Nonlinear Phenomena: Homework 1

Reading: §1.0-1.3, 2.0-2.2, 2.4, 8.7, 2.5
Due April 11, 2013

Problem 1  (§2.2 : Fixed Points and Stability)

[Adapted from Problems 2.2.1-2.2.7 in Strogatz.]
Analyze the following differential equations for first-order (one-dimensional) systems graphically. For each equation: sketch the vector field (1D), find all fixed points, classify their stability, and sketch the graph of $x(t)$ for different initial conditions.

a) $\dot{x} = -x^3 + 16x$

b) $\dot{x} = 1 - \tan(x)$

Problem 2  (§2.2 : Fixed Points and Stability)

[Adapted from Strogatz Problem 2.2.8.]
In Problem 1, you were asked to start with an equation, to reason about dynamics over time. Now, work backwards to find an equation, given the fixed points below, on the real line. ("There are an infinite number of correct solutions – and wrong ones, too." Extra credit will be awarded for finding significantly “unique” solutions...)

Problem 3  (§2.4 : Linear Stability Analysis)

[Adapted from Problems 2.4.1-2.4.7 in Strogatz.]
Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to determine stability.

a) $\dot{x} = \sin^2 x$

b) $\dot{x} = (x + 3)^2(x - 1)$

c) $\dot{x} = ax + x^2$, where $a$ can be negative, positive, or zero. Discuss all three cases.
Problem 4  (§8.7 : Poincaré Maps)

Do Problem 8.7.2 (in Strogatz).

Problem 5  (§8.7 : Poincaré Maps)

Analyze the following difference equations for first-order (one-dimensional) systems graphically. For each equation: sketch the return map (also call an iterated map), find all fixed points, classify their stability, and sketch the cobweb graph of $x_n$ for different initial conditions. For any stable fixed point, give the set of all initial $x_0$ values that will converge to this fixed point.

a) $x_{n+1} = 5 - x_n$

b) $x_{n+1} = (x_n - 2)^2 + 2$

c) $x_{n+1} = (x_n - 3)^2 + 1$

Be sure to draw the cobweb for (at least) one point between the two fixed points for part c)!

Problem 6  §2.5 : Existence and Uniqueness)

Do Problem 2.5.1 (in Strogatz).