Nonlinear Phenomena: Recitation 1

Reading: §1.0-1.3, 2.0-2.2, 2.4, 8.7, 2.5

Discussion Problems, April 5 and 8, 2013.

Discussion 1 (§2.2 : Fixed Points and Stability)

Strogatz Problem 2.2.1.

Strogatz Problem 2.2.8.
To discuss: Consider also a generalized case, in which each of the three fixed points ($x = -1$, $x = 0$, and $x = 2$) can be either black, white, white-black, or black-white. What is required for $\dot{x} = f(x)$ to be a continuous function? Note that if $f(x)$ is piece-wise, finding a variety of solutions becomes trivial. How can one systematically create a continuous function, for any case that satisfies your earlier requirement(s)?

Discussion 2 (§2.4 : Linear Stability Analysis)

Strogatz Problems 2.4.4, 2.4.7.

Discussion 3 (§8.7 : Poincaré Maps)

For each difference equation: sketch the return map, find all fixed points, classify their stability, and sketch cobweb graphs of x_n. For any *stable* fixed point, define all initial x_0 values that converge to this fixed point. To discuss: If no fixed points are stable, must any initial condition eventually become unbounded? Why or why not?

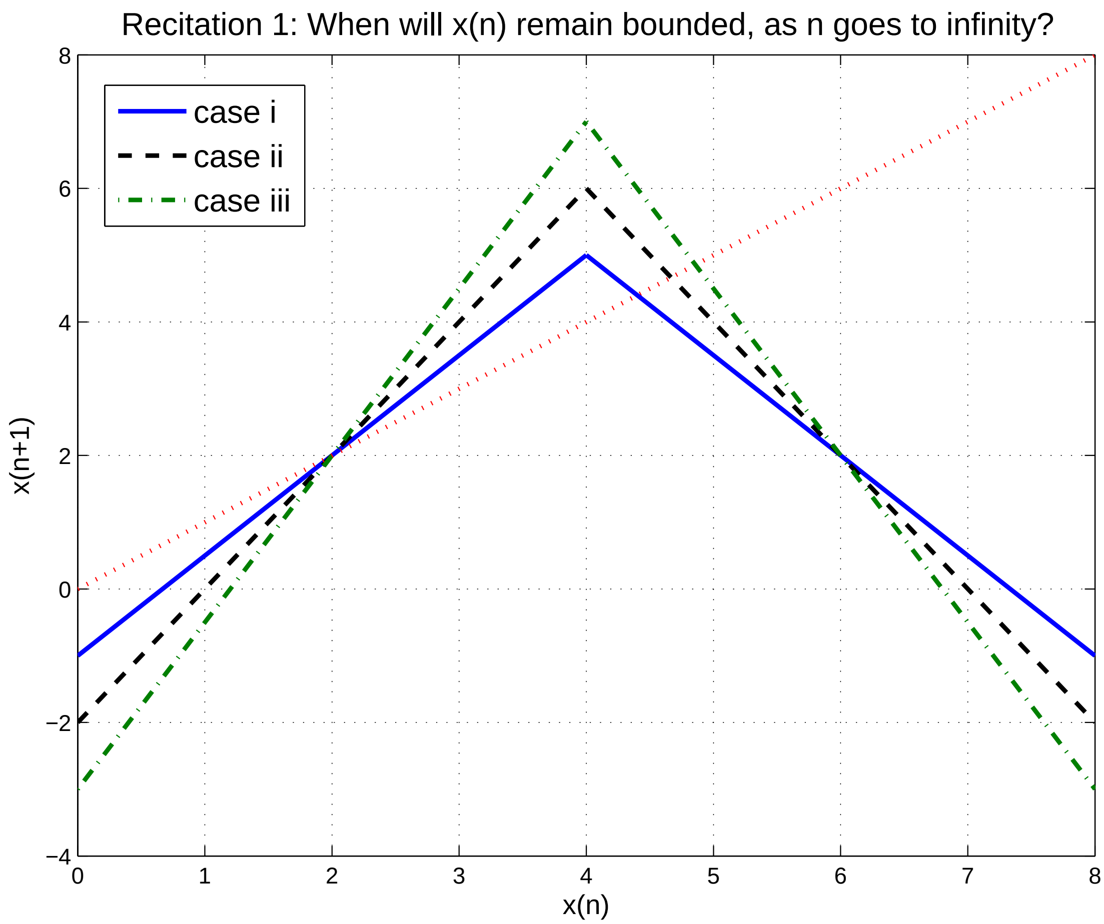
i) $x(n+1) = \begin{cases} -1 + (3/2)x(n), & \text{if } x(n) < 4 \\ 5 - (3/2)x(n), & \text{otw} \end{cases}$

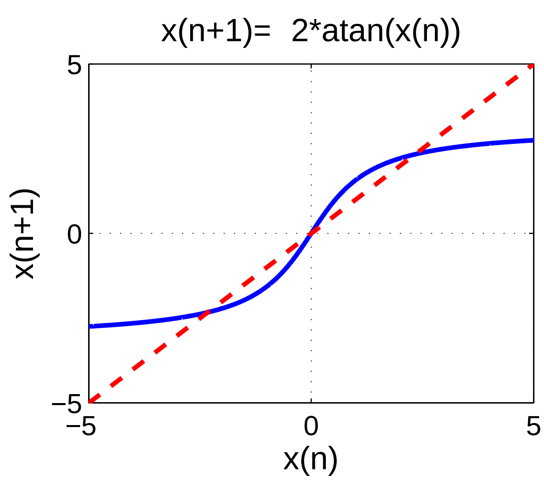
ii) $x(n+1) = 2 \cdot atan(x(n))$

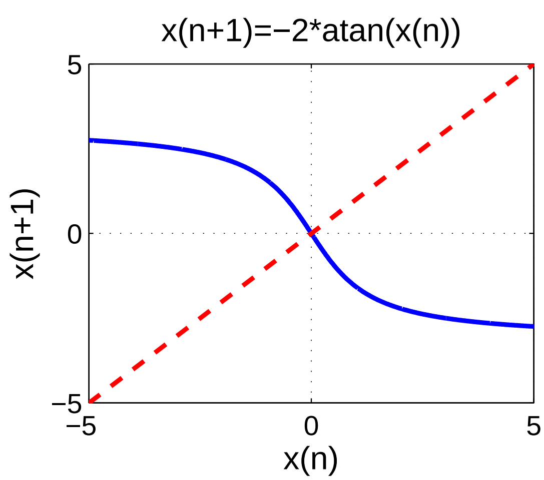
iii) $x(n+1) = -2 \cdot atan(x(n))$

Discussion 4 §2.5 : Existence and Uniqueness)

Strogatz Problem 2.5.5.


Recitation 1: When will x(n) remain bounded, as n goes to infinity?
case i
case ii
case iii
x(n+1)
x(n)
8
6
4
2
0
−2
−4
0
1
2
3
4
5
6
7
8


x(n+1)= 2*atan(x(n))
x(n+1)
x(n)
5
0
−5
−5
0
5


x(n+1)=−2*atan(x(n))
x(n+1)
x(n)
5
0
−5
−5
0
5