Name ______________________________

Midterm Exam

May 2, 2013

__________ Problem 0 (5 points) : Turn in one-page cheat sheet.

__________ Problem 1 (10 points)

__________ Problem 2 (10 points)

__________ Problem 3 (15 points)

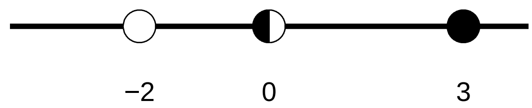
__________ Problem 4 (20 points)

__________ Problem 5 (15 points)

__________ Problem 6 (25 points)

__________ Total (out of 100)

Problem 1



The figure above shows fixed points on the line for a 1D system.
a) Write a valid expression for $f(x)$ that agrees with the figure.
b) Sketch $f(x)$ and draw arrows along the x-axis to indication the direction of flow on the line.

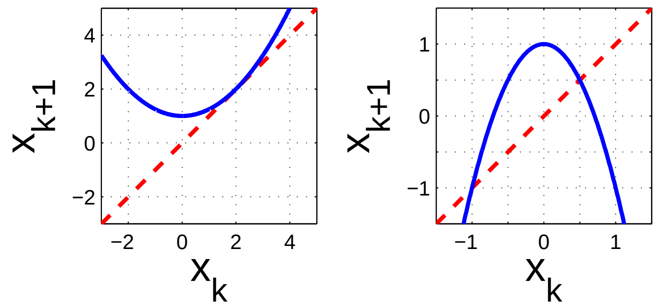
Problem 2

$f(x) = 25x - x^3$

a) Find all fixed points for f(x).
b) Sketch f(x), indicating fixed point locations and their stability.

Problem 3

$$P(x) = x_{k+1} = ax_k^2 + 1$$



Above is a return map, $P(x)$, for a 1D discrete-time system (at left), with instances of $P(x)$ for two different values of a (right).

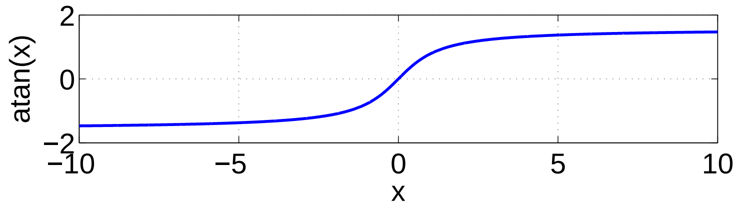
a) Write an expression to solve for the fixed points. (Do not bother to simplify this expression!)
b) Write an expression for the slope of $P(x)$.
c) For what values of a does the system have 1 stable and 1 unstable fixed point?

Problem 4

$f(x) = f_1(x) - f_2(x) = x^4 - 1 + r(x-1)$
Define: $f_1(x) = x^4 - 1$
and: $f_2(x) = -r(x-1)$

a) Sketch $f_1(x)$, as a first step.
b) Now, find r_c and x_c^* at the point of bifurcation.
c) Identify the type of bifurcation.
d) Sketch the bifurcation diagram (x^* vs r). Do not try to capture the exact shape, but do clearly illustrate the directions and stability of all branches. (Take care that your bifurcation actually occurs at the r_c and x_c^* location you previously found!)

Problem 5



Above is a sketch of the arctangent function.
(Recall that $tan(\delta) \approx \delta$, for small delta.)

$f(x) = rx + \text{atan}(x)$

a) Solve for r_c and x_c^* at the bifurcation point.
b) Identify the type of bifurcation.
c) Sketch the bifurcation diagram. As in Problem 3: Do not try to capture the exact shape, but do clearly illustrate the directions and stability of all branches. (Take care that your bifurcation actually occurs at the r_c and x_c^* location you previously found!)

Problem 6

$$\mathbf{A} = \begin{bmatrix} -3 & 5 \\ 1 & 1 \end{bmatrix}$$

a) Find eigenvalues and eigenvectors of $\mathbf{A}$.

b) Write the linear dynamic equations for the 2 states, x and y.

c) On the axes below, sketch the eigenvectors as solid lines.

d) Then, sketch nullclines (i.e., $\dot{x} = 0$ and $\dot{y} = 0$) as dashed lines.

e) And sketch flow pattern (sample trajectories) in 2D phase space.