Home Work 2

Due on October 6, 2008

Reading assignments don't have to be turned in.

1. **Reading assignment.** Read chapters 1, 2 and 3 of Dym's book.

2. **Reading assignment.** Finish reading chapter 2 of the notes posted on the class web-site.

3. **Reading assignment.** Read chapter 3 of the notes posted on the class web-site.

4. Suppose $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, and that $\phi(x) = \frac{1}{2}x^T Ax - x^T b$. Show that the gradient of ϕ is given by $\nabla\phi(x) = \frac{1}{2}(A^T + A)x - b$.

5. Let $A = \begin{pmatrix} 1/3 & 1/3 & 1/3 \end{pmatrix}^T$. Think of A as an operator from $\mathbb{R}^1$ to $\mathbb{R}^3$ via matrix-vector multiplication. Show that the operator is one-to-one. Find two linear left-inverses for A. Find a left-inverse for A that is not linear.

6. Find all matrices X that satisfy the equation

$$AXB^T = C,$$

   in terms of the LU factorizations of A and B. When are there no solutions?

7. Let U_1 and U_2 be two upper-triangular matrices. Let Z be an $m \times n$ matrix. Let X be an unknown matrix that satisfies the equation

$$U_1 X + X U_2 = Z.$$

   A. Give an algorithm to find X in $O(mn(m + n))$ flops (floating-point operations).

   B. Find conditions on U_1 and U_2 which guarantee the existence of a unique solution X.

   C. Give a non-trivial example ($U_1 \neq 0$, $U_2 \neq 0$, $X \neq 0$) where those conditions are not satisfied and

$$U_1 X + X U_2 = 0.$$