Nonlinear Phenomena: Recitation 1

Reading: §1.0-1.3, 2.0-2.2, 2.4, 8.7, 2.5

Discussion Problems, April 5 and 8, 2013.

Discussion 1  (§2.2 : Fixed Points and Stability)

Strogatz Problem 2.2.1.

Strogatz Problem 2.2.8.

To discuss: Consider also a generalized case, in which each of the three fixed points
\((x = -1, x = 0, \text{ and } x = 2)\) can be either black, white, white-black, or black-white. What is required for \(\dot{x} = f(x)\) to be a continuous function? Note that if \(f(x)\) is piece-wise, finding a variety of solutions becomes trivial. How can one systematically create a continuous function, for any case that satisfies your earlier requirement(s)?

Discussion 2  (§2.4 : Linear Stability Analysis)

Strogatz Problems 2.4.4, 2.4.7.

Discussion 3  (§8.7 : Poincaré Maps)

For each difference equation: sketch the return map, find all fixed points, classify their stability, and sketch cobweb graphs of \(x_{n}\). For any stable fixed point, define all initial \(x_{0}\) values that converge to this fixed point. To discuss: If no fixed points are stable, must any initial condition eventually become unbounded? Why or why not?

i) \(x(n + 1) = \begin{cases} -1 + (3/2)x(n), & \text{if } x(n) < 4 \\ 5 - (3/2)x(n), & \text{otw} \end{cases} \)

ii) \(x(n + 1) = 2 \cdot atan(x(n))\)

iii) \(x(n + 1) = -2 \cdot atan(x(n))\)

Discussion 4  §2.5 : Existence and Uniqueness)

Strogatz Problem 2.5.5.
Recitation 1: When will $x(n)$ remain bounded, as $n$ goes to infinity?

**Case i**

**Case ii**

**Case iii**

$x(n+1) = -2 \cdot \text{atan}(x(n))$