ξ_i, the total non-conservative torque affecting each equation of motion.