Finals

Due at noon, December 8, 2008, in the homework box.

All work must be done independently. You are allowed to consult the textbook and class lecture notes only.

1. Relate the singular values and vectors of $A = B + iC$, where $B, C \in \mathcal{R}^{m\times n}$, to those of

$$\begin{pmatrix} B & -C \\ C & B \end{pmatrix}.$$

2. Find the Jordan decomposition of

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Let A be a real symmetric $n \times n$ matrix with non-negative entries. Prove that A has an eigenvector with non-negative entries.

4. Let A and B be $n \times n$ real matrices. Show that

   a. If $A = A^T$ and all eigenvalues of A are strictly positive then $AX + XA = B$ has a unique solution X.

   b. Moreover, if $B = B^T$ and the eigenvalues of B are non-negative, then $X = X^T$ and the eigenvalues of X are also non-negative.

5. Let A be an $n \times n$ complex matrix. Show that if $\mathrm{trace}(A^i) = 0$ for all i from 1 to n, then A must be nilpotent.

6. Let A be an $n \times n$ Hermitian matrix satisfying the condition

$$A^5 + A^3 + A = 3I.$$

   Show that $A = I$.