Nonlinear Phenomena: Homework 1

Reading: §1.0-1.3, 2.0-2.2, 2.4, 8.7, 2.5

Due April 11, 2013

Problem 1 (§2.2 : Fixed Points and Stability)

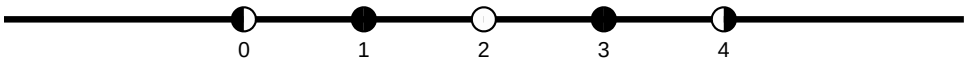
[Adapted from Problems 2.2.1-2.2.7 in Strogatz.]
Analyze the following differential equations for first-order (one-dimensional) systems *graphically*. For each equation: sketch the vector field (1D), find all fixed points, classify their stability, and sketch the graph of $x(t)$ for different initial conditions.

a) $\dot{x} = -x^3 + 16x$

b) $\dot{x} = 1 - tan(x)$

Problem 2 (§2.2 : Fixed Points and Stability)

[Adapted from Strogatz Problem 2.2.8.]
In Problem 1, you were asked to start with an equation, to reason about dynamics over time. Now, *work backwards* to find an equation, given the fixed points below, on the real line. ("There are an infinite number of correct solutions – and wrong ones, too." Extra credit will be awarded for finding significantly "unique" solutions...)



Problem 3 (§2.4 : Linear Stability Analysis)

[Adapted from Problems 2.4.1-2.4.7 in Strogatz.]
Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to determine stability.

a) $\dot{x} = sin^2 x$

b) $\dot{x} = (x+3)^2(x-1)$

c) $\dot{x} = ax + x^2$, where a can be negative, positive, or zero. Discuss all three cases.

Problem 4 (§8.7 : Poincaré Maps)

Do Problem 8.7.2 (in Strogatz).

Problem 5 (§8.7 : Poincaré Maps)

Analyze the following difference equations for first-order (one-dimensional) systems *graphically*. For each equation: sketch the return map (also call an *iterated map*), find all fixed points, classify their stability, and sketch the cobweb graph of x_n for different initial conditions. For any *stable* fixed point, give the set of all initial x_0 values that will converge to this fixed point.

a) $x_{n+1} = 5 - x_n$

b) $x_{n+1} = (x_n - 2)^2 + 2$

c) $x_{n+1} = (x_n - 3)^2 + 1$

Be sure to draw the cobweb for (at least) one point between the two fixed points for part c)!

Problem 6 §2.5 : Existence and Uniqueness)

Do Problem 2.5.1 (in Strogatz).